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edited by Jan Plate

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Quo Vadis, Metaphysics of Relations?

Introduction to the Special Issue 'The Metaphysics of Relational States'

Jan Plate

A many-faceted beast, the metaphysics of relations can be approached from many angles. One could begin with the various ways in which relational states are expressed in natural language. If a more historical treatment is wanted, one could begin with Plato, Aristotle, or Leibniz.[1] In the following, I will approach the topic by first drawing on Russell's *Principles of Mathematics* (1903) (still a natural-enough starting point), and then turn to a discussion mainly of *positionalism*. The closing section contains an overview of the six contributions to this Special Issue.

1. A Trilemma

Assuming that one goes in for talk of states of affairs (as I shall), the following may be considered a non-negotiable datum (cf., e.g., MacBride 2007, 27):

> D1. The state of affairs that Abelard loves Héloïse is identical with the state of affairs that Héloïse is loved by Abelard.

It also seems *prima facie* hard to deny that

1 Recent discussions of Plato's views on relations (in a liberal sense) may be found in Scaltsas (2013), Duncombe (2020, chaps. 2–4), and Marmodoro (2021, chap. 6). For Leibniz, see, e.g., Mugnai (2012). Aristotle's *Categories* form the principal starting point for medieval theorizing about relations, on which see, e.g., Martin (2016) and Brower (2018). Two other topics that I shall set aside in this introduction are the debate about realism vs. anti-realism about relations and the internal/external distinction. Introductory discussion of these latter topics can be found in Heil (2009, 2021) and MacBride (2020). For more extensive discussion of Russell's views on relations, see, e.g., Hochberg (1987), Lebens (2017), and MacBride (2018, chap. 8).

D2. 'Loves' expresses a relation distinct from the one expressed by 'is loved by.'

But this last statement might give rise to linguistic qualms; for, given that 'is loved by' is not even a complete phrase, it does not look like an appropriate target for the attribution of a semantic value. We can get around this by adopting the notational expedient of λ-expressions. Instead of 'loves' and 'is loved by,' we might speak of '$\lambda x, y\,(x$ loves $y)$' and '$\lambda x, y\,(x$ is loved by $y)$,' and lay down a semantics of λ-expressions under which $\ulcorner \lambda x, y\,(x\ \varphi\text{s}\ y)\urcorner$ denotes whatever dyadic relation is such that the instantiation of that relation by any entities x and y, in this order, is just the state of affairs that x φs y.[2] Under such a semantics, '$\lambda x, y\,(x$ loves $y)$' denotes the dyadic relation whose instantiation by any entities x and y (in this order) is the state of affairs that x loves y. Analogously for '$\lambda x, y\,(x$ is loved by $y)$,' which may also be said to denote the *converse* of $\lambda x, y\,(x$ loves $y)$.

Using λ-expressions as names for relations, (D2) becomes:

D2′. The relation $\lambda x, y\,(x$ loves $y)$ is distinct from $\lambda x, y\,(x$ is loved by $y)$.

And this *is* hard to deny. As the argument is both straightforward and tedious, I delegate it to a footnote.[3] (D2) closely reflects what Bertrand Russell implies

2 Here I am *provisionally* taking the locution 'is an instantiation of … by …, in this order' as primitive. I also take it to be understood that every instantiation is a state of affairs. The second ellipsis in 'is an instantiation of … by …, in this order' is supposed to be filled by a list of two or more arguments, and, relatedly, the 'and' in 'is an instantiation of … by x and y' should not be read as a term-forming operator but as a delimiter. (Cf. van Inwagen 2006, 461.) Worries about the semantic determinacy of this locution, of the sort raised by Williamson (1985), and concerns about its intelligibility, of the sort raised by van Inwagen (2006), will have to be addressed sooner or later; but for now I will adopt the working hypothesis that they can be answered *somehow*. (For recent discussion of Williamson's argument, see, e.g., Gaskin and Hill 2012, sec. V; and Trueman 2021, sec. 10.4.2.)

3 By the semantics of λ-expressions adumbrated in the previous paragraph, we have that

(1) The instantiation of $\lambda x, y\,(x$ loves $y)$ by Abelard and Héloïse, in this order, is the state of affairs that Abelard loves Héloïse,

whereas the instantiation of $\lambda x, y\,(x$ is loved by $y)$ by Abelard and Héloïse (again, in this order) is the state of affairs that Abelard is loved by Héloïse. Given that (as seems obvious) the state of affairs that Abelard loves Héloïse is distinct from the state of affairs that Abelard *is loved by* Héloïse, it follows that when he, in his *Principles of Mathematics* (1903), speaks of an "indubitable distinction between *greater* and *less*," adding that

> These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and [what they mean] are certainly relations. (1903, 228)

So far, no problem. (D1) and (D2′) can both be maintained without giving rise to any obvious contradiction. But a problem does arise once we adopt a further assumption, to the effect that

> U. For any two relations R_1 and R_2: any instantiation of R_1 fails to be an instantiation of R_2.

In other words, nothing is an instantiation of two relations. In Kit Fine's seminal "Neutral Relations" (2000), this assumption (formulated using somewhat different terminology) is referred to as 'Uniqueness.' And now—at least assuming that there exists an instantiation of $\lambda x, y$ (x loves y) by Abelard and Héloïse (in this order) as well as an instantiation of $\lambda x, y$ (x is loved by y) by Héloïse and Abelard—we have a problem. For, by the semantics of λ-expressions suggested above, the former instantiation is the state of affairs that Abelard loves Héloïse, just as the latter instantiation is the state of affairs that Héloïse is loved by Abelard. By (D1), these 'two' states of affairs are one and the same. So, by (D2′), we have here a single state of affairs that is an instantiation of two distinct relations. So we have a counter-example to (U). But, at least at first blush, (U) may seem an attractive thesis. For instance, the above-quoted passage from Russell's *Principles* continues as follows:

> Hence if we are to hold that "*a* is greater than *b*" and "*b* is less than *a*" are the same proposition, we shall have to maintain that both *greater* and *less* enter into each of these propositions, **which seems obviously false**; or else we shall have to hold that what really occurs is neither of the two [...]. (1903, 228, boldface emphasis added)

(2) The instantiation of $\lambda x, y$ (x is loved by y) by Abelard and Héloïse, in this order, is *not* the state of affairs that Abelard loves Héloïse.

From (1) and (2) we can conclude, by Leibniz's law, that $\lambda x, y$ (x loves y) is distinct from $\lambda x, y$ (x is loved by y).

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What seems to bother Russell here is (i) the thought that the relation *less* should "enter into" an instantiation of the distinct relation *greater* and (ii) the analogous thought that *greater* should enter into an instantiation of *less*. According to MacBride (2020, sec. 4), adherents of (U) may offer the following motivation (cf. also Fine 2000, 4):

> States are often conceived as complexes of things, properties and relations. They are, so to speak, metaphysical molecules built up from their constituents, so states built up from different things or properties or relations cannot be identical. Hence it cannot be the case that the holding of two distinct relations give rise to the same state. (MacBride 2020, sec. 4)

However, the picture of a relational state (i.e., of an instantiation of a relation) as a "metaphysical molecule," admitting only a *single* way in which such a state is put together from its constituents, can seem slightly naïve or at least under-motivated. A possible way to motivate it may be to hold, on the one hand, that, if one and the same relational state is an instantiation of two relations, then there needs to be some explanation of how this can be (cf. Fine 2000, 15; MacBride 2007, 55; 2014, 4; Ostertag 2019, 1482), and, on the other hand, that it is not easy to see what such an explanation might look like. But this argument will be persuasive only as long as no plausible candidate explanation has been produced. So it seems appropriate to take a skeptical attitude towards (U), as MacBride does at the end of his (2007). More recently, David Liebesman notes that *prima facie* "the motivation for Uniqueness looks suspect" (2014, 412) and that "the intuitions elicited by Fine fail to establish Uniqueness" (2014, 413).

Given that the case for (U) looks fairly weak, and given how blatantly this thesis conflicts with (D1) and (D2′), one may naturally expect that the literature on relations would have come down rather strongly against (U). However, this is not what we find.

In the *Principles*, Russell's way out of the conflict between (U) on the one hand and (D1) and (D2′) on the other was in effect to opt for the denial of (D1). Using Peirce's notation for the converse of a relation, he concluded that "R and $\breve{R}$ must be distinct, and 'aRb implies $b\breve{R}a$' must be a genuine inference"

(1903, 229).[4] This last remark suggests that the state of affairs that Abelard loves Héloïse would on Russell's view be distinct from the state of affairs that Héloïse is loved by Abelard. A decade later, however, we find him endorsing the existence of entities that, following Fine, have become known as *neutral relations*. The text in question is his manuscript on the *Theory of Knowledge* (1984), which is worth quoting from at some length:

> The subject of "sense" in relations is rendered difficult by the fact that the words or symbols by which we express a dual complex always have a time-order or a space-order, and that this order is an essential element in their meaning. When we point out, for example, that "x precedes y" is different from "y precedes x", we are making use of the order of x and y in the two complex symbols by which we symbolize our two complexes. [...] Nevertheless, we decided that there are not two different relations, one called *before* and the other called *after*, but only one relation, for which two words are required because it gives rise to two possible complexes with the same terms. (1984, 86)

A few paragraphs further down, the terms '*before*' and '*after*' are recycled for the purpose of naming two special relations that Russell refers to as *positions*:

> Let us suppose an a and a b given, and let us suppose it known that a is before b. Of the two possible complexes, one is realized in this case. Given another case of sequence, between x and y, how are we to know whether x and y have the same time-order as a and b, or the opposite time-order?
>
> To solve this problem, we require the notion of *position* in a complex with respect to the relating relation. With respect to time-sequence, for example, two terms which have the relation of sequence have recognizably two different positions, in the way that makes us call one of them *before* and the other *after*. Thus if, starting from a given sequence, we have recognized the two positions, we can recognize them again in another case of sequence, and say again that the term in one position is *before* while the term in the

4 Peirce introduced the '$\breve{R}$' notation in his "Algebra of Logic" (1880, 50). It has subsequently also been used by Schröder (1895), from whom Russell borrowed it in the *Principles* (1903, 25). That aRb is distinct from $b\breve{R}a$ has also been held by Hochberg (1999, 161; 2000, 47).

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> other position is *after*. That is, generalizing, if we are given any relation R, there are two relations, both functions of R, such that, if x and y are terms in a dual complex whose relating relation is R, x will have one of these relations to the complex, while y will have the other. The other complex with the same constituents reverses these relations. (1984, 87–88)

In this relatively brief passage, Russell introduces a member of what has become one of the most prominent families of views on the metaphysics of relations, namely *positionalism*. (The term is due to Fine, who coined it in his "Neutral Relations"; but I here use it in a slightly relaxed sense, on which a form of positionalism need not involve a commitment to what Fine calls 'neutral relations.') It has received more or less tacit endorsements by Segelberg (1947, 190), Armstrong (1978, 1997), Williamson (1985), Svenonius (1987, sec. 4), Barwise (1989, 180–181), Grossmann (1992, 57), Paul (2012, 251), Gilmore (2013), and Dixon (2018), among others. Where Russell speaks of 'positions,' these other authors speak in related senses of 'sides,' 'relation places,' 'gaps,' 'empty places,' 'argument places,' 'slots,' 'ends,' or 'pockets' of, or in, a relation.[5] Castañeda (1972, 1975, 1982) attributes a form of positionalism to both Plato and Leibniz.[6] More recently, Francesco Orilia (2008, 2011, 2014, 2019a, 2019b) has defended a form of positionalism under which positions, referred to as 'onto-thematic roles,' are widely shared among relations. These 'roles' are thought of as ontological counterparts of the *thematic roles* known from linguistics.

2. Positionalism

Most of the positionalists just cited conceive of relations as unordered or—using Fine's term—'neutral,' i.e., as not imposing any order on the positions with which the respective relations are associated. (The only clear exceptions seem to be Gilmore and Dixon.) Nor has the appeal of unordered relations been limited to positionalists. The so-called *antipositionalist* views defended by Fine (2000, 2007) and Leo (2008a, 2008b, 2010, 2013, 2014, 2016) also

5 Armstrong uses the term 'relation place' in his (1997, 121–122), but not in his (1978). In the latter work, he instead only speaks of the "roles" that particulars can play in a given "relational situation" (1978, 94). This use of 'role' is similar to that found in Sprigge (1970, 69–70).

6 For some discussion critical of Castañeda's interpretation of Plato, see Scaltsas (2013, 34–35).

conceive of relations as unordered, as does the 'primitivist' view proposed by MacBride (2014).[7]

Let us now look back at (D2′). What would a proponent of unordered relations make of that thesis?

According to Williamson (1985), any relation R is identical with its converse, so that we have the equation '$R = \breve{R}$.'[8] But, he says, in this equation 'R' functions as a singular term, whereas, in 'Rxy,' it instead functions as a *relational expression*, and this is supposed to block the inference from 'Rxy' to '$\breve{R}xy$' which one might otherwise have felt entitled to on the strength of '$R = \breve{R}$.' Crucially, while 'R' "stands for the relation R, this does not exhaust its semantic significance: it stands for R *with a particular convention as to which flanking name corresponds to which gap in R*" (italics in the original). He adds that " 'Ř' as a relational expression uses the opposite convention" (1985, 257). On a certain flat-footed way of applying this treatment to the case of $\lambda x, y$ (x loves y), one would say that this relation is in fact *identical* with its converse $\lambda x, y$ (x is loved by y) and that (D2′) is therefore false. But this would be to ignore the stipulatively specified semantics of λ-expressions on which that thesis was based (and with the help of which it was justified in footnote 3). What the Williamsonian positionalist should really say is that (D2′) is not false but *meaningless*, due to a crippling mistake in the underlying semantics of λ-expressions. For under that semantics, " '$\lambda x, y$ (x loves y)' denotes the dyadic relation whose instantiation by any entities x and y (in this order) is the state of affairs that x loves y." To the Williamsonian positionalist, this talk of instantiation can make no sense, because it can make no sense, by his lights, to speak of a *relation* as having an instantiation by some entities x and y in a given order. After all, the Williamsonian positionalist conceives of relations as unordered. Mention to someone a certain unordered relation R, together with some entities x and y and an ordering of x and y: the receiver of this information cannot possibly deduce *which* of the two positions of R (or 'gaps,' in Williamson's terminology) is supposed to be filled with x and which with y. Any information about an ordering of x and y is simply irrelevant. What is needed is not a function from some set of ordinals to x and y, but rather a function from *the set of R's positions* to x and y.[9]

7 Something like the primitivist view seems to have also been held by Armstrong (1993, 430–431) before he reverted to a form of positionalism in his later book (1997) with the same title.

8 For conformity of notation, I use italics where Williamson uses upright letters.

9 By similar reasoning, it can be seen that Williamson's own definition of 'converse' at the outset of his paper ("for x to have one [of a relation and its converse] to y is for y to have the other to

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We have now encountered one way in which the conflict between (D1), (D2′), and (U) might be resolved while holding onto (U): namely, to treat (D2′) as meaningless. Another option, which does *not* require the positing of unordered relations, would be to deny that relations have converses, so that, e.g., there only exists the relation $\lambda x, y\,(x \text{ loves } y)$ or the relation $\lambda x, y\,(x \text{ is loved by } y)$, but not both.[10] There is also a third way, which requires that 'relation' may be said in at least two ways. Thus it might be thought that, in one of its senses, the term 'relation' applies to unordered relations while, in another sense, it applies to what one might call 'ordered' or (using another phrase coined by Fine) 'biased' relations. One might then go on to suggest that this latter sense is operative in (D2′) and the former in (U). In this way the conflict between the three theses would be resolved through the power of equivocation, as it were, without having to abandon any of the three. But now there arises a question: How exactly should the believer in *un*ordered relations conceive of *ordered* relations? We might be content with thinking of unordered relations as unanalyzable metaphysical whatnots, but the question of how ordered relations come by their peculiar directedness still deserves an answer.

According to one such answer, suggested by Fine, the positionalist might

> think of each biased relation as the result of imposing an order on the argument-places [i.e. positions] of an unbiased relation. Thus, each biased relation may be identified with an ordered pair (R, O) consisting of an unbiased relation R and an ordering O of its argument-places. *Loves*, for example, might be identified with the ordered pair of the neutral amatory relation and the ordering of its argument-places in which *Lover* comes first and

x") must also be considered meaningless by the lights of the Williamsonian positionalist. For it cannot make any more sense to speak of an entity x as 'having' an unordered relation 'to' another entity y than to say that an unordered relation is instantiated by x and y 'in that order.' (It is worth noting that positionalistic tendencies are absent from Williamson's more recent metaphysical work.)

10 For recent discussion of such a view, see Bacon (2023). Bacon adopts a "broadly Fregean picture of properties and relations as unsaturated propositions," which may be thought of "as propositions with holes poked into some of the argument places" (2023, sec. 2). While these "unsaturated propositions" may *prima facie* seem to be properties and *un*ordered relations, Bacon holds that "there is a language independent ordering of the constituents a and b" in a given proposition Rab (2023, sec. 2). The assumption of such a language-independent ordering is also a component of Hochberg's theory of relational facts. For critical discussion of Hochberg's view, see MacBride (2012).

> *Beloved* second; and similarly for *is loved by*, though with the argument-places reversed. (2000, 11, original italics)

If we let $\mathcal{A}$ be the "neutral amatory relation" and understand an "ordering of its argument-places in which *Lover* comes first and *Beloved* second" to be the ordered pair (*Lover*, *Beloved*), then this amounts to the suggestion that the ordered relation *loves* is the ordered pair ($\mathcal{A}$, (*Lover*, *Beloved*)) while its converse is the ordered pair ($\mathcal{A}$, (*Beloved*, *Lover*)). On a common construal of ordered *triples*, one might also put this by saying that *loves* is the ordered triple ($\mathcal{A}$, *Lover*, *Beloved*) while its converse is the ordered triple ($\mathcal{A}$, *Beloved*, *Lover*).

On this proposal, then, ordered relations are certain set-theoretic constructions. Such a proposal is apt to provoke resistance in anyone who is used to conceiving of ordered relations as the objectively determined semantic values of such verbs as 'loves' or 'stabs,' which these latter verbs stand for "without need of philosophical stipulation" (Williamson 1985, 254). It is also apt to provoke resistance in anyone who conceives of relations as "*fundamental* entities, not mere projections onto the world of idiosyncratic facts about human language" [Dorr (2004), 187; emphasis in the original]. However, the thesis that transitive verbs have determinate semantic values, outside of any more or less arbitrary assignment scheme, is a strong assumption that it is not *a priori* easy to see how to defend. And the idea that relations, whatever they are, can only be "fundamental" entities looks far from incontrovertible in light of the fact that it was once not unusual to conceive of relations as mere *entia rationis* (see, e.g., Brower 2018, sec. 5.2).

Once we have reached a point at which we are prepared to take seriously the identification of *loves* with ($\mathcal{A}$, *Lover*, *Beloved*), it becomes natural to ask whether we might not, in the interest of both ontological and ideological parsimony, get rid of unordered relations altogether and take ordered n-adic relations to be simply ordered n-tuples of *positions*. On this view, *loves* would be the ordered pair (*Lover*, *Beloved*) and its converse would be (*Beloved*, *Lover*). In the case of certain symmetric relations, one might even make do with a single position. Thus the dyadic relation of adjacency might be construed as the ordered pair (*Next*, *Next*).[11] A great advantage of this construction lies in the fact that it immediately reveals this relation to be identical with its converse and thereby offers a satisfying explanation of *why* adjacency is symmetric.

11 Some positionalistically-minded theorists, such as Yi (1999), would regard adjacency not as a relation at all but as a property that has 'plural' bearers. However, cf. Pruss and Rasmussen (2015).

However, presumably not *every* ordered pair of positions should count as a relation; and it might be argued that here is where unordered relations earn their keep. For instance, it might be thought that the pair (*Lover*, *Giver*) should not count as an ordered relation because there are no states of affairs in which both *Lover* and *Giver* are occupied; and the non-existence of such states may in turn be thought to be due to the putative fact that *Lover* and *Giver* do not belong to the same unordered relation.[12] Thus, more generally, unordered relations may be thought of as organizing positions into groups such that only members of the same group can have occupants in the same states of affairs. But again one might wonder why the work that is thus ascribed to unordered relations cannot be done more cheaply. After all, together with the category of unordered relations, we would need to have in our conceptual inventory a non-symmetric relational notion of 'belonging' that applies to unordered relations and their respective positions. Yet if unordered relations merely serve to 'collect together' certain sets of positions, then why not adopt instead a symmetric notion of *connectedness* that holds directly between positions? Rather than to say that *Lover* and *Beloved* are the only two positions that 'belong' to a certain unordered relation, we might then, for example, say that *Lover* and *Beloved* form a maximal clique of connected positions. Some other options will be mentioned in section 4.

3. The Instantiation Problem

Whether one keeps unordered relations in the picture or not, the task of working out the details of a positionalist theory of relations is not trivial. Above all, the positionalist will have to specify what exactly is required for a given ordered relation to be instantiated by some entities $x_1, \ldots, x_n$, in this order. While it may *in principle* be open to the positionalist to leave the concept of *being instantiated by … (in this order)* unanalyzed, this would be profoundly unsatisfactory. After all, on the positionalist view, at least of the sort now under discussion, ordered relations are fairly artificial set-theoretic constructs,

12 In an Orilia-style positionalism, unordered relations also perform a vital additional role in the individuation of relational states. For example, since the relations of *loving* and *admiring* are in Orilia's metaphysic both associated with the roles of *Agent* and *Patient*, there would in his system be no way to distinguish Antony's *loving* Cleopatra from Antony's *admiring* Cleopatra if there did not exist an unordered amatory relation that in some sense 'enters into' the first state but not into the second or an unordered *admiratory* relation that enters into the second state but not the first.

and one would not expect that any metaphysically fundamental notion, other than the 'formal' notions of set-membership and identity (and perhaps mereological notions, if one follows Lewis (1991) in thinking of sets as fusions of singletons), would apply directly to ordered relations, any more than one would expect a set to have mass or charge other than in a derivative sense.[13] Consequently the notion of instantiation, given that it *does* apply directly to ordered relations, would not plausibly be thought of as metaphysically fundamental. What we would like to have, then, is an account of what it takes for a given ordered relation to be instantiated by such-and-such entities in a given order.[14]

Can this *instantiation problem*, to give it a name, be avoided by abjuring (with Williamson, for example) all talk of ordered relations and acknowledging only *un*ordered ones? Strictly speaking, yes. But the believer in unordered relations will then still be faced with the problem—which I shall call the *contribution problem*—of explaining what metaphysical work those unordered relations are supposed to do; and since their only reasonably clear hope for employment lies in contributing to the truth-conditions of relational predications, our theorist will thus be confronted with the task of specifying just what that contribution consists in. For example, someone who posits a 'neutral amatory relation' will need to tell some story, in the terms of her favored metaphysic, of what it takes for it to be the case that Abelard loves Héloïse; and that amatory relation had better play a prominent part in that story. (Or

13 McDaniel (2004, 145) makes a similar point.

14 An argument for the view that the notion of *being instantiated by … (in this order)*—call it '$\mathcal{I}$'—fails to be metaphysically fundamental can also be found in Dorr (2004, sec. 3–4). An important intermediate result that Dorr seeks to establish in the course of his argument is the claim that, if $\mathcal{I}$ *were* fundamental, then the following thesis would be neither metaphysically necessary nor knowable with *a priori* certainty:

> C. For any dyadic relation R_1 there exists a relation R_2 such that, for any x and y: R_1 is instantiated by x and y (in this order) iff R_2 is instantiated by y and x (in this order).

(I have adapted Dorr's thesis to the terminology of the present essay. For the original version, see Dorr 2004, 161.) Dorr thinks that we have good *a priori* reason to think that (C) expresses a metaphysical necessity: if we took it to be possibly false, we would have to expect there to be "spurious structural distinctions between possible worlds" (2004, 167). Hence, in light of the aforementioned intermediate result, we have (according to Dorr) good *a priori* reason to think that $\mathcal{I}$ is not metaphysically fundamental.

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at least, so one may argue.[15]) Moreover, since for it to be the case that Abelard loves Héloïse is patently not the same as for it to be the case that Héloïse loves Abelard, the unordered-relations theorist will need to be able to tell a *different* story of what it takes for it to be the case that Héloïse loves Abelard, or at the very least allow that the relational state of Abelard's loving Héloïse is distinct from that of Héloïse's loving Abelard.

Arguably, however, mere numerical distinctness is not quite sufficient. Consider a 'minimalist' view that takes any two states Rab and Rba (for distinct a and b) to be merely numerically distinct 'completions' of some unordered relation R: "two indiscernible 'atoms' within the space of states," in Fine's memorable phrase. If such a view were correct, it would be more perspicuous to write '$(R\{a,b\})_1$' and '$(R\{a,b\})_2$' instead of 'Rab' and 'Rba,' using the subscripts '1' and '2' as nothing more than arbitrary tags. With the help of this amended notation, the minimalist view can be seen to suffer from the following difficulty: Suppose we have three particulars a, b, and c, giving rise to six possible instantiations of R, namely $(R\{a,b\})_1$, $(R\{a,b\})_2$, $(R\{b,c\})_1$, $(R\{b,c\})_2$, $(R\{a,c\})_1$, and $(R\{a,c\})_2$. Suppose further that, of these six states, only the following three obtain: $(R\{a,b\})_1$, $(R\{b,c\})_1$, and $(R\{a,c\})_2$. Question: Is R transitive on the set $\{a,b,c\}$? There appears to be no fact of the matter, or maybe one should say that the question is ill-posed. In either case, the minimalist has no ready way of capturing the distinction between transitive and non-transitive relations.[16]

How might the Finean antipositionalist address the contribution problem? A crucial feature of antipositionalism, as developed towards the end of "Neutral Relations," is that it conceives of the 'completions' of neutral relations as interrelated by substitution, where the relevant notion of substitution is taken as primitive. Positions and ordered relations do not enter the picture at

15 Put quite simply: If the amatory relation were to play no part in the metaphysics of Abelard's loving Héloïse (or Antony's loving Cleopatra, say), it would *prima facie* be hard to see what point there could be in positing such a relation in the first place.

16 At first blush the view that has here been called 'minimalism' might be thought to be similar to the one recommended at the end of MacBride (2014), which is to the effect that "we should just take the difference between aRb and bRa as primitive" (2014, 14). However, this identification would be a mistake, for MacBride holds that the difference between aRb and bRa is not mere numerical distinctness but a difference "which arises from how the constituents of these states are arranged, where how they are arranged is a primitive matter" (2014, 14), and he also explicitly allows that "[*s*]*ometimes* it may be helpful to appeal to the notion of an *agent* or *patient* to elucidate the distinction between (for instance) *loves* applying one way rather than another" (2014, 15). (Thanks to Fraser MacBride for alerting me to this point and for valuable additional discussion.)

the ground level (as it were) but are rather conceived of as abstractions and set-theoretic constructions. While the antipositionalist is able—unlike the minimalist—to distinguish between transitive and non-transitive relations, she is *un*able to characterize the difference between, say, Abelard's loving Héloïse and Héloïse's loving Abelard without appeal to a reference state, such as as that of Antony's loving Cleopatra (cf. Fine 2000, 29–30). As a result, the antipositionalist is unable to say what it takes for it to be the case that Abelard loves Héloïse *independently* of who else loves whom. This need not by itself constitute a problem. The antipositionalist might maintain that in fact there is nothing interesting to be said in response to the question of what it takes for Abelard to love Héloïse: she might regard Abelard's loving Héloïse as a "basic relational fact (at least in the relevant respect)," as Fine (2007, 62) puts it. However, this view still leaves us in a curious position: plausibly there exist precisely two completions (or *possible* completions) of the neutral amatory relation in which Abelard and Héloïse function as relata. But antipositionalism offers no explanation as to *why* there should be exactly two such completions, rather than only one (as in the case of the adjacency relation), or three, or a hundred. Under antipositionalism, the fact that, for any given pair of distinct entities, there are exactly two completions of the amatory relation with those two entities as relata appears to be effectively treated as brute.[17]

While there is certainly more to be said about antipositionalism, I will have to leave the matter here.

4. Positionalism Developed

Let us now return to the positionalist's instantiation problem, which (as may be recalled) was to provide "an account of what it takes for a given ordered relation to be instantiated by such-and-such entities in a given order." This problem is inseparable from the question of how facts concerning positions—and, where applicable, unordered relations—determine what ordered relations there are. In addition, it is inextricably linked to the positionalist's selection of basic notions and to the question of what role positions play in the individ-

17 Gaskin and Hill (2012) make essentially the same point with regard to the adjacency relation. They also claim, however, that *positionalism* has to "concede that whether a relation is symmetric or not is a brute fact" (2012, 185). This seems to me mistaken; cf. the previous section's example of (*Next*, *Next*). Additional discussion of *anti*positionalism may be found in §IV of Gaskin and Hill's paper, as well as in MacBride (2007, 44–53; 2014, 14). For responses to MacBride, see Fine (2007) and Leo (2014, sec. 6).

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uation of relational states (where a relational state is just an instantiation of a relation). The menu of available options is marked by at least five noteworthy choice points.

Choice point #1: The occupation predicate. Arguably the central notion in the positionalist's ideology is that of *occupation*, which in its simplest form applies to an entity, a position, and a relational state. While more complicated notions of occupation are conceivable, in the following we will only be discussing forms of positionalism that operate with this simple triadic concept, expressed by the predicate 'occupies … in …'.

Choice point #2: Unordered relations. As already noted, positionalists have traditionally assumed that there are such things as unordered or 'neutral' relations with which positions are in some sense associated. However, at least in those forms of positionalism that (unlike the view put forward by Orilia) do not allow for positions to be shared among relations, the only theoretically significant work performed by unordered relations seems to lie in organizing positions into different 'groups,' where the theoretical role of these groups in turn lies in determining what relational states there are. Thus it might be said that it is because *Lover* does not 'belong' to the same unordered relation as *Giver* that there does not exist a state in which Antony occupies *Lover* and Cleopatra occupies *Giver*. To the positionalist who rejects unordered relations, by contrast, it is open to dispense with the concept of an unordered relation as well as with that of 'belonging,' and to work instead with a concept of *connectedness* that applies directly to positions (cf. section 2 above). She will then be able to say that it is simply because *Lover* is not *connected* to *Giver* that there does not exist a state in which Antony occupies *Lover* and Cleopatra occupies *Giver*.[18]

In following this route, the positionalist can further choose among several options. For example, she might assume that connectedness is transitive. But likewise she might hold that it isn't, and allow that there are positions p, q, and r such that p is connected to q and q to r, but p is not connected to r, and that, correspondingly, there exist relational states in which both p and q are

18 An important question that arises at this point is how best to understand this 'because.' (Is there some form of 'metaphysical necessity' afoot? Are we dealing with a case of 'metaphysical grounding'?) According to Dorr (2004, sec. 7), the positionalist is in this connection committed to 'brute necessities,' which Dorr regards as a serious liability of the view. It is not clear, however, that the positionalist is under any pressure to posit 'necessities' rather than merely general truths—such as a principle to the effect that no two (fundamental) positions are occupied in the same state of affairs unless they are connected.

occupied, and also states in which both q and r are occupied, but *no* states in which both p and r are occupied. Another possibility would be to hold that what matters for the question of whether there exists a state in which two given positions p and q are occupied is not whether p and q are *directly* connected but rather whether they are directly *or indirectly* connected, i.e., whether there exist any positions $p_1, \ldots, p_n$ such that (i) $p = p_1$, (ii) $q = p_n$, and (iii) for each i with $1 \leq i < n$, p_i is connected to p_{i+1}. Or again, she might hold that what matters is whether p and q are both members of the same maximal clique of connected positions.

Another interesting option would be to understand *being connected* as a *multigrade* notion, i.e., as a relational concept that can apply to different numbers of arguments. Equipped with such a concept, the positionalist might propose that the question of whether there exists a relational state in which some given positions $p_1, p_2, \ldots$, and no others, are occupied depends on whether $p_1, p_2, \ldots$ are connected, where this is *not* analyzable in terms of whether any two of them are connected.

Choice point #3: Non-obtaining states. The third choice point we have to consider concerns the question of whether to allow for non-obtaining relational states. Let us use the term *state-positivism* for the view that every state of affairs obtains (or in other words: for the view that every state of affairs is a *fact*).[19] According to the state-positivist, there is no distinction to be drawn between obtainment and existence: Abelard loves Héloïse if and only if the state of Abelard's loving Héloïse exists. The state-*anti*positivist, by contrast, will allow that this latter state exists even if Abelard does not love Héloïse.

Choice point #4: Multiply occupiable positions. To see how the positionalist might address the instantiation problem, let us focus on that form of positionalism that (i) employs a simple triadic notion of occupation, (ii) dispenses with unordered relations in favor of a multigrade notion of connectedness, and (iii) rejects state-positivism. On such a view, the question of how facts about positions determine what relations there are may be answered as follows:

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19 A corollary of this view is that no state of affairs is a *negation* of another, since in that case both the former and the latter (of which the former is a negation) would have to obtain, which would be absurd. So it might be said that, on this view, every state of affairs is 'positive,' which provides the motivation for the second part of the proposed label (viz., 'positivism'). A concise statement of state-*anti*positivism—i.e., the denial of state-positivism—may be found in Pollock (1967, sec. 2).

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R. An entity x is an (ordered) *relation* iff there exist some positions $p_1, \ldots, p_n$ (for some $n > 1$) such that (i) $p_1, \ldots, p_n$ are connected and (ii) $x = (p_1, \ldots, p_n)$.[20]

It may further be natural to adopt the following uniqueness claim for relational states:

US. For any $n > 1$, any positions $p_1, \ldots, p_n$, and any entities $x_1, \ldots, x_n$: if $p_1, \ldots, p_n$ are connected, then there exists at most one state of affairs s that is such that, for each i with $1 \leq i \leq n$: x_i occupies p_i in s.[21]

However, if the positionalist wishes to allow for positions to be *multiply occupiable*, a weaker claim is needed:

US′. For any $n > 1$, any positions $p_1, \ldots, p_n$, and any entities $x_1, \ldots, x_n$: if $p_1, \ldots, p_n$ are connected, then there exists at most one state of affairs s that is such that, for each i with $1 \leq i \leq n$ and any x: x occupies p_i in s iff $x = x_j$ for some j with $1 \leq j \leq n$ and $p_j = p_i$.

Finally, the instantiation problem may be addressed in two steps. In the first and main step, the positionalist may adopt a thesis that characterizes instantiations of ordered relations:

I1. For any $n, m > 1$, any positions $p_1, \ldots, p_n$, any entities $x_1, \ldots, x_m$, and any y: y is an *instantiation* of $(p_1, \ldots, p_n)$ by $x_1, \ldots, x_m$, in this order, iff (i) $m = n$, (ii) $p_1, \ldots, p_n$ are connected, and (iii) y is a state of affairs such that, for each i with $1 \leq i \leq n$ and any x: x occupies p_i in y iff $x = x_j$ for some j with $1 \leq j \leq n$ and $p_i = p_j$.

Note that, together with (R) and (US′), it follows from this that any ordered relation has only at most one instantiation by a given sequence of entities. One

20 For simplicity's sake, I will be ignoring the question of how to accommodate infinitary relations.

21 To see the need for the antecedent ("$p_1, \ldots, p_n$ are connected"), suppose that there are three positions *Giver*, *Gift*, and *Recipient*, and suppose moreover that these three are connected (in that irreducibly multigrade sense) while *Giver* and *Recipient* are not connected. Thanks to the antecedent, (US) does then *not* have the consequence that, for any entities x_1 and x_2, there exists at most one state of affairs s that is such that x_1 and x_2 respectively occupy in s the positions of *Giver* and *Recipient*.

can now specify what it takes for a given ordered relation to be *instantiated* by some such sequence:

> I2. For any $n > 1$, any ordered relation R, and any entities $x_1, \dots, x_n$: R is *instantiated* by $x_1, \dots, x_n$, in this order, iff there exists an obtaining instantiation of R by $x_1, \dots, x_n$, in this order.

This solves the instantiation problem for the form of positionalism that we have here been considering.

Choice point #5: The place of relations in the world. So far it has been left largely implicit what thesis positionalism amounts to: just what it is that positionalists want us to believe about the world. To remedy this situation, one could employ the concept of a *relational phenomenon*. For present purposes, a relational phenomenon may be understood to be simply any state of affairs that can be felicitously expressed with the help of 'relational' vocabulary—notably, transitive verbs and prepositions, as in 'the cat is on the mat' or 'Abelard loves Héloïse.' Unlike the concept of a relational *state* (i.e., of an instantiation of a relation), the concept of a relational phenomenon is not directly tied to that of a relation. Once we settle on a specific conception of relations, and also clarify the notion of an *instantiation* of a relation, we will have specified what a relational state is; but we will not thereby have specified how relational states relate to relational *phenomena*. Among the options that the positionalist is presented with in this regard, we can usefully identify two extremes, which might be called the *strong* and the *weak* thesis, respectively:

> ST. Every relational phenomenon is a relational state.

> WT. At least one relational phenomenon is 'partially grounded' in a relational state (or the negation of such a state).[22]

22 For present purposes, we may understand a state of affairs s_1 to be *partially grounded in* a state of affairs s_2 iff s_1 obtains and s_2 is a member of the smallest class C that satisfies the following four conditions:

(i) $s_1 \in C$.
(ii) For any $s \in C$ and any state of affairs s': if s is a conjunction of two or more states of affairs, and s' is one of the conjuncts of s, then $s' \in C$.
(iii) For any $s \in C$ and any state of affairs s': if s is a disjunction of two or more states of affairs, and s' is one of the *obtaining* disjuncts of s, then $s' \in C$.
(iv) For any $s \in C$ and any state of affairs s': if s is an existential quantification and s' one of its obtaining instances, then $s' \in C$.

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Of course, neither (ST) nor (WT) by itself amounts to a form of positionalism. However, we obtain a form of positionalism if we combine either (ST) or (WT) with a positionalistic conception of relations and relational states; and one such conception is given by (R) and (I1) above. A form of positionalism that entails (ST) may be called 'strong positionalism,' while a theory that entails only (WT) may be called 'weak positionalism.' Unlike the strong positionalist, the weak positionalist may well deny that the sentence 'Abelard loves Héloïse' expresses a relational state (although she will presumably agree that it expresses a relational *phenomenon*) and, correspondingly, that there exists such a thing as the relation $\lambda x\,(x \text{ loves } y)$. For the sake of the example, however, I will in the following continue to assume that there is such a relation.

On the background of the above solution to the instantiation problem, let us now return one last time to the conflict observed in section 1 between (D1), (D2′), and (U). To recapitulate, (D2′) states that the (ordered) relation $\lambda x, y\,(x \text{ loves } y)$ is distinct from $\lambda x, y\,(x \text{ is loved by } y)$. The positionalist who wishes to analyze relational states like that of Abelard's loving Héloïse in terms of the occupation of two positions *Lover* and *Beloved* will, if she also accepts (R), identify the relations $\lambda x, y\,(x \text{ loves } y)$ and $\lambda x, y\,(x \text{ is loved by } y)$ with, respectively, the ordered pairs (*Lover*, *Beloved*) and (*Beloved*, *Lover*). That these are distinct follows straightforwardly from the assumed distinctness of *Lover* and *Beloved*. So (D2′) holds true. By contrast, (U)—the thesis that nothing is an instantiation of two relations—looks now more questionable than ever. For if one thinks of an ordered relation as an ordered tuple of positions, one will hardly be inclined to think of its instantiations as 'metaphysical molecules' in which it figures as a constituent. But then it becomes difficult to see the intuitive appeal of (U). With (U) accordingly given up, nothing prevents us from accepting (D1), i.e., the thesis that Abelard's loving Héloïse is the same state as that of Héloïse's being loved by Abelard. And indeed, if one identifies $\lambda x, y\,(x \text{ loves } y)$ with (*Lover*, *Beloved*) and $\lambda x, y\,(x \text{ is loved by } y)$ with (*Beloved*, *Lover*), then (D1) can be seen to follow from (US′) and (I1).[23]

Clauses (ii)–(iv) correspond to commonly accepted 'introduction' rules for grounding claims. (Cf. Fine 2012, 58–59) The concept of partial ground thus defined differs from more traditional ones (like Fine's notion of 'strict partial' ground) by the fact that it does *not* require a state of affairs to be distinct from its grounds. This constitutes a simplification that seems, at least for present purposes, to be harmless.

23 In particular, by the semantics of λ-expressions hinted at in section 1, the instantiation of $\lambda x, y\,(x \text{ loves } y)$ by Abelard and Héloïse, in this order, is the state of affairs that Abelard loves Héloïse. Given the identification of $\lambda x, y\,(x \text{ loves } y)$ with (*Lover*, *Beloved*), this same state is, by (US′) and (I1), the unique state in which *Lover* and *Beloved* are only occupied by

5. Potential Objections

Still, it is not all smooth sailing for the positionalist. A first worry is akin to 'Bradley's regress.' As we have seen, the positionalist (at least of the sort considered in this essay) characterizes relational states in terms of what positions are occupied in them by what entities. If now *s* is the state of Abelard's loving Héloïse, shouldn't there also be a further state of affairs to the effect that, in *s*, the position *Lover* is occupied by Abelard—as well as a state of affairs to the effect that the position *Beloved* is in *s* occupied by Héloïse? If the positionalist is to apply her approach to these further states, she has to introduce three additional positions, of *State*, *Occupant*, and *Position*.[24] With their help the state of Abelard's occupying *Lover* in *s*—call it *s′*—can be characterized as a state in which *s* occupies the position of *State*, *Lover* occupies *Position*, and Abelard occupies *Occupant*. (See figure 1.) But now we seem to have three further states on our hands, one of which may be characterized by saying that *s′* occupies in it the position of *State*, *s* the position of *Occupant*, and *State* the position of *Position*. And so the regress takes its course.[25] It is not obvious, however, that this regress is vicious. For it is not as if the state of Abelard's loving Héloïse is in any sense *grounded in* (or 'explained by') the fact that Abelard occupies in it the role of *Lover*; rather, the former state is merely (in some suitable sense) "characterized" by the latter. We thus have a "regress of characterization," not of grounding or explanation.

To be sure, the positionalist should presumably allow that

(1) There exists an obtaining state of affairs in which Abelard, and nothing else, occupies *Lover* and in which Héloïse, and nothing else, occupies *Beloved*

is in a certain sense a more perspicuous representation of Abelard's loving Héloïse than the simpler and more familiar 'Abelard loves Héloïse': because

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Abelard and Héloïse, respectively. And by parallel reasoning, this state is also the instantiation of $\lambda x, y\,(x \text{ is loved by } y)$ by Héloïse and Abelard, in this order, and is hence the state of affairs that Héloïse is loved by Abelard.

24 In the following, I will assume that the positionalist has to introduce these positions as primitive posits. An alternative approach (which I will not explore here) might be to construe them as 'abstractions' of some sort, in a sense more or less analogous to lambda-abstraction.

25 Cf. MacBride (2005, 585–586; 2012, 99; 2014, 12). A similar regress has been discussed by Russell (1984, 111–112). Orilia (2014, sec. 9) offers a reply to MacBride in the terms of Orilia's own brand of positionalism. For an introduction to Bradley's regress, see Perovic (2017). Also cf., e.g., Eklund (2019) and Heil (2021, sec. 6).

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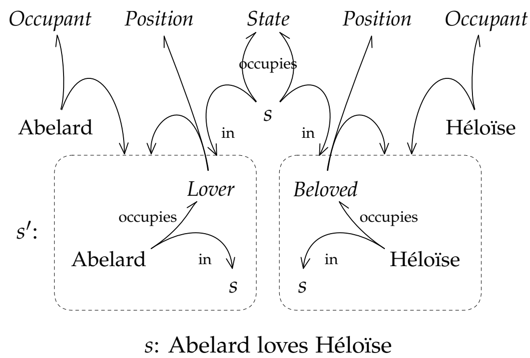

Figure 1: Various states related to Abelard's loving Héloïse. (See text for details.)

(1), but not 'Abelard loves Héloïse,' lets us know about the existence of the two positions of *Lover* and *Beloved*. By the same token, a positionalist who posits the aforementioned positions of *State*, *Occupant*, and *Position* should presumably allow that

(2) There exist three obtaining states of affairs s, s', and s'' such that: (i) s' is the only obtaining state in which s occupies *State* and *Lover* occupies *Position*; (ii) in s', nothing other than s occupies *State*, nothing other than *Lover* occupies *Position*, and only Abelard occupies *Occupant*; (iii) s'' is the only obtaining state in which s occupies *State* and *Beloved* occupies *Position*; and (iv) in s'', nothing other than s occupies *State*, nothing other than *Beloved* occupies *Position*, and only Héloïse occupies *Occupant*

is more perspicuous than (1); but this is *only* because from (2)—and not from (1)—we can infer the existence of those three positions. Hence it is *not* the case that the positionalist has now embarked on some infinite 'regress of perspicuity.' Nor has she embarked on an infinite regress of *analysis*, in the form of some incompletable attempt at providing a metaphysical analysis of the 'occupies … in …' locution. To think that she has would be to presuppose that (2) is put forward as an attempt at such an analysis; but this would be highly uncharitable, given that (2) itself is rife with instances of that locution. The positionalist, at least of the stripe considered here, is 'stuck' with that locution in the same way in which a more traditional proponent of universals is stuck with 'instantiates' or 'is an instantiation of ... by ....' But this in itself is not an objection.

So much for potential worries about a vicious regress. In his "Neutral Relations," Fine has raised a number of additional concerns about positionalism. According to one of his objections, positionalism is guilty of "ontological excesses" (2000, 16–17). This objection, however, appears to rest largely on the claim that "surely we would not [...] wish to be committed to the existence of argument-places [a.k.a. positions] as the intermediaries through which the exemplification of the relations was effected" (2000, 16–17).

Fine has also maintained that positionalism is unable to accommodate strictly symmetric or multigrade ('variably polyadic') unordered relations (2000, 17, 22), where " [a]n unbiased binary relation *R* is said to be *strictly symmetric* if its completion by the objects *a* and *b* is always the same regardless of the argument-places to which they are assigned" (2000, 17). This claim relies on a special feature of the particular form of positionalism discussed by Fine, namely that no position is ever occupied by more than one entity in the same state. There seems to be nothing incoherent, however, in embracing an alternative form of positionalism that *does* allow for multiple occupancy.[26]

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26 Cf. (US′) in the previous section. For an explicit defense of a view that admits multiply occupiable positions, see Orilia (2011) or Dixon (2018). The view that Donnelly (2016) refers to as 'Naïve Positionalism' is also of this kind. The possibility of allowing positions to be multiply occupiable has first (to my knowledge) been considered by Fine (2000, fn10). His celebrated objection to this approach will be discussed in the next section.

It is further worth noting that, by allowing for multiply occupiable positions, the positionalist is (at least in principle) able to address a problem that has been raised by Joop Leo (2008a, 2008b, 2010) for a certain way of "modelling relations." Leo considers a relation $\mathfrak{R}$ "in which $\mathfrak{R}abc$ represents the state that *a* loves *b* and *b* loves *c*" (2008a, 374). In present terminology, this may be understood as referring to a triadic relation *R* whose instantiation by any entities *x*, *y*, and *z* (in this order) is the conjunction of *x*'s loving *y* and *y*'s loving *z*. At first blush, a positionalistic treatment of this relation requires three positions p_1, p_2, p_3 such that an instantiation of *R* by any entities x, y, z is the unique state in which p_1 is occupied only by *x*, p_2 is occupied only by *y*, and p_3 is occupied only by *z*. However, as a consequence of this treatment, for any entities *a* and *b*, the state *Raba* is distinct from *Rbab*. This is arguably implausible, for, on an intuitively reasonable, at least moderately coarse-grained conception of relational states, 'both' *Raba* and *Rbab* are just the state of affairs that *a* and *b love each other*. Multiply occupiable positions may be thought to solve this problem. In particular, positing only two positions p_1 and p_2, the positionalist can say that the instantiation of *R* by any three entities x, y, z is the unique state in

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Admittedly, a positionalist who, contrary to the form of positionalism discussed by Fine, *does not admit any unordered relations* will *a fortiori* not be able to accommodate unordered relations that are strictly symmetric or multigrade. However, the idea that there are strictly symmetric or multigrade unordered relations is less of a datum than a metaphysical hypothesis. A theorist might be drawn to the idea that there are *strictly symmetric* unordered relations because it helps to accommodate certain intuitive identities between relational phenomena, such as the identity of a's being next to b with b's being next to a. And a theorist might be drawn to the idea that there are *multigrade* unordered relations because it helps to accommodate certain analogies between relational phenomena, such as the analogy between, on the one hand, the state of affairs that a and b jointly support c and, on the other hand, the state of affairs that a, b, and c jointly support d. But neither of these considerations constitutes a compelling argument for invoking unordered relations. The first intuition—that a's being next to b is the same state of affairs as b's being next to a—can be accommodated by adopting a form of positionalism under which a's being next to b and b's being next to a are 'both' characterized as a state in which a certain position *Next* is occupied by both a and b. And the intuitive analogy between the state of affairs that a and b jointly support c and the state of affairs that a, b, and c jointly support d can be accommodated by positing two connected positions, *Supporter* and *Supportee*, of which at least the first is multiply occupiable (cf. Marmodoro 2021, 173).

which p_1 is occupied only by x and y and in which p_2 is occupied only by y and z. As a result, the state $Raba$ turns out to be the unique state in which both p_1 and p_2 are occupied only by a and b; and exactly the same description is given of $Rbab$. In this way $Raba$ and $Rbab$ come out identical, as desired.

Whether this proposal is ultimately satisfactory is, however, another matter. First of all (though this is not an objection), it is worth noting that the proposal does not sit well with the conception of relations as tuples of positions; instead it appears to favor a conception under which relations are tuples of *sets* of positions. (Thus R might under this proposal be conceived of as the ordered triple $(\{p_1\}, \{p_1, p_2\}, \{p_2\})$, with the previous section's thesis (I1) modified accordingly.) It might also be asked how the proposal can be generalized to higher-adic analogues of Leo's relation. (Thanks to Joop Leo for pressing this point.) For example, let S be the tetradic relation whose instantiation by any entities x, y, z, and w, in this order, is the conjunction of x's loving y, y's loving z, and z's loving w. The positionalist might then postulate two positions q_1 and q_2 such that an instantiation of S by any entities x, y, z, w is a state in which q_1 is occupied only by x, y, and z, while q_2 is occupied only by y, z, and w. On this approach, the state $Sabca$ would be given exactly the same characterization as the distinct state $Sacba$, but this need not be seen as a fatal problem. A more pressing concern would be the question of how to formulate a general principle that would lead to the particular positionalistic treatment of the relations in question.

6. Symmetries

Nonetheless, at least under a sufficiently 'abundant' view as to what (ordered) relations there are, some of them—in particular ones that exhibit a 'cyclical' symmetry—do not easily lend themselves to the positionalist approach.[27] To elaborate this point, we first have to go over some technical preliminaries.

Let us say that a function f is a *symmetry* of an n-adic ordered relation R iff f is a permutation of the set $\{1, \dots, n\}$ such that, for any sequence of entities $x_1, \dots, x_n$ and any y: y is an instantiation of R by $x_1, \dots, x_n$, in this order, iff y is an instantiation of R by $x_{f(1)}, \dots, x_{f(n)}$, in this order.[28] It is easy to verify that, for any n-adic unigrade ordered relation R, the symmetries of R form a *group* with respect to function composition. That is to say, where S_R is the set of R's symmetries, the following three conditions are satisfied:

(i) For any permutations $f, g \in S_R$, S_R also contains the permutation $g \circ f$ that applies g to the result of applying f.
(ii) S_R contains the function id_n that maps each member of $\{1, \dots, n\}$ to itself (and which therefore acts as an identity element within S_R).
(iii) For any permutation $f \in S_R$, S_R also contains the unique permutation g that is such that $f \circ g = g \circ f = id_n$ (i.e., the inverse of f).

This set S_R is also called the *symmetry group* of R.[29] Further, for any group G of functions defined on a common set, let us say that the latter is the *domain* of G. For example, if a given group consists of permutations of the set $\{1, \dots, n\}$ (for some $n > 0$), then this set is the domain of that group.

Consider now an n-adic ordered relation R (for some $n > 2$) whose symmetry group satisfies the following condition:

> C. It contains a permutation f such that, for some k in its domain: (i) $k \neq f(k)$, and (ii) it contains no permutation that merely transposes k and $f(k)$ and maps all other members of the domain to themselves.

27 The previous footnote describes a related difficulty.

28 An adherent of the view that has above been called 'state-positivism' (which rejects non-obtaining states of affairs) might criticize this definition for giving rise to 'spurious symmetries.' For example, if R happens to be uninstantiated, it has no instantiations (by the state-positivist's lights); and as a result any permutation of $\{1, \dots, n\}$ will under the present definition be classified as a symmetry of R. A possible solution would be to insert a 'necessarily' after the 'such that.' Another definition, which also appeals to modal notions, can be found in Svenonius (1987, 37–38).

29 Leo (2008b, 344) speaks in a similar case of 'permutation groups.'

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A well-known example of such a relation is due to Fine (2000, 17, n.10): "the relation R that holds of a, b, c, d when a, b, c, d are arranged in a circle (in that very order)". Fine goes on to say that "the following represent the very same state s: (i) $Rabcd$; (ii) $Rbcda$; (iii) $Rcdab$; (iv) $Rdabc$." If this list is supposed to be exhaustive, then the relation in question will have to be understood as a relation of circular arrangement that is either clockwise or counter-clockwise *relative to some vantage point*; for otherwise the state s may also be represented as (v) $Rdcba$, (vi) $Rcbad$, (vii) $Rbadc$, and (viii) $Radcb$.[30] Given that Fine specifies neither a vantage point nor a direction (clockwise or counter-clockwise), let us take R to be 'direction invariant' in this latter sense, i.e., so that the state $Rabcd$ is identical not only with $Rbcda$ (etc.), but also with $Rdcba$. R's symmetry group will then have eight members, which may be respectively represented as (i) id_4, (ii) (1 4 3 2), (iii) (1 3)(2 4), (iv) (1 2 3 4), (v) (1 4)(2 3), (vi) (1 3)(2)(4), (vii) (1 2)(3 4), and (viii) (1)(2 4)(3).[31]

This set is also known as a 'dihedral group of order eight.' To verify that it satisfies (C), it is enough to note that it, on the one hand, contains the permutation (1 4 3 2), which for instance maps 1 to 4, but on the other hand does *not* contain the permutation (1 4)(2)(3) that merely transposes 1 and 4. As Maureen Donnelly (2016, 88–89) points out, relations whose symmetry groups are of this kind—i.e., such as to satisfy (C)—tend to pose a problem for positionalism. More specifically, they pose a problem for the sort of positionalism that operates with a simple triadic occupation predicate and individuates relational states exclusively in terms of what entities occupy in them which positions. To see this, let us focus on the particular form of positionalism that conceives of relations in accordance with the statement (R) in section 4 above, and which conceives of *instantiations* of relations in accordance with the statements (US′) and (I1) in the same section.

To begin with, we can note that the question of what position(s) an entity a occupies in the instantiation of R by some given sequence of entities $x_1, \ldots, x_4$ (at least one of which is a itself) depends, apart from R, only on where a

30 For example, if a, b, c, and d are four cups arranged in a circle on a glass table, they might be said to be arranged in the clockwise order a, b, c, d as seen from *above* the table; but, seen from *below* the table, they will appear to be arranged in the clockwise order a, d, c, b. The expressions '$Rabcd$,' '$Rbcda$,' etc., should here be understood in the obvious way as names of instantiations of R.

31 In this representation scheme, non-trivial permutations are represented by their 'orbits.' For example, the permutation (1 3)(2)(4) has three orbits: one consisting of 1 and 3, and the other two consisting of, respectively, 2 and 4. It accordingly transposes 1 and 3 and maps 2 and 4 to themselves.

appears in this sequence.[32] From this it follows that *a* has to occupy exactly the same position(s) in *Radbc* as it does in *Rabcd*. Further, since the former state is identical with *Rdbca* (as is reflected in the fact that *R*'s symmetry group contains the permutation (1 2 3 4)), it follows that *a* occupies exactly the same position(s) in *Rdbca* as it does in *Radbc*. Putting the previous two statements together, we have that *a* occupies the same position(s) in *Rdbca* as it does in *Rabcd*. By analogous reasoning, it can be shown that *d* occupies the same position(s) in *Rdbca* as it does in *Rabcd*. Hence, the two states *Rabcd* and *Rdbca* cannot differ with respect to which positions are in them respectively occupied by *a* and *d*. And clearly they cannot differ, either, with respect to which positions are in them respectively occupied by *b* and *c*. Accordingly, since, under the form of positionalism now in question, relational states are characterizable up to uniqueness in terms of what entities occupy in them which positions, it follows that the two states are identical. But they aren't, as is reflected in the fact that *R*'s symmetry group fails to contain the permutation (1 4)(2)(3). So we have a contradiction.

To have a name for this difficulty, let us refer to it as the *symmetry problem*. How might a positionalist respond to it? The first thing to note is that it is not obviously a problem for what has above (in section 4) been called *weak* positionalism. This is because—as has in essence already been pointed out by MacBride (2007, 41)—it is open to the weak positionalist to deny the existence of relations whose symmetry groups satisfy (C).[33] In the particular case of Fine's example, the weak positionalist may maintain that, for any entities *a*, *b*, *c*, and *d*, the state of affairs that *a*, *b*, *c*, and *d*, in this order, are arranged in a circle is only a relational phenomenon rather than a relational *state*: in other words, that it is not an instantiation of a relation. (It is compatible with this claim that the state of affairs in question is grounded in, or analyzable in terms of, states of affairs that *are* relational states.) Thus the positionalist may hope to obviate the symmetry problem by retreating to some form of weak positionalism and, with it, to a 'sparse' ontology of relations. Admittedly,

32 More formally: for any entities $x_1, \ldots, x_4$ and $y_1, \ldots, y_4$: if the set $\{i \mid x_i = a\}$ is identical with $\{i \mid y_i = a\}$, then a occupies in $Rx_1x_2x_3x_4$ (i.e., in the instantiation of R by x_1, x_2, x_3, and x_4, in this order) exactly the same position(s) as it does in $Ry_1y_2y_3y_4$. This can be seen to follow from (R) and (I1).

33 In addition, MacBride argues that the positionalist may question whether Fine's relation, "even if it exists, constitutes any kind of counter-example" (2007, 41). However, see Fine's (2007, 59) reply.

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however, this move is not likely to appeal to a theorist who is unwilling to give up the advantages of an abundant ontology of intensional entities.[34]

Alternatively, the positionalist might opt for giving up the assumption that relational states are characterizable up to uniqueness in terms of what entities occupy in them which positions. She might then for instance allow that the states *Rabcd* and *Rdbca*, although distinct, are both such that a, b, c, and d occupy in them one and the same position p. The idea that all four relata thus occupy the same position can be readily motivated by the symmetry of R. This line of thought is not available, however, in the case of Leo's (2008a, 2008b, 2010) example of a triadic relation S whose instantiation by any entities x, y, and z (in this order) is the state of affairs that x loves y and y loves z. Given that this relation is thoroughly non-symmetric—its symmetry group contains only the identity permutation—the positionalist should find it hard to avoid positing three positions p_1, p_2, and p_3 such that, for any x, y, and z, the instantiation of S by x, y, and z (in this order) is a state in which p_1 is occupied only by x, p_2 only by y, and p_3 only by z. But if she follows this approach, she will not be able to accommodate the idea that, for any x and y, the state *Sxyx* is identical with *Syxy*. Plausibly *Sxyx* and *Syxy* are 'both' the state of affairs that x and y love each other, yet on the approach in question, p_2 is in *Sxyx* occupied only by y, while, in *Syxy*, p_2 is occupied only by x.[35]

A very different view has recently been proposed by Donnelly (2016). According to her *relative* positionalism, there exist unordered relations, associated with which there are 'relative properties.' At least from a formal point of view, these relative properties behave much like ordered relations: just as an ordered relation may be instantiated by some entities $x_1, \ldots, x_n$ (in this order), so a relative property may be instantiated by an entity x_1 "relative to" an entity x_2, ..., "relative to" an entity x_n.[36] Relatedly, Donnelly's view is not limited with regard to the symmetry groups it can accommodate; but this flexibility comes at a steep price in ontological commitment. Suppose R is a tetradic ordered relation whose symmetry group contains only id_4. In place of R, the relative positionalist would posit $4! = 24$ different relative properties. A *non*-relative positionalist, by contrast, would only posit four different positions $p_1, \ldots, p_4$. It is true that, given standard set theory, there would then also exist 24 different tuples (p_i, p_j, p_k, p_l) for pairwise distinct

34 MacBride himself (2007, 41) considers the present maneuver unsatisfactory, criticizing it as "insufficiently systematic to really address the concern Fine has raised."

35 For further discussion of this example, see footnote 26 above.

36 See Donnelly (2021) for discussion of how to understand this locution.

$i, j, k, l \in \{1, \ldots, 4\}$; and, as proposed above, these tuples could play the role of ordered relations. But the ontological commitment to these tuples would be a consequence of set theory, given the existence of $p_1, \ldots, p_4$. They would be 'derivative' entities. By contrast, the 24 relative properties posited by the relative positionalist would presumably have to be regarded as ontologically fundamental; for it is not easy to see (and Donnelly doesn't specify) how they might be derived from anything more basic.[37]

7. The Contributions to this Special Issue

Four of the papers of this Special Issue have first been presented at a workshop on "Properties, Relations, and Relational States" that has taken place in Lugano in October 2020.

Scott Dixon presents an extensive defense of what is often called the 'standard view' of relations, or 'directionalism,' against objections recently raised by Maureen Donnelly. A central thesis of directionalism is to the effect that a relation "applies to its relata in an order, proceeding from one to another." Donnelly (2021, 3592) has criticized this conception as "obscure" and as failing "to connect with ordinary thinking about" the semantic difference between such statements as 'Abelard loves Héloïse' and 'Héloïse loves Abelard.' She also argues that directionalism "does not have the right structure to explain the differential application of partly symmetric relations like *between* or *stand clockwise in a circle*" (2021, 3592). Dixon responds to these criticisms and moreover argues that directionalism has advantages over a number of competing views, including Donnelly's own.

Joop Leo describes a new form of positionalism, dubbed 'thin positionalism,' which can be regarded as a middle ground between traditional forms of positionalism on the one hand and antipositionalism on the other.[38] Thin positionalism, like its more traditional counterparts, accords a central place to

37 Further discussion of Donnelly's view can be found in MacBride (2020, sec. 4). In an interesting objection to positionalism that has not so far been discussed, Ralf Bader (2020) considers the "weak betterness relation" R, which is "the disjunction of the symmetric 'equally as good' relation and the asymmetric 'strictly better than' relation" (2020, 37). He holds that, when a and b are equally good, the state Rab is identical with Rba, due to their 'both' being grounded in the fact that a and b are equally good. The positionalist, by contrast, will have to *distinguish* the two states, due to a's (as well as b's) occupying a different position in Rab than in Rba. To avoid this problem, the positionalist may feel compelled to reject Bader's grounding-theoretic way of individuating states of affairs.

38 Cf. Remark 4.1 in his (2014, 272).

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the notion of a position. But positions are here conceived of as "substitutable places in a structure or form." The substitution of entities for such positions yields *relational complexes*, which are also related among each other by substitution relationships. As in Fine's antipositionalism, the relevant notion of substitution is taken as primitive. And, like Fine's antipositionalism, thin positionalism is immune to the symmetry problem discussed in the previous section.

Fraser MacBride argues that quantification into predicate position, as one finds it in second-order logic, cannot be understood as quantification over "relations conveived of as the referents of predicates." He argues for this thesis by constructing a dilemma. On the one hand, if converse predicates—understood as open sentences, such as 'ξ is on top of ζ' and 'ξ is underneath ζ'—co-refer, then we fail to understand the higher-order predicates that are involved in quantification into relational predicate position: predicates (understood, again, as open sentences) such as 'Alexander Φ Bucephalus.' On the other hand, if converse predicates do *not* co-refer, then we can still not make sense of those higher-order predicates unless we "impute implausible readings to lower-order constructions." For instance, even a symmetric predicate, such as 'ξ differs from ζ,' would have to be read as applying to its relata in a given order, which, MacBride argues, would be implausible.

Francesco Orilia offers a sophisticated form of positionalism, dubbed *dualist role positionalism*, that on the one hand embraces very finely individuated 'biased' relations (and their abundant converses) at the 'semantic' level while, on the other hand, rejecting them "at the truthmaker or ontological level of sparse attributes." At this more fundamental level, Orilia allows only *neutral* relations, whose exemplification he conceives of as being mediated through 'roles' such as *agent* and *patient* or *inferior* and *superior*. For instance, where V is a neutral relation of vertical alignment with respect to the Earth's surface, Orilia would write (in boldface) 'V(superior(a), inferior(b))' to represent the state of affairs of a plane a's being above a bird b.

MacBride and Orilia, in their joint contribution, respond to van Inwagen's (2006) argument for the conclusion that we do not have any "formal and systematic" names for non-symmetric relations. They concede the plausibility of supposing that, if non-symmetric relations had distinct converses, then it would be impossible to introduce such names for them. But they do not follow van Inwagen in holding that non-symmetric relations *do* have distinct converses. They point out that there are alternative conceptions of non-symmetric relations under which the existence of distinct converses—and hence the conclusion of van Inwagen's argument—can be avoided. And they moreover argue, *contra* van Inwagen, that it is possible (either in English or a modest extension of English) to introduce names for non-symmetric relations of an adicity greater than 2.

Finally, Edward Zalta replies to two papers by MacBride. More specifically, he replies (i) to MacBride's argument, in his contribution to the present issue, for the conclusion that second-order quantifiers cannot be interpreted as ranging over relations and (ii) to the argument in MacBride (2014) for the conclusion that (as Zalta puts it) "unwelcome consequences arise if relations and relatedness are *analyzed* rather than taken as *primitive*" (emphases in the original). Both arguments are examined in the light of Zalta's theory of relations, as developed in the context of his object theory.[39] The resources of this theory are brought to bear on the individuation of states of affairs, an issue which Zalta identifies as central to both of MacBride's arguments.

As I hope can be seen from this brief overview, the metaphysics of relations and relational states continues to be a fertile field of inquiry.[*]

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39 On the latter, cf. Zalta (1983, 1988, 1993).

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Directionalism and Relations of Arbitrary Symmetry

Scott Dixon

Maureen Donnelly has recently argued that directionalism, the view that relations have a direction, applying to their relata in an order, is unable to properly treat certain symmetric relations. She alleges that it must count the application of such a relation to an appropriate number of objects in a given order as distinct from its application to those objects in any other ordering of them. I reply by showing how the directionalist can link the application conditions of any fixed arity relation, no matter its arity or symmetry, and its converse(s) in such a way that directionalism will yield the correct ways in which it can apply. I thus establish that directionalism possesses the same advantage Donnelly's own account of relations, relative positionalism, has over traditional positionalist accounts of relations, which do not properly treat symmetric relations. I then note some advantages that directionalism has over its closest competitors. This includes Donnelly's relative positionalism, since directionalism is not, like relative positionalism, committed to the involvement of relative properties in every irreducibly relational claim. I close by conceding that, as Donnelly notes, directionalism is committed to the primitive relation of order-sensitive relational application. But I don't find this notion as mysterious as Donnelly does. I conclude that, even if one construes this feature of directionalism as a drawback, the two views are at worst at a draw, other things being equal, since this drawback is mitigated by the advantage directionalism has over relative positionalism.

Since Timothy Williamson's (1985) and Kit Fine's (2000) critiques of Bertrand Russell's (1903) view about the nature of relations, directionalism, according to which relations are understood as having a direction, applying to their relata in an order, philosophers have largely turned away from it.[1] They have turned toward views according to which relations are adirectional, or *neutral*.

1 The view is also known as "the standard view" and "the standard account."

One popular sort of theory of neutral relations is absolute positionalism, according to which relations have positions or roles associated with them which their relata occupy or have, respectively Dixon (2018).[2] As Fine (2000) argues, however, absolute positionalist views face the *problem of symmetric relations*; they are unable to properly treat relations with certain symmetries. That is, they are unable to deliver the correct possible *completions* of such a relation, where a completion of a relation is anything which results from that relation applying to some things in a certain way, e.g., a fact, a state of affairs, or a proposition.[3] I will characterize a way a relation can apply formally in what follows, but for now, a couple of examples will serve to elucidate the idea. The binary relation *loving*, for example, seems able to apply to two objects in two ways. Goethe's loving Charlotte Buff is a different state of affairs from Buff's loving Goethe. The binary relation *being next to*, on the other hand, seems able to apply to two objects in only one way. Goethe's being next to Buff is the same state of affairs as Buff's being next to Goethe.

The difficulty absolute positionalism has with symmetric relations has led to the development of other, neutral, views of relations which can solve this problem, including Fine's (2000) antipositionalism, Fraser MacBride's (2014) relational primitivism, and Maureen Donnelly's (2016, 2021) relative positionalism. These views properly treat any fixed arity relation, no matter its particular symmetry structure. Donnelly has recently argued that directionalism is, like absolute positionalism, also unable to properly treat symmetric relations. I begin, in the remainder of this section, by explaining the difficulty Donnelly alleges directionalism has with symmetric relations, which emerges clearly even in the case of binary relations, and state how my reply on behalf of directionalism goes in that case. I then remind the reader of fixed arity relations of arity greater than two, which can have more complex symmetries, and which any account of relations, including directionalism, ought to be able to to treat properly.

In section 1, I develop a way of formally representing the symmetry structure of any fixed arity relation, similar to Donnelly's (2016), and a way of

2 Following Donnelly (2016), I qualify these forms of positionalism as *absolute* to distinguish them from her positionalist view, which she qualifies as *relative*.

3 Fine (2000, 17–18, including fn.10) first articulates this problem, and Fine (2000, 4–5) introduces the notion of a completion. Of course, there are important differences between completions of these three different types. Presumably, for example, if the fact that Goethe loves Buff exists then Goethe loves Buff. This is usually thought not to be so in the case of the state of affairs of Goethe's loving Buff, or in that of the proposition that Goethe loves Buff. For simplicity, I restrict my attention primarily to states of affairs in what follows.

formally modeling the ways a fixed arity relation can apply. Along the way, I discuss several relations with various symmetry structures, some of which are known to cause problems for absolute positionalism. In section 2, I explain how Donnelly takes her objection to generalize to n-ary relations for all $n \geq 2$, and I develop my reply to this generalized criticism by showing how the directionalist can link the application conditions of any fixed arity relation, no matter its arity or symmetry structure, and its converse(s) in such a way that directionalism yields the correct manners in which it can apply.[4] I thus establish that directionalism possesses the same advantage Donnelly's, Fine's, and MacBride's accounts of relations have over absolute positionalism, which, it is well known, cannot handle all such relations.

In section 3, I turn to the task of evaluating directionalism, with my previous results in mind, in relation to other accounts of relations that avoid the problem of symmetric relations, viz., Fine's, MacBride's, and Donnelly's. I argue that directionalism has advantages over each of these views. In the case of Donnelly's relative positionalism, directionalism's advantage is that it is not, like relative positionalism, committed to the involvement of relative properties in every irreducibly relational claim (i.e., in every relational claim which cannot be construed as a claim involving the instantiation of only ordinary non-relative properties). I close by conceding, in section 4, that, as Donnelly notes, directionalism is committed to the primitive relation of ordered relational application. But I don't find this notion as mysterious as Donnelly does. I conclude that, even if one construes this feature of directionalism as a drawback, the two views are at worst at a draw, other things being equal, since this drawback is mitigated by the advantage directionalism has over relative positionalism. Unfortunately, I won't have the space to properly address all of the objections that have been leveled against directionalism over the years, including Williamson's and Fine's, and instead leave replies to these objections for another occasion.

4 Like Donnelly, in her development of relative positionalism, I consider only relations of fixed *finite* arity.

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Directionalism is usually formulated in terms of binary relations only.[5] It is typically taken to consist of three central theses.

> D1. Every relation has a *direction* (what Russell calls a "sense"). It applies to its relata in an order, proceeding from one to another.

The relation *loving*, for example, is understood by the directionalist as applying first to Goethe then to Buff when Goethe loves Buff, or, alternatively, proceeding from Goethe to Buff.[6]

5 See Russell's own (1903, sec. 94–95, 218–219) formulations of directionalism, as well as those of Fine (2000, sec. 1), MacBride (2007, 25; and 2014, 1–2), Gaskin and Hill (2012, sec. 1), Leo (2014, 263), Liebesman (2014, 409), Donnelly (2016, sec. 5.2), and Ostertag (2019, sec. 2.1). Fine (2000, 3) and Donnelly (2016, 83–85) discuss some elements of a generalization of the view, though, as I note below, Donnelly suggests that directionalism *can't* be generalized. Others, including Gaskin and Hill (2012, 167) and MacBride (2014, 4), appear to acknowledge that directionalism can be generalized to cover relations of any arity, though they provide few details about how they think such a generalization could be carried out. Russell (1913, 123) himself appears to recognize the relevance of algebra to the question of individuating completions of relations, but he did not himself give a general statement of directionalism. Thanks to Gregory Landini (personal communication) for bringing this passage to my attention. As suggested at the outset of the article, directionalism is not particularly popular, at least in the literature on the metaphysics of relations. But it appears to be standardly assumed, or at least major components of it are, in the tradition of higher-order metaphysics, at least implicitly. Many working in this tradition employ a higher-order language, often simple type theory with lambda abstraction Bacon (2020), that allows one to attribute to higher-order entities even higher-order properties and relations. To express the idea that a binary relation R applies to objects a and b in that order, one would say in such a language that $(\lambda X^{\langle e,e\rangle}.Xa^e b^e)R^{\langle e,e\rangle}$, which says of the binary relation R whose domain encompasses first-order objects (type e entities) that it applies to a and b. But the fact that "a" and "b" must appear in a specific order in such an expression forces an interpretation of relational application in such a language as being order-sensitive. There is a semantic difference between the expression above and "$(\lambda X^{\langle e,e\rangle}.Xb^e a^e)R^{\langle e,e\rangle}$." In addition to this, many working in higher-order metaphysics distinguish between each (non-symmetric) relation and its converse, as does the directionalist (see D3 below), since a necessary condition on the identity of second-order entities is that they are coextensive. And the extensions of a (non-symmetric) relation and its converse *are* distinct; the ordered pairs which populate them consist of pairs of the same objects but those objects oppositely ordered in those pairs in the two extensions. See Trueman (2021, 141–142) and Skiba (2021, 3).

6 While relations are characterized as having *directions* or *senses*, or applying in an *order*, according to directionalism, this needn't be understood as involving the reification of any of these things. What is important is that, according to D1, a relation applies *first* to one relatum *then* to the other, or, alternatively, it proceeds *from* one *to* the other.

> D2. Every relation R has a *converse*, which applies to x and y in the opposite order to that in which R applies whenever R applies to x and y.

The converse of *loving*, for example, is *being loved by*. It applies first to Buff and second to Goethe when Goethe loves Buff—in the opposite order or direction to that in which *loving* applies to them under the same condition.

> D3. Every necessarily symmetric relation is identical to its converse, while every other relation is distinct from its converse,

where a (binary) relation R is necessarily symmetric if and only if, necessarily, Rxy if and only if Ryx, and is non-symmetric otherwise. So while *loving* is distinct from its converse *being loved by*, a symmetric binary relation, like *being next to*, is its own converse.

Donnelly's criticism of directionalism emerges clearly even in the case of binary symmetric relations. She says,

> If the different ways R can hold among $x_1, \ldots, x_n$ amount to just different orders of application of R to $x_1 \ldots, x_n$, then *any* difference in the order of $x_1, \ldots, x_n$ should correspond to a different way for R to hold among $x_1, \ldots, x_n$. (2021, 6, ital. orig.) [7]

Donnelly is concerned that, because the directionalist imparts a direction to *every* (binary) relation, not just non-symmetric ones, she will be forced to say that, just as a non-symmetric binary relation like *loving* can apply to two objects in two ways, a symmetric binary relation like *being next to* will have to too. Note, however, that D2 saves the directionalist from this consequence. Since *being next to* is necessarily symmetric, by D3, it is its own converse, and so D2 demands that, when it applies to two objects like Goethe and Buff in that order, it must also apply to them in the opposite order. So there is only one way for it to apply to Goethe and Buff: the way in which it applies to Goethe and Buff both in that order *and* the opposite order. Contrast that with how directionalism treats *loving*. Since it is non-symmetric, by D3, it is distinct from its converse *being loved by*. D2 demands that, when *loving* applies to Goethe and Buff in that order, *being loved by* must apply to them in the opposite order

7 See Donnelly (2016, 83) for an earlier statement of the objection. Gaskin and Hill (2012, 175) also take directionalism to be incapable of properly treating relations with partial symmetries (defined below).

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(and vice versa). But this yields *two* ways for *loving* (and *being loved by*) to apply to Goethe and Buff: the way in which *loving* applies to Goethe and Buff in that order and *being loved by* does so in the opposite order, and the way in which *loving* applies to Buff and Goethe in *that* order and *being loved by* does so in the opposite order. Of course many countenance relations of arity greater than two, and such relations exhibit a variety of different symmetry structures, and as I will discuss later, Donnelly takes her concern to generalize to many of these structures. So the directionalist's response can't be as simple as this. To understand Donnelly's criticism in full, and the directionalist's response to it, we first need to see the full picture of the possible symmetry structures relations can have. This task I undertake in the next section.

1. Relations of Arbitrary Symmetry

Following Donnelly (2016), I represent a relation's symmetry (structure) by its *symmetry group*. A *group* is a set of objects that is closed under an associative operation $\cdot$, the *group operation*, which has a unique *identity element* e such that x, $e \cdot x = x \cdot e = x$ and, for each element x, a unique *inverse element* x^{-1} such that $x \cdot x^{-1} = x^{-1} \cdot x = e$. A symmetry group of an n-ary relation is a group of permutations of $\{1, 2, \dots, n\}$ (i) whose group operation is function composition, $\circ$, (ii) whose identity element is the identity permutation (i.e., the permutation that maps 1 to 1, 2 to 2, …, and n to n), and (iii) for which the inverse of each element is that element's inverse permutation. In particular,

> Definition of Symmetry Groups. The *symmetry group* of an n-ary relation R, where $n \in \{2, 3, \dots\}$, is the set Sym_R of permutations of $\{1, \dots n\}$ such that, for each member p of Sym_R, necessarily, for all $x_1, \dots, x_n$, $Rx_1 \dots x_n$ iff $Rx_{p(1)} \dots x_{p(n)}$.[8]

As Donnelly notes (2016, 83, incl. fn.10), the symmetry group of any n-ary relation will be a *subgroup* of the group of *all* possible permutations of $\{1, \dots, n\}$, i.e, of the *symmetric group of degree n*, or S_n.[9]

A question arises at this point, for each n-ary relation R, whether the fact that, necessarily, for all $x_1, \dots, x_n$, $Rx_1 \dots x_n$ iff $Rx_{p(1)} \dots x_{p(n)}$ really is suf-

8 Henceforth, when I introduce an arbitrary n-ary relation, I leave it implicit that $n \in \{2, 3, \dots\}$ unless specified otherwise.

9 That is, the set is a subset of that group and itself forms a group under the group operation of permutation composition.

ficient for p to be in R's symmetry group, as the above definition of symmetry groups stipulates, or whether instead $[Rx_1 \dots x_n]$ must be *identical to* $[Rx_{p(1)} \dots x_{p(n)}]$ to guarantee this to be the case, where $[Rx_1 \dots x_n]$ and $[Rx_{p(1)} \dots x_{p(n)}]$ are completions of the same type (viz., facts, states of affairs, or propositions). But since "R" appears on both sides of the biconditional in the definition, there will presumably be no cases in which $[Rx_1 \dots x_n]$ is distinct from $[Rx_{p(1)} \dots x_{p(n)}]$. It is plausible that, for any relations R and R', when $R = R'$, if necessarily, $Rx_1 \dots x_n$ iff $R'x_{p(1)} \dots x_{p(n)}$, then $[Rx_1 \dots x_n] = [R'x_{p(1)} \dots x_{p(n)}]$, even if this is implausible when $R \neq R'$. So an intensional definition of symmetry groups should be adequate. For this reason, I'll allow myself to move back and forth between talk of (non-)identity of completions and (non-)equivalence of relational claims in what follows.

The discussion of relations' symmetry groups has been pretty abstract so far, so I'll consider some examples. I'll begin with the symmetry groups of the binary relations *being next to* and *loving*. Since, necessarily, for any x_1 and x_2, x_1 is next to x_2

- iff x_1 is next to x_2 (equivalently: $x_{[1\,2](1)}$ is next to $x_{[1\,2](2)}$),

and

- iff x_2 is next to x_1 (equivalently: $x_{[2\,1](1)}$ is next to $x_{[2\,1](2)}$),

where ⌜$[i_1\, i_2 \dots\, i_n]$⌝ denotes the permutation of $\{1, 2, \dots, n\}$ that maps 1 to i_1, 2 to i_2, …, and n to i_n, the symmetry group of *being next to*,

$$Sym_{being\ next\ to} = \{[1\,2], [2\,1]\}.$$

In other words, every permutation of x_1 and x_2 results in an equivalent claim. But since (i) necessarily, for any x_1 and x_2, x_1 loves x_2 iff

- x_1 loves x_2 (equivalently: $x_{[1\,2](1)}$ loves $x_{[1\,2](2)}$)

but (ii) it is not the case that, necessarily, for any x_1 and x_2, x_1 loves x_2 iff

- x_2 loves x_1 (equivalently: $x_{[2\,1](1)}$ loves $x_{[2\,1](2)}$),

the symmetry group of *loving*,

$$Sym_{loving} = \{[1\,2]\}.$$

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In other words, the only permutation of x_1 and x_2 that results in an equivalent claim is the identity permutation, i.e., the permutation that leaves the two terms where they are.

An n-ary relation such that, necessarily, for all $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $Rx_{p(1)} \ldots x_{p(n)}$ for *every* permutation $p \in S_n$, is *completely symmetric*, while one that is such that this is true *only* when p is the identity permutation of S_n, $[1\,2\,\ldots\,n]$, is *completely non-symmetric*. *Being next to* is an example of the former, and *loving* the latter. Indeed, any binary relation can only be either completely symmetric or completely non-symmetric, since there are only two subgroups of the group of S_2, viz., S_2 itself, and the group that consists of just the identity permutation of S_2, i.e., $\{[1\,2]\}$. There are, of course, also completely symmetric and completely non-symmetric n-ary relations for $n > 2$ as well, though I will not consider any here.

Fine (2000, 17–18, incl. fn.10) argues that absolute positionalism is unable to properly treat fixed arity relations with certain symmetries (see also Donnelly 2016, sec. 5.3). According to absolute positionalism, relations are neutral (directionless), but feature positions, which have been interpreted as worldly correlates of thematic roles in linguistics that their relata fill Orilia (2014), or as entities akin to holes which their relata occupy Dixon (2018). Such views properly treat relations with some symmetries just fine. But there are relations with other symmetries that they cannot properly treat. They can properly treat any completely symmetric or completely non-symmetric relation one might throw at them.

For a theory of relations to properly treat a given n-ary relation, I mean that the theory has the resources to ensure that that relation can apply in the ways that we think it should be able to apply. But what is a way for an n-ary relation to apply? And, for a given n-ary relation, what are the ways that it *should* be able to apply? The ways such a relation can apply can be identified with the *left cosets* of that relation's symmetry group. For a given ordering of n objects, yielding a certain completion of an n-ary relation R, Sym_R includes exactly those permutations of that ordering that yield the same completion of R, which of course include the identity permutation. This amounts to one way the relation can apply. For some relations (any relation that is not completely symmetric), there will be non-identity permutations of that initial ordering (in S_n but not in Sym_R) that yield distinct completions of a given sort (facts, states of affairs, or propositions). Consider such a relation R and such a non-identity permutation q. Then $[Rx_1 \ldots x_n] = [Rx_{[1\ldots n](1)} \ldots x_{[1\ldots n](n)}] \neq [Rx_{q(1)} \ldots x_{q(n)}]$. And $[Rx_{q(1)} \ldots x_{q(n)}]$ will be identical to every other completion (of the same sort) that results from permuting the arguments of R in $[Rx_{q(1)} \dots\ x_{q(n)}]$ by some permutation in Sym_R. The sets of permutations identified by considering every $q \in S_n$ form the *left cosets* of Sym_R in S_n, and represent the ways R can apply to n fixed objects. More formally,

> Definition of Left Cosets of the Symmetry Group of a Relation. For any n-ary relation R, the *left cosets* of Sym_R in S_n are the sets $\{q \circ p\ :\ p \in Sym_R\}$ for each $q \in S_n$.

The left cosets of the symmetry group of an n-ary relation R partition S_n into between 1 and n! equally-sized sets of permutations, depending on R's symmetry group. And by Lagrange's theorem, which implies that the number of left cosets of a subgroup H of a group G equals $|G| \div |H|$, the number of left cosets of $Sym_R = |S_n| \div |Sym_R|$. So there are $|S_n| \div |Sym_R|$ ways for R to apply to n objects.[10,11]

A completely symmetric n-ary relation R will therefore be able to apply to n objects in only $|S_n| \div |Sym_R| = |S_n| \div |S_n| = 1$ way, corresponding to the single coset of Sym_R in S_n. The single way *being next to* can apply to two objects, for example, corresponds to the single left coset of $Sym_{being\ next\ to} = \{[1\,2], [2\,1]\}$ in $S_2 = \{[1\,2], [2\,1]\}$, viz., $\{[1\,2], [2\,1]\}$ itself.[12] ($|S_2| \div |Sym_{being\ next\ to}| = 2 \div 2 = 1$.) A completely non-symmetric n-ary relation, on the other hand, will be able to apply in $|S_n| \div |Sym_R| = |S_n| \div |\{[1\ 2\ \dots\ n]\}| = |S_n| \div 1 = n!$ ways to n objects, corresponding to the n! cosets of Sym_R in S_n. The two ways *loving* can apply to two objects, for example, correspond to the two left cosets of $Sym_{loving} = \{[1\,2]\}$ in $S_2 = \{[1\,2], [2\,1]\}$, viz., $\{[1\,2]\}$ and $\{[2\,1]\}$. ($|S_2| \div |Sym_{loving}| = 2 \div 1 = 2$.)

The absolute positionalist can say that a completely symmetric relation has just one position which can take up to n arguments. This results in there being

10 See Gallian (2013, 147–148) for a statement and proof of Lagrange's theorem.

11 An n-ary relation R can apply to m objects in fewer ways when $m < n$. Certain combinatorial possibilities collapse in such cases because a relation's/predicate's argument cannot be permuted with itself and yield a new completion/non-equivalent claim. See Donnelly (2016, 83–84, fn.11).

12 The left coset $[1\,2] \circ Sym_{being\ next\ to} = \{[1\,2] \circ p\ :\ p \in Sym_{being\ next\ to}\} = \{[1\,2] \circ [1\,2], [1\,2] \circ [2\,1]\} = \{[1\,2], [2\,1]\}$. The left coset $[2\,1] \circ Sym_{being\ next\ to} = \{[2\,1] \circ p\ :\ p \in Sym_{being\ next\ to}\} = \{[2\,1] \circ [1\,2], [2\,1] \circ [2\,1]\} = \{[2\,1], [1\,2]\}$. These cosets are identical and exhaustive of the permutations in S_2, and so $Sym_{being\ next\ to}$ has only a single coset in S_n. Remember that $\circ$ is function composition. For permutations p and q of $\{1, \dots, n\}$, $p \circ q$ is the permutation that maps each $i \in \{1, \dots, n\}$ to $p(q(i))$. In other words, it is the result of first applying q to i, getting the result, and then applying p to that result. So $[1\,2] \circ [2\,1] = [2\,1]$, for example, because (i) $([1\,2] \circ [2\,1])(1) = [1\,2]([2\,1](1)) = [1\,2](2) = 2$ and (ii) $([1\,2] \circ [2\,1])(2) = [1\,2]([2\,1](2)) = [1\,2](1) = 1$.

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just one way for such a relation to apply to n objects: that constituted by each of those objects being assigned to that single position. So, for example, the absolute positionalist would say that *being next to* has one position, p_1, which can take up to two arguments, and so there is only one way for it to apply to two objects like Goethe and Buff. Goethe and Buff can only both be assigned to p_1. And, as mentioned, there is indeed only one way for *being next to* to apply to two objects like Goethe and Buff. Goethe's being next to Buff is the same state of affairs as Buff's being next to Goethe. The absolute positionalist can say that a complete non-symmetric n-ary relation has n positions, each of which can take just a single argument. This results in there being $n!$ ways for such a relation to apply to n objects, each corresponding to a different assignment of those n objects to those n positions. For example, the absolute positionalist would say that *loving* has two positions, p_2 and p_3, each of which can take just a single argument, and so there are two ways for it to apply to two objects, such as Goethe and Buff. Goethe can be assigned to p_2 and Buff to p_3, or Buff can be assigned to p_2 and Goethe to p_3. And, as mentioned, there are indeed two ways for *loving* to apply to two objects like Goethe and Buff: one in which Goethe is doing the loving, and Buff is being loved, and one in which Buff is doing the loving, and Goethe is being loved.

In addition to *completely* symmetric and non-symmetric n-ary relations for $n > 2$, however, there are also *partially (non-)symmetric* such relations. The symmetry group of a partially symmetric n-ary relation is a proper non-trivial subgroup of S_n. That is, it will contain some, though not all, non-identity permutations of $\{1, \dots, n\}$. The ternary relation *being between* is an example of such a relation. Since (i) necessarily, for any x_1, x_2, and x_3, x_1 is between x_2 and x_3

- iff x_1 is between x_2 and x_3 (equivalently: $x_{[1\,2\,3](1)}$ is between $x_{[1\,2\,3](2)}$ and $x_{[1\,2\,3](3)}$),

and

- iff x_1 is between x_3 and x_2 (equivalently: $x_{[1\,3\,2](1)}$ is between $x_{[1\,3\,2](2)}$ and $x_{[1\,3\,2](3)}$),

but (ii) this is false of every other permutation of $\{1, 2, 3\}$, the symmetry group of *being between*,

$$Sym_{being\ between} = \{[1\,2\,3], [1\,3\,2]\}.$$

Absolute positionalist views can properly treat *some* partially symmetric relations, like this one. The absolute positionalist can say that such a relation, while ternary, has only two positions, p_4 and p_5, the first of which can take only a single argument, while the other can take up to two (see Dixon 2018, 208). This results in there being three ways for such a relation to apply to three objects, such as Larry, Curly, and Moe. Larry can be assigned to p_4 and the other two to p_5, or Curly can be assigned to p_4, and the other two to p_5, or Moe can be assigned to p_4, and the other two to p_5. And there are, indeed, three ways for such a relation to apply to Larry, Curly, and Moe. Larry could be between the other two, or Curly could be, or Moe could be. These three ways correspond to the three left cosets of $Sym_{being\ between} = \{[1\,2\,3], [1\,3\,2]\}$ in $S_3 = \{[1\,2\,3], [1\,3\,2], [2\,1\,3], [2\,3\,1], [3\,1\,2], [3\,2\,1]\}$, viz., $\{[1\,2\,3], [1\,3\,2]\}$, $\{[2\,1\,3], [2\,3\,1]\}$, and $\{[3\,1\,2], [3\,2\,1]\}$. ($|S_3| \div |Sym_{being\ between}| = 6 \div 2 = 3$.)

But absolute positionalist views cannot handle *all* partially symmetric relations. The ternary relation *being arranged clockwise in that order* is such a relation.[13] Since (i) necessarily, for any x_1, x_2, and x_3, x_1, x_2, and x_3 are arranged clockwise in that order

- iff x_1, x_2, and x_3 are arranged clockwise in that order (equivalently: $x_{[1\,2\,3](1)}$, $x_{[1\,2\,3](2)}$, and $x_{[1\,2\,3](3)}$ are arranged clockwise in that order),
- iff x_2, x_3, and x_1 are arranged clockwise in that order (equivalently: $x_{[2\,3\,1](1)}$, $x_{[2\,3\,1](2)}$, and $x_{[2\,3\,1](3)}$ are arranged clockwise in that order),

and

- iff x_3, x_1, and x_2 are arranged clockwise in that order (equivalently: $x_{[3\,1\,2](1)}$, $x_{[3\,1\,2](2)}$, and $x_{[3\,1\,2](3)}$ are arranged clockwise in that order),

but (ii) this is false of every other permutation of $\{1, 2, 3\}$, the symmetry group of *being arranged clockwise in that order*,

$$Sym_{being\ arranged\ clockwise\ in\ that\ order} = \{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}.$$

13 This nominalization and the corresponding predicate "..., ..., and … are arranged clockwise in that order" presuppose a particular vantage point on one side of the plane in which the objects are arranged. The nominalization also makes essential reference to the order of terms with respect to the argument places of the predicate . A name for the relation that avoids the latter issue (though not the former) is "*being clockwise in front of from the perspective of*." See Donnelly (2016, 92–94).

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The absolute positionalist appears to have only four options for treating *being arranged clockwise*. But none of these options yields the correct number of possible ways for it to apply to three objects. If it has one position, then it can apply in only one way. If it has two positions, one which can take only a single argument while the other can take up to two, it can apply in three ways (as was the case with in the previous example). If it has two positions, either of which can take up to two arguments, then it can apply in six ways. And if it has three positions, it can apply in six ways. But there are *two* ways for such a relation to apply to three objects, like Larry, Curly, and Moe. Larry, Curly, and Moe could be arranged clockwise in that order. Or Larry, Moe, and Curly could be arranged in *that* order instead. These two ways correspond to the two left cosets of $Sym_{\textit{being arranged clockwise in that order}}$ in $S_3 = \{[1\,2\,3], [1\,3\,2], [2\,1\,3], [2\,3\,1], [3\,1\,2], [3\,2\,1]\}$, viz., $\{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}$ and $\{[1\,3\,2], [3\,2\,1], [2\,1\,3]\}$. ($|S_3| \div |Sym_{\textit{being arranged clockwise in that order}}| = 6 \div 3 = 2$.)

2. Generalizing Directionalism

It is the shortcoming of absolute positionalism just related which has motivated others to develop alternative accounts of relations. This includes Donnelly, who develops relative positionalism, which provably yields the correct possible completions of any fixed arity relation. She recognizes that the problem of symmetric relations is at its heart an algebra problem, and uses this to draw insights about what relations would have to be like to avoid the problem. But she thinks that directionalism is unable to do the same. Donnelly (2016, 83–85; and 2021, 6) takes her concern about directionalism's ability to deal with symmetric relations, which I explicated above, to generalize to any relation that is anything but completely non-symmetric. Stated generally, her concern is that, because each n-ary relation R applies to n relata in a total order, there will always be $n!$ ways for R to apply to n relata, clashing with our intuitive judgement about n-ary relations that are anything but completely non-symmetric that they apply in m ways where $m < n!$. In this section, I show how directionalism can properly treat relations of *any* fixed arity relation.

It is clear that directionalism, as formulated earlier in the text, like absolute positionalism, cannot properly treat the relation *being arranged clockwise in that order*. This is for the simple reason that directionalism was formulated there in terms of binary relations only, and a relation expressed by the predicate "..., ..., and ... are arranged clockwise in that order" is presumably ternary.

(For the same reason, directionalism, as formulated above, can't even handle *being between*—something that I noted absolute positionalism *can* do.) But all the directionalist needs to do is construe D1 as allowing for some relations to take more than two relata. The direction of such a relation can be understood as the ordering of those relata, proceeding from the first relatum to the second, to the third, ..., to the nth.

Then, once a couple more adjustments are made to the original formulation of directionalism, it becomes clear that directionalism can treat these relations, and indeed relations of *any* fixed arity, and that it can do so properly, no matter these relations' symmetries. First, for any n-ary relation R and each possible ordering of n relata, $x_1, \dots, x_n$, the directionalist posits a unique converse for R which applies to $x_1, \dots, x_n$ in that ordering of them exactly when R applies to $x_1, \dots, x_n$ in *that* order. More precisely,

> p-Converse Existence. For any n-ary relation R and any permutation p of $\{1, \dots, n\}$, R has exactly one *p-converse*,

where

> Definition of p-Converses. For any n-ary relations R and R' and any permutation p of $\{1, \dots, n\}$, R' is the *p-converse* of R, i.e., $R' = R_p =_{df}$ (i) R' is a converse of R, and (ii) necessarily, for all $x_1, \dots, x_n$, $Rx_1 \dots x_n$ iff $R'x_{p(1)} \dots x_{p(n)}$.

p-Converse Existence effectively replaces D2.

I will not define the notion of a converse of a relation, as it appears in clause (i) of the above definition. A straightforward way to do so—in terms of a p-converse of that relation (given a definition of p-converses which omits clause (i) of the above definition)—is as follows:

> For any n-ary relations R and R' and any permutation p of $\{1, \dots, n\}$, R' is a *converse* of $R =_{df} R'$ is a p-converse of R for some permutation p of $\{1, \dots, n\}$.

But if one thinks that there are distinct though intensionally equivalent relations, this definition would be too permissive. For example, it would seem that, necessarily, for any x_1 and x_2, x_1 is triangular and taller than x_2 iff x_2 is shorter than x_1 and x_1 is trilateral. But presumably *being a y and z such that y is shorter than z and z is triangular*—and not *being a y and z such that*

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y is shorter than z and z is trilateral—is the single distinct converse of the completely symmetry binary relation *being triangular and larger than*. Such cases would also prevent one from supposing that the p-converse of a relation is unique, as I stipulate in the above definition of p-converses.

I will prevent such cases from causing problems by instead taking the notion of a converse as primitive, regarding facts about which relations are which relations' converses as brute, and adopting the following principle:

> Converse-p-Converse Link. For any n-ary relations R and R' and any permutation p of $\{1, \dots, n\}$, if R' is a converse of R, then R' is a p-converse of R for some permutation p of $\{1, \dots, n\}$.

I assume that every relation is (one of) its own converse(s), so that $R = R_{[1 \dots n]}$ for every n-ary relation R. Thus the notion of a converse I have in mind, and which I will employ in what follows (mainly to simplify the discussion), is different than that given by D3—the claim that every necessarily symmetric binary relation is identical to its converse, while every other binary relation is distinct from its converse. But even if revised according to this new terminology, D3 will still entail a difference between necessarily symmetric relations and all other fixed arity relations; each of the former is its own *only* converse, while each of the latter has at least one converse distinct from itself.

To be able to properly treat any n-ary relation R, no matter its symmetry, all the directionalist needs to do is identify those p-converses of R whose orderings of relata $x_1, \dots, x_n$, when $Rx_1 \dots x_n$, "can be transformed into one another by a permutation in the symmetry group" of R (Donnelly 2016, 94). More precisely,

> p-Converse Identity. For any n-ary relation R, R's q-converse $=$ R's q^*-converse ($R_q = R_{q^*}$) iff there is some $p \in Sym_R$ such that $q^* = p \circ q$.

p-Converse Identity effectively replaces D3.[14] I assume that the symmetry structure of any n-ary relation R is represented by some subgroup of S_n.[15] Whatever subgroup of S_n Sym_R turns out to be, p-Converse Identity

14 It corresponds to Donnelly's (2016, 94) principle ($\bigtriangledown$), which provides analogous identity conditions for relative properties.

15 This follows assuming that every relation can be expressed by a predicate which is *order-determined*, i.e., by a predicate that is such that implications of a relational claim that involve

guarantees that $R = R_p$ iff $p \in Sym_R$. ($R = R_{[1 \dots n]}$, and so *p*-CONVERSE IDENTITY implies that $R = R_{q^*}$ iff there is some $p \in Sym_R$ such that $q^* = p \circ [1 \dots n] = p$.) This ensures that the ways R can apply to n objects correspond to the left cosets of Sym_R, as they should. This is because, by the definition of *p*-converses, $R = R_p$ iff, necessarily, for any $x_1, \dots, x_n$, $Rx_1 \dots x_n$ iff $Rx_{p(1)} \dots x_{p(n)}$, and so the directionalist has ensured that $p \in Sym_R$ iff, necessarily, for any $x_1, \dots, x_n$, $Rx_1 \dots x_n$ iff $Rx_{p(1)} \dots x_{p(n)}$, which is in agreement with the definition of symmetry groups. And I explained in the previous section why the ways an n-ary relation R can apply correspond to the left cosets of Sym_R in S_n. This in turn ensures, of course, that the number of ways R can apply to n objects equals $|S_n| \div |Sym_R|$, as it should.

But directionalism must also imply that the symmetry group of any converse of an n-ary relation R is isomorphic to that of R. What I've just said establishes that any n-ary relation will, according to directionalism, be able to apply to n objects in the ways we think it should. But we also expect R's (non-identical) converses (if it has any) to apply to n objects in the same ways as R (or, at least, in ways that are *structurally* the same). To show that this isomorphism holds, consider any n-ary relation R and any permutation p of $\{1, 2, \dots\}$. There is bijective function f_p from Sym_R to Sym_{R_p} such that, for any $i, j \in Sym_R$, $f_p(i \circ j) = f_p(i) \circ f_p(j)$. $f_p(q) = p \circ q \circ p^{-1}$ fits this bill. (Recall that p^{-1} is the inverse of p. See section 1 above.) In other words, $f_p(q)$ is the permutation in S_n that maps a to b iff q maps $p^{-1}(a)$ to $p^{-1}(b)$,

the predicate "concerning the order of relational application are completely determined in some fixed way by the order of the terms denoting the relata" relative to the predicate (Donnelly 2016, 84, fn.13). Donnelly makes this assumption in her development of relative positionalism as well. It means that, according to directionalism, every relation must be expressible by a relational predicate that has a fixed number of singular argument places, and relates to directionalism's (and relative positionalism's) inability to accommodate variable arity relations. See fn.26 in section 3 below.

where $a, b \in \{1, \dots, n\}$, i.e., $f_p(q)(a) = b$ iff $q(p^{-1}(a)) = p^{-1}(b)$.[16] In general, $Sym_{R_p} = \{p \circ q \circ p^{-1} : q \in Sym_R\}$.[17]

This discussion has been very abstract, so to provide the reader with a better idea of how directionalism handles relations of different arities and symmetries, and to highlight some interesting differences between directionalism, understood as applying to relations of any fixed arity, as compared to the binary formulation of it that I gave on pages four and five, I'll show how directionalism, as formulated above, treats the examples I discussed in section 1. I've already noted that, according to directionalism, a (completely) symmetric binary relation is its own only converse, while a (completely) non-symmetric binary relation has a single converse distinct from it. (Though now even a non-symmetric binary relation is a converse of itself.) But it will be instructive to see how p-Converse Existence and p-Converse Identity result in these treatments. Consider first the binary (completely) symmetric relation *being next to*. p-Converse Existence implies that *being next to* has p-converses *being next to*$_{[1\,2]}$ and *being next to*$_{[2\,1]}$. Since $[1\,2], [2\,1] \in Sym_R = \{[1\,2], [2\,1]\}$, by p-Converse Identity, *being next to* $=$ *being next to*$_{[1\,2]}$ $=$ *being next to*$_{[2\,1]}$. So by the definition of p-converses, *being next to* has a single converse, viz., itself. This means that, according to directionalism, *being next to* can apply to two things, such as Goethe and Buff, in only one way. If *being next to* applies to Goethe and Buff in that order, then *being next to*'s converse must apply to them in the opposite order. And if *being next to*'s converse applies to Goethe and Buff in that order, then *being next to* must apply to them in the opposite order. But since *being next to* is its own converse, there is no difference between these two possibilities, which are depicted in figure 1.

16 Because every permutation is a bijection and the composite of bijections is a bijection, f_p is a bijection. To show it is an isomorphism, consider arbitrary $i, j \in Sym_R$. $f_p(i \circ j) = p(q(p^{-1}(i \circ j)))$. Then

$f_p(i \circ j)$	$=$	$p \circ i \circ j \circ p^{-1}$	by the definition of f_p
	$=$	$p \circ i \circ e_n \circ j \circ p^{-1}$	recall that e_n is the identity element of S_n
	$=$	$p \circ i \circ p^{-1} \circ p \circ j \circ p^{-1}$	$e_n = p^{-1} \circ p$
	$=$	$f_p(i) \circ f_p(j)$	by the definition of f_p.

17 Sym_R and Sym_{R_p} are *conjugate subgroups*. Many thanks to Maureen Donnelly and Jan Plate (personal communications) for helpful suggestions about the reasoning in this section.

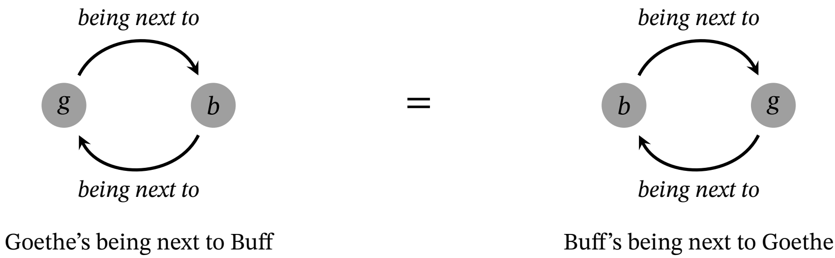

Figure 1: The single possible application of *being next to* to Goethe and Buff. In this diagram and the one to follow, a relation applying to x_1 and x_2 in that order is represented by an arrow going from x_1 to x_2.

By assigning Goethe to 1 and Buff to 2, it is clear that this single manner of application corresponds to the single left coset $\{[1\,2], [2\,1]\}$ of $\mathrm{SYM}_{\textit{being next to}}$ in S_2.

Things go the same in the case of a non-symmetric relation like *loving*, except that, because $[2\,1] \notin \mathit{Sym}_{\mathit{loving}} = \{[1\,2]\}$, it follows by *p*-CONVERSE IDENTITY that $\mathit{loving} = \mathit{loving}_{[1\,2]} \neq \mathit{loving}_{[2\,1]}$. So by the definition of *p*-converses, *loving* has two converses, one of which is itself, the other, presumably, being *being loved by*. This means *loving* can apply to two things, such as Goethe and Buff, in two ways. If *loving* applies to Goethe and Buff in that order, then *loving*'s distinct converse, *being loved by*, must apply to them in the opposite order. And if *being loved by* applies to Goethe and Buff in that order, then *loving* must apply to them in the opposite order. But since *loving* ≠ *being loved by*, these are two different possibilities, which are depicted in figure 2.
By assigning Goethe to 1 and Buff to 2, it is clear that these two manners of application correspond to the two left cosets $\{[1\,2]\}$ and $\{[2\,1]\}$ of $\mathrm{SYM}_{\mathit{loving}}$ in S_2.

Things become more complicated for ternary relations. Consider *being between*. Recall that

$$\mathit{Sym}_{\textit{being between}} = \{[1\,2\,3], [1\,3\,2]\}.$$

By *p*-CONVERSE EXISTENCE, the directionalist would say that *being between* (R for now) has *p*-converses $R_{[1\,2\,3]}$ ($= R$), $R_{[1\,3\,2]}$, $R_{[2\,1\,3]}$, $R_{[2\,3\,1]}$, $R_{[3\,1\,2]}$, and $R_{[3\,2\,1]}$. And because

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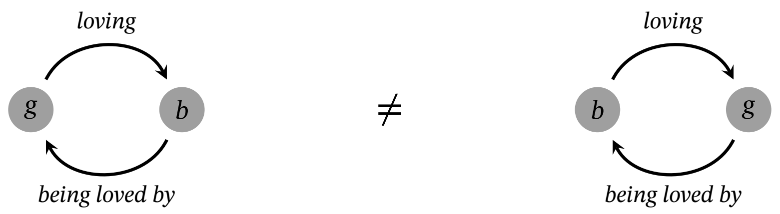

Figure 2: The two possible applications of *loving* and its single (distinct) converse to Goethe and Buff

(i) $[1\,3\,2] \in Sym_{being\ between}$ and $[1\,3\,2] \circ [1\,2\,3] = [1\,3\,2]$,
(ii) $[1\,3\,2] \in Sym_{being\ between}$ and $[1\,3\,2] \circ [2\,1\,3] = [3\,1\,2]$,

and

(iii) $[1\,3\,2] \in Sym_{being\ between}$ and $[1\,3\,2] \circ [2\,3\,1] = [3\,2\,1]$,

the directionalist would say, by *p*-Converse Identity, that (i) $R_{[1\,2\,3]} = R_{[1\,3\,2]}$, (ii) $R_{[2\,1\,3]} = R_{[3\,1\,2]}$, and (iii) $R_{[2\,3\,1]} = R_{[3\,2\,1]}$. But because

(iv) there is no permutation $p \in Sym_{being\ between}$ such that, e.g., $p \circ [2\,1\,3] = [1\,2\,3]$,
(v) there is no permutation $p \in Sym_{being\ between}$ such that, e.g., $p \circ [2\,3\,1] = [1\,2\,3]$,

and

(vi) there is no permutation $p \in Sym_{being\ between}$ such that, e.g., $p \circ [2\,3\,1] = [2\,1\,3]$,

the directionalist would say, by *p*-Converse Identity, that $R_{[1\,2\,3]} \neq R_{[2\,1\,3]}$, $R_{[1\,2\,3]} \neq R_{[2\,3\,1]}$, and $R_{[2\,1\,3]} \neq R_{[2\,3\,1]}$ (and so $R_{[1\,3\,2]} \neq R_{[2\,3\,1]}$ and $R_{[3\,1\,2]} \neq R_{[3\,2\,1]}$).

By the definition of *p*-converses, this means that *being between*, according to the directionalist, has three converses, one of which is itself. To identify plausible interpretations of the two converses distinct from *being between*, suppose Larry is between Curly and Moe, and consider the following diagram.

Figure 3: Larry's being between Curly and Moe

Being between applies to Larry, Curly, and Moe in that order, and also to Larry, Moe, and Curly in *that* order (see (i) above). But what relation applies to Curly, Larry, and Moe in that order and to Moe, Larry, and Curly in *that* order (as (ii) above demands)? A plausible interpretation of this relation is *being on the far side of from the perspective of*. Curly is on the far side of Larry from the perspective of Moe, and Moe is on the far side of Larry from the perspective of Curly. And what relation applies to Curly, Moe, and Larry in that order and to Moe, Curly, and Larry in *that* order (as (iii) above demands)? A plausible interpretation of this relation is *being on the opposite side as from the perspective of*. Curly is on the opposite side as Moe from the perspective of Larry, and Moe is on the opposite side as Curly from the perspective of Larry.[18]

Given the way the application conditions of these three relations are connected to one another, there are, according to directionalism, three possible ways for each of them to apply to three objects, like Larry, Curly, and Moe. These three manners of application are depicted in figure 4.[19]
The reader can check, by assigning Larry to 1, Curly to 2, and Moe to 3, that these three manners of application correspond to the three left cosets {[1 2 3], [1 3 2]}, {[2 1 3], [2 3 1]}, and {[3 1 2], [3 2 1]} of $Sym_{being\ between}$ in S_3.

Being between is noteworthy because it has more than one converse distinct from it, which undermines the idea, expressed in D2, that the order in which a relation *R*'s converse applies to its relata is *opposite* to that in which *R* does; they are merely *different*. When Larry is between Curly and Moe, *being between* applies in the orders $[l\,c\,m]$ and $[l\,m\,c]$, *being on the far side of from the perspective of* applies in the orders $[c\,l\,m]$ and $[m\,l\,c]$, and *being on the opposite side as from the perspective of* applies in the orders $[c\,m\,l]$ and $[m\,c\,l]$.

18 Donnelly's (2021, 16) interpretations of the three relative properties associated with the predicate "… is between … and …" are similar.

19 In this diagram, a relation applying to x_1, x_2, and x_3 in that order is represented by an arrow going from x_1 to x_2, then to x_3. Light grey arrows depict applications of *being between*, black arrows depict applications of *being on the far side of from the perspective of*, and dark grey arrows depict applications of *being on the side opposite as from the perspective of*.

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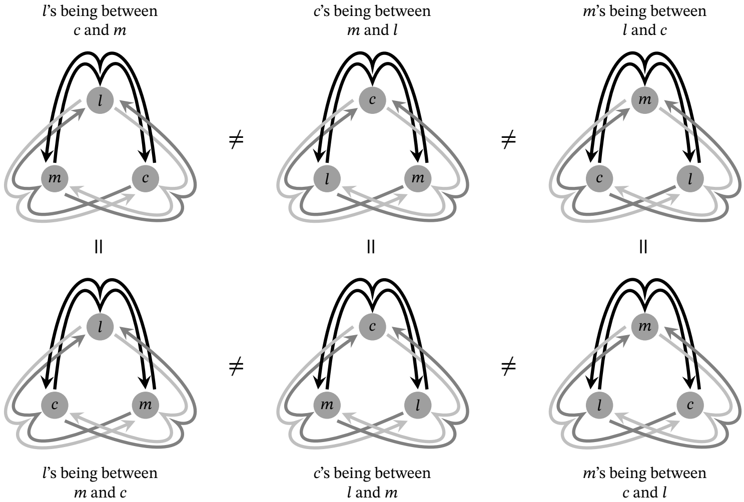

Figure 4: The three possible applications of *being between* and its two distinct converses to Larry, Curly, and Moe

But neither of these two pairs of orders seem to be *opposite* the two orders in which *being between* applies; they appear only to be *different*. It is somewhat plausible that the first and third of these relations apply in opposite orders. It is, after all, Larry who is the one who is privileged in the scenario under consideration (i.e., when Larry is between Curly and Moe). And Larry is at opposite ends of the light grey and dark grey arrows in the leftmost column of figure 4, which depicts this scenario. But neither of the first and third relations could plausibly be understood to apply in orders opposite to those in which the *second* relation (depicted by the black arrow) applies, and the second relation is nonetheless a converse of each of the other two. This, in conjunction with my choice to count every relation—even every completely non-symmetric relation—as its own converse means that the most we can hang onto as far as D2 goes is that, except in cases of completely non-symmetric relations, a relation *R*'s converse (even in the case when it is its own converse) applies to its relata in an order that is *different*, not *opposite*, from the order in which *R* applies to them.

Our discussion of *being between* also helps to illustrate why D3, which covers only (completely) symmetric and (completely) non-symmetric binary relations, needs to be replaced with something, like *p*-CONVERSE IDENTITY, that can accommodate complete non-symmetries and partial symmetries which arise in relations of higher arities. According to D3, a symmetric binary relation is identical to its converse, while a non-symmetric one is distinct from its converse. But a completely non-symmetric n-ary relation, where $n \in \{3, 4, \ldots\}$, will have more than one converse distinct from it, $n! - 1$, to be exact. And a partially symmetric relation will have more than one converse (some factor of $n!$ between 1 and $n!$), though one of those converses will be identical to it. Of completely symmetric relations of any arity, the directionalist can say that it has a single converse, viz., itself.

Consider last the ternary relation *being arranged clockwise in that order*—the relation with a symmetry structure that causes problems for the absolute positionalist. Recall that

$$Sym_{\textit{being arranged clockwise in that order}} = \{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}.$$

By *p*-CONVERSE EXISTENCE, the directionalist would say that *being arranged clockwise in that order* (R for now) has p-converses $R_{[1\,2\,3]}$ ($= R$), $R_{[1\,3\,2]}$, $R_{[2\,1\,3]}$, $R_{[2\,3\,1]}$, $R_{[3\,1\,2]}$, and $R_{[3\,2\,1]}$. And because

(i) $[2\,3\,1] \in Sym_{\textit{being arranged clockwise in that order}}$ and $[2\,3\,1] \circ [1\,2\,3] = [2\,3\,1]$

and

(ii) $[3\,1\,2] \in Sym_{\textit{being arranged clockwise in that order}}$ and $[3\,1\,2] \circ [1\,2\,3] = [3\,1\,2]$,

the directionalist would say, by *p*-CONVERSE IDENTITY, that (i) $R_{[1\,2\,3]} = R_{[2\,3\,1]}$ and (ii) $R_{[1\,2\,3]} = R_{[3\,1\,2]}$ (and so $R_{[2\,3\,1]} = R_{[3\,1\,2]}$). Similarly, because

(i) $[2\,3\,1] \in Sym_{\textit{being arranged clockwise in that order}}$ and $[2\,3\,1] \circ [1\,3\,2] = [2\,1\,3]$

and

(ii) $[3\,1\,2] \in Sym_{\textit{being are arranged clockwise in that order}}$ and $[3\,1\,2] \circ [1\,3\,2] = [3\,2\,1]$,

the directionalist would say, by *p*-CONVERSE IDENTITY, that (i) $R_{[1\,3\,2]} = R_{[2\,1\,3]}$ and (ii) $R_{[1\,3\,2]} = R_{[3\,2\,1]}$ (and so $R_{[2\,1\,3]} = R_{[3\,2\,1]}$). And finally, because

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> there is no permutation $p \in Sym_{being\ arranged\ clockwise\ in\ that\ order}$ such that, e.g., $p \circ [1\,3\,2] = [1\,2\,3]$,

we know by *p*-Converse Identity that $R_{[1\,2\,3]} \neq R_{[1\,3\,2]}$ (and so $R_{[1\,2\,3]} \neq R_{[2\,1\,3]}$, $R_{[1\,2\,3]} \neq R_{[3\,2\,1]}$, $R_{[1\,3\,2]} \neq R_{[2\,3\,1]}$, and $R_{[1\,3\,2]} \neq R_{[3\,1\,2]}$).

By the definition of *p*-converses, this means that *being arranged clockwise in that order* has two converses, one of which is itself. A plausible interpretation of the converse of *being arranged clockwise in that order* distinct from it is *being arranged counterclockwise in that order*.[20] Given the way the application conditions of these two relations are coordinated, there are, according to directionalism, two possible ways for each of them to apply to three objects, like Larry, Curly, and Moe. These three manners of application are depicted in figure 5.[21]

The reader can check, by assigning Larry to 1, Curly to 2, and Moe to 3, that these two manners of application correspond to the two left cosets $\{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}$ and $\{[1\,3\,2], [3\,2\,1], [2\,1\,3]\}$ of $Sym_{being\ arranged\ clockwise\ in\ that\ order}$ in S_3.

3. Directionalism's Advantages Over Its Closest Competitors

I've shown how directionalism avoids Donnelly's charge, in that it is able to properly treat any fixed arity relation with any symmetry such a relation can have. As such, it possesses the same advantage over absolute positionalist theories that is enjoyed by Donnelly's relative positionalism, Fine's (2000) antipositionalism, and MacBride's (2014) relational primitivism. In this section, I describe some advantages that directionalism has over each of these three accounts of relations. First, directionalism, unlike primitivism, supplies an explanation of why a given relation can apply in the ways it can.

20 The name for this relation and the associated predicate are subject to the same issues I mentioned in connection with "*being arranged clockwise in that order*" in fn.13 above. It presupposes a vantage point on one side of the plane in which the objects are arranged, and it makes essential reference to the order of terms with respect to the argument places of the corresponding predicate, in this case "…, …, and … are arranged clockwise in that order." It could be analogously replaced with "*being clockwise behind from the perspective of*" to avoid the latter issue (though not the former).

21 In this diagram, a relation applying to x_1, x_2, and x_3 in that order is represented by an arrow going from x_1 to x_2, then to x_3. Grey arrows depict applications of *being arranged clockwise in that order*, while black arrows depict applications of *being arranged counterclockwise in that order*.

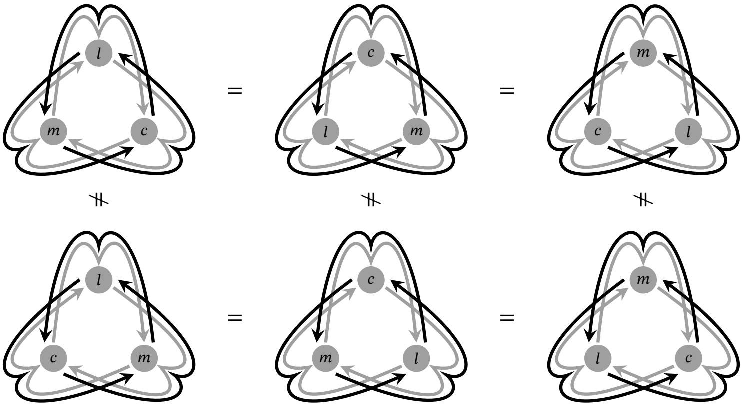

Figure 5: The two possible applications of *being arranged clockwise in that order* and its single distinct converse to Larry, Curly, and Moe

Second, directionalism, unlike antipositionalism and primitivism, supplies an explanation of why two relations can apply in the same or different ways (as the case may be). And third, directionalism, unlike relative positionalism, isn't committed to the involvement of relative properties in every irreducibly relational claim (i.e., in every relational claim which cannot be captured by a claim involving the instantiation of only ordinary non-relative properties). I'll describe each of these advantages in that order, explaining the views along the way as necessary.

Relations can apply in a variety of ways. But *why* is a given relation able to apply in the ways in can? Not all accounts of relations answer this question. Directionalism does. For example, directionalism explains why the binary relation *being next to* can apply to two objects in the single way it can. This is because it can apply to up to two objects (i.e., it is a binary relation), it is its own unique converse, and necessarily, for any x_1 and x_2, if it applies to x_1 and x_2 in that order (i.e., if x_1 is next to x_2), then its converse applies to x_2 and x_1 in *that* order (i.e., x_2 is next to x_1). The binary relation *loving*, on the other hand, can apply to two objects in the two ways it can, according to the

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directionalist, because (i) it can apply to up to two objects, (ii) it has a single converse distinct from it, viz., *being loved by*, and (iii) necessarily, for any x_1 and x_2, if *loving* applies to x_1 and x_2 in that order (i.e., if x_1 loves x_2), then its distinct converse applies to x_2 and x_1 in *that* order (i.e., x_2 is loved by x_1). This ensures that it is possible for *loving* to apply to x_2 and x_1 in that order *whether or not* it applies to x_1 and x_2 in *that* order, and vice versa, yielding two ways in which it can apply to two objects. In general, for any n-ary relation R, R can apply to n objects in the ways it can because R can take up to the number of relata it can, it has the number of converses it does, and the application conditions of it and its p-converses are necessarily connected in the ways that they are.

Contrast this with MacBride's relational primitivism, which is the view that, in general, there is no explanation for why any given relation can apply in the ways it can. It is, according to the primitivist, a matter of brute fact, for example, that *being next to* can apply in the single way it can, and that *loving* can apply in the two ways it can. By refusing to explain such facts, the primitivist avoids postulating any machinery that might treat a relation improperly (as does the absolute positionalist's machinery). Thus primitivism can properly treat any relation directionalism can properly treat. But primitivism has a pro tanto disadvantage compared to directionalism, in that it does not supply an explanation of the behavior of each relation it can properly treat, whereas directionalism does.

Now to directionalism's second advantage. Some relations seem able to apply in the same ways as one another, while others seem able to apply in different ways from one another. Consider *loving* and *hating*. Each of these relations can apply to two objects in two ways. Moreover, they seem to be applicable in the *same* two ways. $Sym_{loving} = Sym_{hating} = \{[1\,2]\}$, and so the two left cosets of each these relations' symmetry groups are the same; they are the two left cosets of $\{[1\,2]\}$ in S_2, viz., $\{[1\,2]\}$ and $\{[2\,1]\}$. The single way in which the binary relation *being next to* can apply to two objects is distinct from each of the two ways in which *loving* or *hating* can do so. That way is represented by the single left coset of $Sym_{being\ next\ to} = \{[1\,2], [2\,1]\}$ in S_2, viz., $\{[1\,2], [2\,1]\}$ itself. Directionalism supplies explanations of the identities and distinctions between the ways any two fixed arity relations can apply to appropriate numbers of objects in terms of the relation's arity, the number of converses it has, and how the application conditions of it and its p-converses are necessarily connected.

In the case of *loving* and *hating*, the directionalist says that these relations can apply in the same two ways because each has arity two, each has two converses, one of which is itself and the other distinct from it, and each is such that, necessarily, for any x_1 and x_2, if it applies to x_1 and x_2 in that order, then its distinct converse applies to x_2 and x_1 in *that* order. The way in which *being next to* can apply to two objects is different from the two ways in which *loving* (or *hating*) can do so, according to directionalism, because, while these relations have the same arity, the former relation is its own only converse, while the latter relation is distinct from one of its converses. As a result, while the latter can apply to two objects in two ways, the former, whenever it applies to x_1 and x_2 in that order, it must, as its own only converse, apply to x_2 and x_1 in *that* order as well, yielding only a single way in which it can apply.

Some relations, while able to apply in the same number of ways, can nonetheless apply in different ways from one another. The ternary relation *being arranged clockwise in that order*, for example, can apply to three objects in two ways. But these two ways are different than the two in which *loving* or *hating* can apply. An intuitive explanation for this is that the first two ways can involve up to three objects, while the latter two can't. In terms of cosets, this can be explained by the fact that the two cosets of $Sym_{\textit{being arranged clockwise in that order}}$ in S_3, viz., $\{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}$ and $\{[1\,3\,2], [3\,2\,1], [2\,1\,3]\}$ are pairwise distinct from the two left cosets that represent the ways *loving* or *hating* can apply. The directionalist can explain these differences by appealing to the fact that *being arranged clockwise in that order* has a different arity than each of *loving* and *hating*.

Some relations have the same arity, apply in the same number of ways, but nonetheless apply in different ways. Such relations, though of the same arity, still have non-isomorphic symmetry groups, and thus the way such relations can apply are still represented by different left cosets. For example, the six ways in which the quaternary *being arranged clockwise in that order*^{4} (as in Alice, Bob, Carol, and Diane are arranged clockwise in that order) can apply to four objects are pairwise distinct from the six ways in which *being closer together than* (as in Alice and Bob are closer together than Carol and Diane) can apply to them.[22] I will not go to the trouble of listing these cosets, but instead just briefly explain why the symmetry groups of these two relations, viz.,

22 As with the ternary version of this clockwise arrangement relation, this name and the associated predicate presuppose a particular vantage point on one side of the plane in which the objects are arranged. See fn.13 above.

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$$Sym_{\text{being arranged clockwise in that order}^4} = \{[1\,2\,3\,4], [2\,3\,4\,1], [3\,4\,1\,2], [4\,1\,2\,3]\}$$

and

$$Sym_{\text{being closer together than}} = \{[1\,2\,3\,4], [1\,2\,4\,3], [2\,1\,3\,4], [2\,1\,4\,3]\},$$

are not isomorphic. This is illustrated by the fact that the latter relation yields the same completion if certain pairs of relata are transposed in its application to them, while the former relation does not. For example,

> Alice and Bob's being closer together than Carol and Diane = Bob and Alice's being closer together than Carol and Diane,

but

> Alice, Bob, Carol and Diane's being arranged clockwise in that order ≠ Bob, Alice, Carol, and Diane's being arranged clockwise in that order

(see Dixon 2019, 68–69 for discussion of this point). These relations have the same number of converses (six). The directionalist will explain these differences in possible applications by appealing to differences in the ways the application conditions of these relations are necessarily connected.[23]

In contrast, neither Fine's antipositionalism nor MacBride's relational primitivism supplies explanations of the identities or differences in the ways distinct relations can apply. The primitivist supplies no explanation for why any

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23 It is worth emphasizing the fact that the explanandum and explanans involved in each of these explanations are distinct. As was hopefully clear in the discussion above concerning the directionalist explanation for why any given n-ary relation R can apply to n objects in the ways it can, the directionalist explains why R can apply in these ways by appealing to R's arity, to the number of converses R has, and to the ways the application conditions of R and its p-converses are necessarily connected. The former fact is distinct from each of these latter facts. The same is going on when explaining why two relations can apply in the same ways (or different ways, as the case may be), except that it involves a comparison between the former and latter sorts of facts for two relations instead of one. The distinctness of the ways in which an n-ary relation R can apply to n objects and the ways in which the application conditions of R and its p-converses are necessarily connected can be further illustrated. The former correspond to the left cosets of Sym_R, as described in section 1, while the latter correspond to the *right* cosets of Sym_R, where

> Definition of Right Cosets of the Symmetry Group of a Relation. For any n-ary relation R, the *right cosets* of Sym_R in S_n are the sets $\{p \circ q : p \in Sym_R\}$ for each $q \in S_n$.

But the left and right cosets of some n-ary relations differ, depending on their symmetry structures. For example, while the ways in which *being between* can apply to three objects are represented by $\{[1\,2\,3], [1\,3\,2]\}, \{[2\,1\,3], [2\,3\,1]\}$, and $\{[3\,1\,2], [3\,2\,1]\}$, the ways in which its application conditions are necessarily connected are best represented by $\{[1\,2\,3], [1\,3\,2]\}, \{[2\,1\,3], [3\,1\,2]\}, \{[2\,3\,1], [3\,2\,1]\}$. The reader can check that these latter three ways are the orders in which *being between* and its two distinct converses apply to Larry, Curly, and Moe when Larry is between Curly and Moe by consulting the left column of figure 4 above. The converses of *being between* must apply to x_1, x_2, and x_3 in certain orders, represented by the right cosets of $Sym_{being\ between}$, exactly when *being between* applies to them in that order. This in turn determines, and, according to the directionalist, explains the ways, represented by the left cosets of $Sym_{being\ between}$, in which *being between* can apply to three objects. The general claim that the ways the application conditions of a relation and its p-converses are necessarily connected correspond to the right cosets of its symmetry group can be shown by considering any n-ary relation R and its q-converse R_q for any permutation q of $\{1, \ldots, n\}$. Whether $q \in Sym_R$ (and so $R_q = R$), or $q \notin Sym_R$ (and so $R_q \neq R$), it follows by the definition of p-Converses that, necessarily, $Rx_1 \ldots x_n$ iff $R_q x_{q(1)} \ldots x_{q(n)}$. By p-Converse Identity, for every $p \in Sym_{R_q}$, necessarily, $R_q x_{q(1)} \ldots x_{q(n)}$ iff $R_{p\circ q} x_{p\circ q(1)} \ldots x_{p\circ q(n)}$. Since Sym_{R_q} and Sym_R are isomorphic (see proof above), for every $p \in Sym_R$, necessarily, $R_q x_{q(1)} \ldots x_{q(n)}$ iff $R_{p\circ q} x_{p\circ q(1)} \ldots x_{p\circ q(n)}$. So the orders in which every converse of R (potentially including itself) applies to $x_1, \ldots, x_n$ exactly when R applies to them in that order constitute one right coset of Sym_R. And because we must consider each of R's q-converse for every permutation q of $\{1, \ldots, n\}$, every right coset of Sym_R contains exactly those orders in which some converse of R (potentially including itself) applies to $x_1, \ldots, x_n$ exactly when R applies to them in that order.

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given relation can apply in the ways it can, and so, ipso facto, can supply no explanation for why two relations can apply in the same or different ways as the case may be.[24] According to antipositionalism, relations do not have positions. What determines the ways in which a given relation can apply to some things—its manners of completion—are not facts about the internal structure of the completions that result from its application. Instead, the ways a relation can apply are determined by identity and distinctness relationships that hold between completions of that relation by different sets of objects. In previous work, I say,

> the manner in which Goethe and Buff complete *loving*... in Goethe's loving Buff is the same, on Fine's view, as exactly one of the two manners in which W. B. Yeats and Maud Gonne complete that relation in Yeats's loving Gonne and Gonne's loving Yeats, and it is distinct from the other. Which identity and distinctness relationships hold of these two possible but mutually exclusive sets of possibilities is, according to the antipositionalist, a matter of brute fact. (Dixon 2019, 65)

Antipositionalism can properly treat any relation that directionalism (and relative positionalism) can treat, as long as any such relation is instantiated by enough distinct sets of objects. But, as I note (see Dixon 2019, 70, fn.17), because Fine defines the identity of manners of completions of relations R and R' only when $R = R'$, the antipositionalist is left without a way to compare manners of completions of distinct relations. Here is Fine's statement of the definition:

> to say that s is a completion of a relation R by $a_1, a_2 \dots, a_m$, in the same manner as t is a completion of R by $b_1, b_2, \dots, b_m$ is simply to say that s is a completion of R by $a_1, a_2 \dots, a_m$ that results from simultaneously substituting $a_1, a_2 \dots, a_m$ for $b_1, b_2, \dots, b_m$ in t (and vice versa). (2000, 25–26)

Moreover, I also note, it is not clear that Fine's definition could be modified in such a way that it could apply when $R \neq R'$. There will be no principled

24 The primitivist might recognize identities and differences between distinct relations' arities, and thus be able to supply the same explanation that the directionalist does of why relations with different arities can apply in different ways. But she will be unable to explain why relations with the same arity that can nonetheless apply in different ways, like *being arranged clockwise in that order*[4] and *being closer together than*, can do so.

way to identify the manner in which a non-symmetric relation *R* applies to some things with any one of the manners in which a distinct non-symmetric relation applies to some other things rather than any of the other ways R' applies to those other things. Why, for example, should Goethe's loving Buff result from simultaneously substituting Goethe and Buff for Yeats and Gonne (and *loving* for *hating*) in Yeats's hating Gonne rather than in Gonne's hating Yeats? Only if this question has an answer will the antipositionalist have a way to explain why the way in which *loving* applies to Goethe and Buff in Goethe's loving Buff is identical to the way in which, say, *hating* applies to Yeats and Gonne in Yeats's hating Gonne and distinct from the way in which *hating* applies to them in Gonne's hating Yeats rather than vice versa. There does not seem to be a non-*ad hoc* way to answer questions like this, and so the antipositionalist seems to be left unable to compare the manners of completions of distinct relations.[25,26]

Directionalism has an advantage over relative positionalism too. Relative positionalism is the view that, when a relation applies to some things, its doing so consists in those things occupying positions of the relation relative to one another. But the positions of a relation are not understood, on relative positionalism, as roles that objects fill, or holes that they occupy, as they are understood on absolute positionalist views. Instead, they are construed as

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25 MacBride (2007, 45–47) raises the issue even for single relations; there is no reason the way in which *loving* applies to Goethe and Buff in Goethe's loving Buff should be identical to the way in which it applies to Yeats and Gonne in Yeats's loving Gonne and distinct from the way in which it applies to them in Gonne's loving Yeats and not vice versa. Admittedly, the antipositionalist may be able to employ the same algebraic analysis of manners of completion as I provide, instead of the substitution-based analysis. And she could accept the idea that the ways in which two distinct *n*-ary relations, such as *loving* and *hating*, can apply to *n* objects are the same, without identifying any pair of ways one of which is a way in which one of the relations can apply while the other is one in which the other can apply. See Dixon (2019, 68, fn.15). But the view faces other problems, e.g., MacBride's (2007, 48; and 2014, 14) objection that the antipositionalist cannot say anything about the ways a relation can apply unless it is instantiated at least twice. See MacBride (2007, sec. 8) and Gaskin and Hill (2012, sec. 3–4) for other objections to antipositionalism.

26 I argue in Dixon (2019) that relative positionalism has these same explanatory advantages over antipositionalism and primitivism, and that they are at least enough to offset the fact that the latter two accounts can accommodate variable arity relations, while relative positionalism cannot. See fn.15. Directionalism, as I have formulated it above, is also unable to handle variable arity relations, and for a perfectly analogous reason that relative positionalism cannot. According to directionalism, some relations with different arities have different numbers of converses, and thus must be distinct. For example, the ternary *being arranged clockwise in that order* has two converses, but the quaternary *being arranged clockwise in that order*^{4} has four. But directionalism's explanatory advantages over antipositionalism and primitivism similarly offset this disadvantage.

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unary *relative properties*, which relata instantiate relative to one another. A relative property is a property that can be instantiated by a thing only relative to a thing or some things, while a non-relative property is a property that can be instantiated by a thing *full stop*. If *being north* is a property, rather than a binary relation, it is presumably a relative property, since something can be north, it would seem, only relative to something or some things. It makes no sense, for example, to say that Washington, D.C. is north. Washington, D.C. is north *relative to something*, such as Kingston, Jamaica. In contrast, many would take a property like *being spherical* to be non-relative. Exceptions to even the latter sort of case are certain endurantists, who regard putative non-relative properties as relative properties that can be instantiated only relative to a time. More on this below.

Structurally, relative positionalism and directionalism are quite similar. The directionalist sees the application of each relation as being order-sensitive, and involving attendant order-sensitive applications of its converse(s). And while the relative positionalist regards each relation as neutral (directionless), she also regards each as having one or more relative properties—equal in number to the number of converses a relation has according to directionalism—which are instantiated by $x_{p(1)}$ relative to $x_{p(2)}$, ..., relative to $x_{p(n)}$ in exactly those orders that the directionalist would have her relation and its distinct converse apply to $x_1, \ldots, x_n$. So according to the relative positionalist, *being next to* has one relative property, which one might interpret as *being adjacent*, that is instantiated by x_1 relative to x_2 and by x_2 relative to x_1 whenever x_1 is next to x_2, yielding only a single way for *being next to* to apply two objects. *Loving*, on the other hand, has two relative properties, *being a lover* and *being beloved*, the first of which is instantiated by x_1 relative to x_2 when x_1 loves x_2 and the second of which is instantiated by x_2 relative to x_1, and vice versa when x_2 loves x_1, yielding two ways for *loving* to apply to two objects. The ternary relation *being between* has three relative properties, resulting in it being able to apply in the three ways discussed in section 1, while the ternary relation *being arranged clockwise in that order* has two, resulting in it being able to apply in the two ways discussed in section 1.[27] Like directionalism, relative

27 For n-ary relations where $n > 2$, the relative properties Donnelly must invoke are, like the two just mentioned in the main text, not instantiated by something relative to just one thing. Instead, they are instantiated by something relative to a thing, relative to a thing, ..., relative to a thing, with the exact number of relativizations equal to $n - 1$. The existence of such *multiply relativized* properties is not wholly implausible. A candidate is that of *closeness*; San Francisco is close relative to (i.e., as compared to) Seattle relative to (i.e., from the perspective of) Los Angeles. In

positionalism can provably properly treat any fixed arity relation with any symmetry such a relation can have (see Donnelly 2016, 94–96).

Directionalism possesses an advantage over relative positionalism in that it is not, while relative positionalism is, committed to the involvement of relative properties in every irreducibly relational claim. An irreducibly relational claim is a claim which cannot be captured by a claim involving the instantiation of only ordinary non-relative properties. For example, the claim that Goethe and Buff are mortal can be captured by the claim that Goethe is mortal and Buff is mortal, which, if it involves the instantiation of properties at all, is most plausibly understood as involving the instantiation of ordinary non-relative properties, viz., the property *being mortal*. I'm putting aside some endurantists' view, mentioned above, that anything that we might have thought is a non-relative property is actually a relative property which can be instantiated only relative to a time. But even the claims I've been discussing at length, like "Goethe is next to Buff" and "Goethe loves Buff," could as easily be regarded as irreducibly relational by such endurantists as by others, since people across that divide think that such claims cannot be adequately paraphrased as claims that involve the instantiation of only non-relative properties.

Relative positionalism's commitment to both relations and relative properties is problematic for the simple reason that it makes that view ontologically less parsimonious than directionalism, as the latter view is committed to only one type of entity, viz., relations. In answer to a different objection, Donnelly (2016, 98–99) considers a version of relative positionalism according to which there *are* no relations, just relative properties; relational predicates are associated immediately with a certain number of relative properties.[28] Adopting

newer work, Donnelly (2021, 13) explicates the instantiation of multiply relativized properties in terms of *embedded standpoints*. According to Donnelly, to embed one object's standpoint within another's "is to supply external structure in terms of which other objects may be, e.g., *front* or *behind*, *closer* or *farther*, *more beloved* or *less beloved*" (2021, 15). From the standpoint of L.A., San Francisco is closer than Seattle. In this example, the standpoint of Seattle is embedded in that of L.A.

28 Donnelly (2016, sec. 5.5) considers the objection that relative positionalism is committed to the primitive relation of relative instantiation, the relation that relative properties stand in to those objects which instantiate them. This relation is to be contrasted with the more familiar non-relative instantiation, the relation that non-relative properties and relations stand in to those objects which instantiate them, to which certain theories of relations are committed. Donnelly concedes that this is a cost of her view, and introduces relationless relative positionalism (see coming discussion in main text) in an effort to answer it. But I think she concedes too much. The matter would be particularly serious if neither of these relations could be defined in terms of the other, thus saddling her view with two *primitive* instantiation relations, in contrast to many other

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this *relationless* relative positionalism would enable the relative positionalist to do away with relations altogether, and be committed to the same number of types of entities as the directionalist. But directionalism possesses an advantage over relationless relative positionalism as well. Directionalism is a theory of non-relative relations only, and makes claims only about *their* application. It explains why a given non-relative relation *R* can apply in the ways it can in terms of the fact that it has a certain number of converses, all of whose application conditions are necessarily connected in a certain way. It says nothing about relative properties. It does not explain the application of non-relative relations in terms of relative properties, and it does not posit relative properties anywhere else. But it is *compatible* with their existence. Directionalism is perfectly compatible with the existence of relational claims that involve the instantiation of relative properties rather than the application of relations; it just won't say anything about *why* these relative properties can be instantiated in the ways they can. That is the job of a theory of relative properties—something which directionalism does not purport to be. Relationless relative positionalism, on the other hand, is committed to the claim that any irreducibly relational claim involves the instantiation of relative properties and not the application of relations.

Thus relationless relative positionalism is compatible with a narrower range of epistemic possibilities than directionalism, and is therefore methodologically inferior in this respect. It is incompatible with the existence of relations, while directionalism is not similarly incompatible with the existence of relative properties. In addition to this, however, there is reason to think that, while some irreducibly relational claims are best understood in terms of the instantiation of relative properties, others are best understood in terms of the application of relations. Jack Spencer (2016) argues that this is the case.

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theories of relations which require only one primitive instantiation relation (see Donnelly 2016, 98). But non-relative instantiation can be defined in terms of relative instantiation as follows:

> Non-Relative Instantiation. $x_1, \ldots, x_n$ instantiate $R =_{df} R$ has between 1 and $n!$ relative properties and (i) each of those relative properties is instantiated by one of $x_1, \ldots, x_n$, relative to another, ..., relative to the remaining one, and (ii) every ordering of $x_1, \ldots, x_n$ is such that at least one of those relative properties is instantiated by the first, relative to the second, ..., relative to the nth. (Adapted from Donnelly 2016, 91.)

Thus the relative positionalist who countenances both non-relative relations and relative properties need only be committed to *one* primitive notion of instantiation—no more than to which many a competing theory of relations is committed.

Spencer is interested in *relativity*, the phenomenon of something's being a certain way relative to a thing or some things.[29] One of Donnelly's examples of a relative property, which I mentioned above, is that something is *north* only relative to a location (or an object in a location). Each example that Spencer has in mind is something that, at least on its face, seems like it can be appropriately construed as the instantiation of a relative property, like *being north*, as Donnelly conceives of it, or of a relative relation like *being closer than*, as in San Francisco's being closer than (i.e., as compared to) Seattle relative to (i.e., from the perspective of) Los Angeles.

Spencer argues that there are at least two ways to cash out talk about relativity, only one of which invokes genuine relative properties (or relative relations). According to the first, *relationalism*, a putative relative property or relation is actually a non-relative relation of greater arity. Instead of there being a genuine relative n-ary property or relation that is instantiated relative to a thing, there is in fact a non-relative $n + 1$-ary relation. *Being north*, on this view, is not a unary property, instantiated relative to a location, but is instead a binary relation, which takes a location as its second argument. According to the second way to cash out talk of relativity, *variabilism*, a relative n-ary property or relation is understood as being genuinely n-ary, and its relativity is captured by the fact that the extension of that property or relation can change when the value of a parameter associated with that property or relation (an *index*) changes. *Being north*, on this view, is a genuine unary property. But its extension function has a location parameter, and can yield different extensions when that parameter takes different values. So, for example, when the location parameter is Lima, Peru, the extension of *being north* includes Kingston, Jamaica, whereas when the location parameter is Washington, D.C., it does not.

On Spencer's account, the difference between relationalism and variabilism, and thus between relative and non-relative properties and relations, is substantive. Relative properties' and relations' extensions vary across parameters, which can take different values, while non-relative properties' and relations' extensions do not, since they don't have such parameters. This means that a relative property or relation is always instantiated relative to at least one thing

29 This is a more general sense of "relativity" than the sort involved in the instantiation of relative properties. As I will discuss, the latter is one way to cash out the former notion. But, as I'll also discuss, there is another way, which invokes only relations and not relative properties. Spencer's notion of relativity is more akin to the irreducible relationality associated with what I've been calling "irreducibly relational claims."

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whenever it is instantiated at all, while a non-relative property or relation is never so instantiated. If instantiation is itself non-relative, then the instantiation relation that n objects stand in to a non-relative n-ary property or relation R can be at most $n + 1$-ary (due to the fact that it will take R as an argument in addition to up to n other arguments). But the instantiation relation that n objects stand in to an n-ary *relative* property or relation R' can be up to $n + k + 1$-ary, where k is the number of parameters relative to which R' may be instantiated. If, on the other hand, the instantiation relation is relative for relative relations and non-relative for non-relative relations, then R and R' will stand in different instantiation relations altogether.

Spencer (2016, 440–444) notes that there are certain tests to which we can subject a putative relative property or relation that can tell us whether it is a genuine example of such an entity, or whether it is in fact a non-relative relation with a higher-than-expected arity. Moreover, these tests deliver examples of both sorts of entity—both genuine relative properties and relations and non-relative relations.[30] The first test Spencer discusses is the *switch-the-index* test. (For simplicity, I'll explain Spencer's tests in terms of relative properties and non-relative binary relations only.) Suppose that x instantiates a property F relative to some putative index i. Now pick a property G that is incompatible with F (i.e., x can't instantiate both F and G relative to the same putative index), and let x instantiate G relative to a different parameter j. If, intuitively, a change in x has taken place, then F and G are genuine relative properties. If, on the other hand, intuitively, no change in x has taken place, then each is a non-relative binary relation.

Consider the examples Spencer uses to illustrate how this test works. David Lewis (1986, 202–204) argues that the endurantist faces a challenge because they are apparently committed to the idea that the very same object is both bent and straight, since they are committed to the view that objects persist by being wholly present at each moment at which they exist. One way of responding to this challenge is to claim that properties like shape are relative, instantiated relative to times (as in Haslanger 1989, 123). That they are in fact relative properties and not disguised binary relations between objects and times can be shown by applying the switch-the-index test. Suppose Lewis instantiates *being bent* at t_1 and *being straight* at t_2. (These two properties are

30 The interested reader can look to Spencer's (2016) paper, which includes treatments of other cases of relativity which I will not discuss. These result in more examples of both relative properties and non-relative relations in addition to the ones I discuss, further substantiating my claim that we have reason to believe that both sorts of entity exist.

incompatible.) Has Lewis undergone a change in properties between these times? Intuitively, yes. So *being bent* and *being straight* are indeed relative properties. Contrast the case of shape with that of size. A big mouse, Remy, is big compared to other mice. But when the average-sized mice surrounding him are replaced with larger animals, say dogs, Remy is no longer big. Has Remy undergone a change through this replacement? Intuitively, no. So *being large* is not instantiated relative to anything, but instead "… is large" expresses a binary relation—presumably something like *being large compared to*—which holds between an object and the objects in certain groups. A difference in properties over time implies genuine change; a thing's simply being related by different relations to different things at different times doesn't. This is what the switch-the-index test is supposed to capture. And it delivers results that imply that both genuine relative properties and non-relative relations exist, assuming that these claims involving shape and size are true and irreducibly relational.

Now to Spencer's second test, the *real similarity* test. Suppose that x instantiates a property F relative to some putative index i. Now switch x out with a different object y, and switch the value of i to a new acceptable value j. If, intuitively, x is exactly similar to y with respect to F, then F is a genuine relative property. If, on the other hand, intuitively, x is *not* exactly similar to y with respect to F, then F is a non-relative binary relation. The rationale for these conclusions is, roughly, that similarity is a matter of sharing properties, not of instantiating relations to different objects (see Spencer 2016, 443). Consider what this test says about the two examples discussed above. Begin by supposing that Lewis instantiates *being bent* at t_1, and then switch Lewis with Haslanger and t_1 with t_2 to yield the result that Haslanger instantiates *being bent* at t_2. Intuitively, Haslanger is exactly similar to Lewis with respect to *being bent*, and therefore *being bent* is a genuine relative property. *Being large*, on the other hand, is a non-relative relation according to the real similarity test. Remy instantiates *being large* relative to mice. Now replace Remy with Jupiter and replace mice with planets of the solar system. Intuitively, Remy is not exactly similar to Jupiter with respect to *being large*. Jupiter is, after all, much larger than Remy.

According to Spencer's account of relativity, there is a real difference between relative properties and relations on the one hand and non-relative properties and relations on the other. And in light of the deliverances of Spencer's tests, I'm happy to grant that relative properties exist. But the relationless relative positionalist is committed to an analysis of *every* instance

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of relativity (i.e., every irreducibly relational claim) in terms of relative properties. Indeed, even the relative positionalist who countenances relations is so committed. Directionalism, on the other hand, can provide an account of relativity in exactly those cases which we have reason to believe involve relations only, and is simply silent in those cases which we have reason to believe involve relative properties only, if any such cases exist.[31]

Consider one of the examples of relativity I have been discussing all along—that connected with someone loving someone. Suppose that Buff is beloved relative to Goethe. Is there a genuine relative property *being beloved*? Or does this case of relativity involve a non-relative relation, *loving*, instead? According to the switch-the-index test, it is the latter that is the case. First, consider the fact that Buff is beloved relative to Goethe. Next, consider the fact that she is not beloved relative to Joseph II. (*Being beloved* and *not being beloved* are incompatible.) However, intuitively, Buff has not undergone a change. To briefly summarize the advantage I have argued directionalism has over relative positionalism: if the relative positionalist countenances relations as well as relative properties, her ontology is more profligate than that of the directionalist. But if she dispenses with relations, then she is forced to posit the involvement of relative properties in relational claims which we have reason to believe involve relations only, while the directionalist is not forced to do the reverse.[32]

31 Spencer's (2016, 441–142, incl. fn.20) view is that variabilists should accept the existence of the corresponding non-relative $n + k$-ary relation along with the relative n-ary property or relation (where k is the number of parameters of the relative property or relation). But whichever way the variabilist decides to go, the relative positionalist, relationless or not, will be in trouble. Even if the variabilist decides to reject the corresponding non-relative relation in cases for which Spencer's tests prescribe a relative relation, this variabilist will still countenance only non-relative relations in cases of relativity for which Spencer's tests prescribe only non-relative relations. And this is incompatible with both varieties of relative positionalism.

32 A believer in relative properties could certainly adopt the view that some apparently irreducibly relational claims actually involve the instantiation of relative properties and not the application of relations, but leave open whether some such claims involve the application of relations and not the instantiation of properties. But this is a different view than relationless relative positionalism, which is committed to the claim that *every* irreducibly relational claim involves the instantiation of relative properties and not the application of relations. The former view is actually the view I prefer, with relations understood as being directed. Spencer's tests will tell us which irreducibly relational claims should be understood to involve the application of (on my view, directed) relations, and which should be understood instead to involve the instantiation of relative properties.

4. Where Directionalism Stands Now

I've shown how directionalism can rise to Donnelly's challenge and properly treat fixed arity relations with any symmetry such a relation can have. Consequently, directionalism has a distinct advantage over absolute positionalist views of relations. Granted, other views, like Fine's antipositionalism, MacBride's relational primitivism, and Donnelly's own relative positionalism can solve this problem as well. But unlike primitivism, directionalism supplies an explanation of why each relation can apply in the ways it can. And unlike both primitivism and antipositionalism, it supplies explanations of why distinct relations can apply in the same or only different ways (as the case may be). Directionalism has an advantage over relative positionalism as well, in that it is not, like relative positionalism, committed to the involvement of relative properties in every irreducibly relational claim.

Still, more remains to be said before we can conclude that directionalism wins the day. I've dealt with only one objection—the problem concerning symmetric relations that Donnelly poses for relative positionalism. But Donnelly gives another objection to directionalism; she charges the directionalist's primitive notion of order-sensitive relational application with being obscure (2016, 82 and 97–98; and 2021, 5–6), since the ordering of a relations' relata by it can't be understood to be "a process which unfolds over time or across space" (2016, 82). She adds,

> [I]t is hard to see how the idea of an order of relational application could be filled out. It is not as though relata are somehow fed into a relation as paper is fed into a printer or wood into a chipper. Relations are not the kinds of things that can "pick up" their relata in a temporal or spatial succession. Perhaps there is some other way for relations to apply to their relata in an order, but no one has tried to explain what this is supposed to be. (2021, 6)

I won't try to explain what order-sensitive relational application is supposed to be, but I'm not as concerned about this as Donnelly is. It's not clear to me that the directionalist is on the hook to provide a general account of this notion, given that relational predicates are themselves order-sensitive. Of course, I doubt Donnelly would be satisfied by this. But this problem strikes me as being no worse than the problem I identified for relative positionalism in the previous section, concerning its commitment to the involvement of relative properties in every irreducibly relational claim. So, other things being equal,

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the two views are at worst on a par. Of course, everything depends on whether other things really are equal between the two views. As I've mentioned, there are other objections to directionalism that still warrant replies, notably Fine's and Williamson's, mentioned in the introduction. There are also important concerns raised by MacBride (e.g., 2014, 5–6) and others. I must leave replies to these objections for another occasion.[33,*]

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33 See Liebesman (2014) and Trueman (2021, ch.10, sec.4) for replies to Fine's objection. See Liebesman (2013) for a reply to Williamson's.

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On Reconciling Positionalism and Antipositionalism

Joop Leo

Positionalism and antipositionalism, two apparently opposing views on relations, give different answers to the question how things can be arranged one way rather than another. In positionalism, relations come with positions to which objects may be assigned; in antipositionalism relations have no positions, but relations consist of a network of complexes interrelated by substitutions. In this paper, a new version of positionalism is proposed, and it is shown that—contrary to what the names suggest—positionalism and antipositionalism are essentially two sides of the same coin.

Abelard's loving Eloise is obviously not the same as Eloise's loving Abelard. A distinguishing feature of non-symmetric relations, like the love relation, is that they admit of *differential application*, i.e., they may apply to the same things in multiple ways. A crucial question is, what makes differential application possible? How can things be arranged one way rather than another?

The answers given depend on the view on relations one adheres to. There are three basic accounts of relations: the *standard view*, the *positionalist view*, and the *antipositionalist view*.

In brief, the standard view says that the arguments of a relation come in a linear order, e.g., Abelard comes first and Eloise comes second in Abelard's loving Eloise. The positionalist view says that a relation comes with positions to which arguments may be assigned, e.g., for the love relation we have the positions *Lover* and *Beloved*. The antipositionalist view says that a relation is a network of complexes interrelated by substitutions, e.g., substituting Anthony for Abelard and Cleopatra for Eloise in Abelard's loving Eloise gives the complex of Anthony's loving Cleopatra.

In his seminal paper "Neutral Relations," Kit Fine made clear that the standard view and the positionalist view give rise to problems (2000). His answer was a new view on relations, the antipositionalist view. However, the antipositionalist view has also been heavily criticized (Donnelly 2016; Gaskin and Hill 2012; MacBride 2007, 2014; Orilia 2011). In my opinion, however, the criticisms arise from a fundamental misunderstanding of the position. In this paper I want to clarify some of the misconceptions. In particular I will show that positionalism and antipositionalism are not really opposite views.

For simplicity I will assume throughout the paper that all relations are of finite degree.

1. Views on Relations

The views presented here contain some aspects that have not been described before. For the positionalist view we make a distinction between thick and thin positionalism, where only in thick positionalism objects may occupy positions.

A note in advance: in Leo (2013), I made a sharp distinction between relational states and relational complexes, and conceived of relational complexes as a structured perspective on relational states. I argued that a state may have more than one corresponding complex. For example, the state of Abelard's loving Eloise corresponds not only with a complex from the binary love relation with two relata, but (among others) also with a complex from the unary relation of loving Eloise with one relatum. For the argumentation in this paper relational states do not play an essential role. However, occasionally I will not only talk about relational complexes but about relational states as well.

1.1. *Standard View*

The standard view assumes that the arguments of a relation always come in a given linear order. For example, in each instance of the love relation one of the arguments comes first and the other comes second. One might also say that relations have a direction. In the instance *aRb* of a relation *R* the relation runs from *a* to *b*, and in *bRa* the relation runs in the opposite direction. Different directions make differential application possible.

A nice feature of the standard view is that it corresponds straightforwardly with natural and most formal languages. For example, for the relation *loves*, we have a direct match with linguistic expressions of the form "___ loves ___."

Unfortunately, there are also problems with the standard view. In the states "out there" there is no linear order or direction between the arguments. The linear order is just a representational artifact. Already in 1913 Russell rejected the idea that all relations have a "natural" direction. For example, this is not the case for right and left, up and down, and greater and less (Russell 1984, 87).

This problem may also be formulated in different terms. The standard view makes it plausible that for each binary relation R there is a *converse relation* R', where aRb holds iff $bR'a$ holds. For example, for the relation *on top of*, we have the converse relation *beneath*, where the state of a's being on top of b is the same as the state of b's being beneath of a. We would like to regard this state as a relational complex consisting of a *single* relation in combination with the two relata. However, this relation can neither be *on top of* nor *beneath*, because there is no good reason to choose one over the other (Fine 2000, 3–4).

1.2. *Positionalism*

According to positionalism, each relation comes with a collection of positions to which objects may be assigned and with no intrinsic order between the positions. Such an assignment results in a relational complex. We distinguish two forms of positionalism: *thick positionalism*, which is the "normal" positionalist view, and *thin positionalism*, a new variant introduced in this paper.

1.2.1. Thick Positionalism

In thick positionalism, a relation comes with positions to which objects may be assigned. Such an assignment may result in a relational complex with objects *occupying* positions.

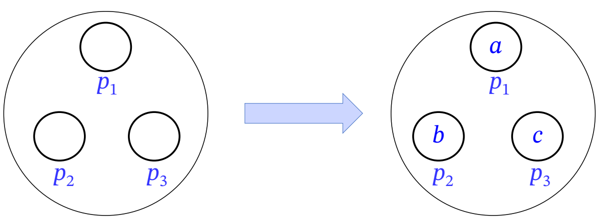

Figure 1: Thick positionalism

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As pointed out by Fine, a problem with this view is that symmetric relations like the adjacency relation have distinct complexes that intuitively should be the same (2000, 17). We would, for example, like to regard *a*'s being next to *b* as the same complex as *b*'s being next to *a*. But suppose that the adjacency relation has two positions *Next* and *Nixt*. Then assigning *a* to *Next* and *b* to *Nixt* gives a complex which is distinct from the complex obtained by assigning *b* to *Next* and *a* to *Nixt* if in the complexes objects occupy positions. In one complex, *a* occupies *Next* and *b* occupies *Nixt*, and in the other complex it is the other way around.[1]

1.2.2. Thin Positionalism

In thin positionalism, a relation comes with positions for which objects may be *substituted*. Such a substitution may result in a relational complex with *occurrences* of the objects involved.

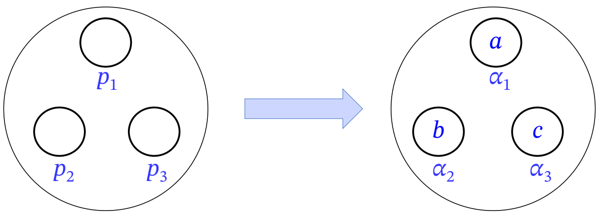

Figure 2: Thin positionalism.

Positions are not boxes in which you can put an object; rather they are substitutable places in a structure or form. The relevance of this distinction can be illustrated with an example.

For the adjacency relation with positions *Next* and *Nixt* substituting *a* for *Next* and *b* for *Nixt* results in a complex with an occurrence of *a* and an occurrence of *b*. The complex is the same as the one that we get when we substitute *b* for *Next* and *a* for *Nixt*. It is as if the positions disappear

1 A proposed way out is to allow objects of a symmetric relation to occupy the same position. This is already done in Russell (1984, 146), and later in Orilia (2011) and in Dixon (2018). Such an approach works for the adjacency relation and many other symmetric relations, but it fails for relations where the objects are arranged clockwise in a circle (Fine 2000, 17, n.10). Another nice example of a relation for which it fails is playing tug-of-war (MacBride 2007, 42–43).

once we assign objects to them.[2] So we don't get too many complexes as in thick positionalism. This makes thin positionalism preferable over thick positionalism.

A relation itself is viewed as an entity and its positions as occurrences of some kind of entity. Though it is not essential, positions might perhaps best be seen as occurrences of arbitrary objects. What is essential is that we may substitute objects for positions. The result of a substitution (if any) is a complex with occurrences of the objects substituted for positions.

The notions of substitution and occurrence are taken as primitive.

In appendix A a general composition principle for substitutions is given. In the principle substitution is conceived of as an operation on occurrences of entities within an entity.

We will assume that thin positionalism endorses the COMPOSITION PRINCIPLE in appendix A.

The COMPOSITION PRINCIPLE does not speak about complexes and positions for which objects may be substituted, but about entities and occurrences of entities for which entities may be substituted. However, because positions are conceived of as occurrences of some kind of entity, and because objects can be substituted for positions, the principle applies in a straightforward way to thin positionalism.

> COMPOSITION PRINCIPLE OF THIN POSITIONALISM. Let s be a substitution of objects for the positions of a relation R resulting in a complex ξ. Then there is a surjective map μ from the positions of R to the occurrences of objects in ξ such that
>
> 1. μ maps every position p to an occurrence of the object substituted by s for p,
> 2. for every substitution s' in ξ, s' results in a complex ξ' iff $\mu \cdot s'$ is a substitution for the positions resulting in ξ',
>
> where $\mu \cdot s'$ denotes the substitution that maps each position p to the object substituted by s' for $\mu(p)$.

If s is taken as a substitution in a complex, then a similar statement holds.

We call μ a *co-map* of substitution s.

2 A comparison could be made with assigning values to variables. Take the formula $x + y = 5$. Then assigning 2 to x and 3 to y results in $2 + 3 = 5$, where in the result the variables are no longer present.

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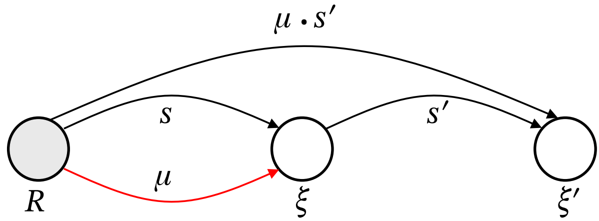

Figure 3: Composition Principle of Thin Positionalism.

The COMPOSITION PRINCIPLE OF THIN POSITIONALISM has the interesting consequence that substitutions in complexes can be derived from the substitutions for the positions and their co-maps.

A single substitution of objects for positions may have more than one co-map. For example, if for a symmetric relation like the resemblance relation substituting an object a for both positions p, p' results in a complex with two occurrences of a, then this substitution has two co-maps; one that maps p to an occurrence α and p' to an occurrence α', and another that maps p to α' and p' to α.

One could in principle allow that a co-map μ is not injective. For example, one could argue that for the love relation with positions *Lover* and *Beloved*, substituting Narcissus for both positions results in a complex with just one occurrence of Narcissus.

If for a given substitution s of objects for positions a co-map μ is not injective, then we say that the substitution results in a *coalescence* of occurrences.

We call a relation *coalescence-free* if it has no coalescence of occurrences. So each complex of an n-ary coalescence-free relation will have n occurrences of objects. If the love relation is coalescence-free, then the complex of Narcissus' loving Narcissus would have one occurrence of Narcissus in the role of lover and another one in the role of beloved.

As we have seen, the adjacency relation is symmetric in a strict sense. Switching the arguments does not change the complex. More generally, we say that R is *strictly symmetric* if there is a non-identity permutation π of its positions such that for every substitution s for the positions resulting in a complex ξ, substitution $\pi \cdot s$ results in ξ as well.[3]

Thin positionalism may appear to be more complicated than thick positionalism. Nevertheless, I think it is a much more natural view than thick positionalism. Having relational complexes in the world as a result of substituting objects for positions seems to make more sense than having complexes "out there" containing objects in a kind of boxes, called positions.

1.3. *Antipositionalism*

Relational complexes have constituents. But this does not necessarily mean that we can directly speak about *how* these constituents occur in a given complex. According to antipositionalism, the structure of a relation can be fully expressed in terms of structure preserving connections between its complexes. There is no need to say anything about the internal structure of the complexes. This may sound a bit vague, so let us look at an example.

For the love relation, one of the complexes could be Paris' loving Helen. In this complex we have one occurrence of Paris and one of Helen. By *substituting* Venus for the occurrence of Paris and Adonis for the occurrence of Helen we get the complex of Venus' loving Adonis. With this substitution corresponds a structure preserving map between the occurrences of Paris and Helen in Paris' loving Helen and the occurrences of Venus and Adonis in Venus' loving Adonis. By taking all possible substitutions into account, we get a network of interrelated complexes.[4]

Networks like this are conceived of as relations. Isomorphic relations are not necessarily identical, as the monadic relations of having a heart and having a kidney make clear.

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3 This definition of strict symmetry is not completely satisfactory in combination with an ontology that is only committed to complexes that actually obtain. In that case, the love relation would according to this definition be strictly symmetric if people would only love themselves. However, by assuming that every substitution resulting in a complex comes with a specific set of one or more co-maps, a more robust definition of strict symmetry can be given by adding the condition that s comes with a co-map μ and $\pi \cdot s$ with a co-map μ' that is distinct from μ. With this addition, the love relation will in no case be labeled as strictly symmetric if every substitution resulting in a complex comes with only one co-map.

4 In Fine's paper "Neutral Relations," objects are substituted directly for objects in a complex, and not for occurrences of objects. However, Fine said (private communication, 2005) that in "Neutral Relations" he was, for simplicity, ignoring the fact that substitution is properly done on occurrences, as is made clear in Fine (1989).

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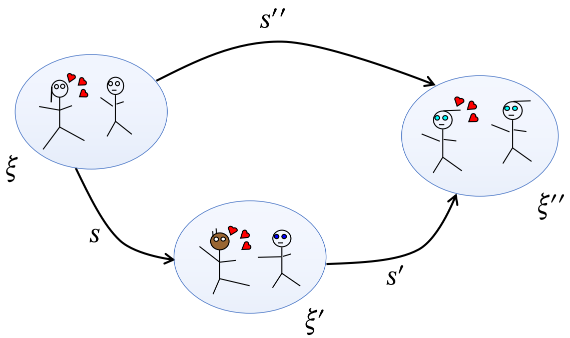

Figure 4: Antipositionalism.

As in thin positionalism, the notions of substitution and occurrence are taken as primitive. Likewise, we assume that antipositionalism endorses the Composition Principle in appendix A.

To make the Composition Principle appropriate for antipositionalism, we only have to make a slight change in terminology. Instead of using a phrase like "a substitution of entities for the occurrences of entities in an entity ξ" we say "a substitution of objects for the occurrences of objects in a complex ξ."[5]

Composition Principle of Antipositionalism. Let s be a substitution of objects for the occurrences of objects in a complex ξ resulting in a complex ξ'. Then there is a surjective map μ from the occurrences of objects in ξ to the occurrences of objects in ξ' such that

1. μ maps every occurrence α in ξ to an occurrence of the object substituted by s for α,
2. for every substitution s' in ξ', s' results in a complex ξ'' iff $\mu \cdot s'$ is a substitution in ξ resulting in ξ'',

5 I do not presuppose that there is a distinction between entities and objects, but it is common to say that a relational complex has (occurrences of) objects as relata.

where $\mu \cdot s'$ denotes the substitution that maps each occurrence α in ξ to the object substituted by s' for $\mu(\alpha)$.

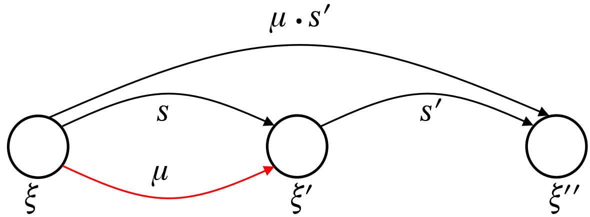

Figure 5: Composition Principle of Antipositionalism.

We call a map μ with this property a *co-map* of substitution s.

We call a complex an *initial complex* if any complex of the relation can be obtained from it by a substitution. If a relation has an initial complex, then it follows from the COMPOSITION PRINCIPLE OF ANTIPOSITIONALISM that for any complex ξ of the relation the substitution in ξ that maps each occurrence α to the object of α results in ξ itself.[6]

More principles could be given. An interesting, but controversial one says that all complexes of a relation are connected via a substitution. This may not hold for certain relations of variable degree, like the relation of *forming a circle*. It is not obvious how to characterize for such relations the unity of its complexes.

Like thin positionalism, antipositionalism does in principle not exclude a *coalescence* of occurrences, i.e., two or more occurrences of objects in a complex may be mapped to the same occurrence of an object in another complex. For example, substituting Narcissus for the occurrence of Paris as well as for the occurrence of Helen in the complex of Paris' loving Helen could result in a complex with one occurrence of Narcissus.

A coalescence of occurrences is very natural for set-like relations. For the relation of *forming a group* we may want the complex for the group consisting

6 To prove this, let ξ_0 be an initial complex and s_0 a substitution in ξ_0 resulting in ξ. If μ_0 is a co-map of s_0, and s a substitution in ξ that maps each occurrence α to the object of α, then $\mu_0 \cdot s$ is the same substitution as s_0. So, by condition 2 of the COMPOSITION PRINCIPLE OF ANTIPOSITIONALISM, s results in ξ itself.

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of Athos, Porthos, and Aramis to have three occurrences and the group of Batman and Robin to have two occurrences. If this is the case, then the second complex may be obtained from the first by a substitution, but there is no substitution the other way around.

Also for the ternary relation *R* where *Rabc* is the complex of *a's loving b and b's loving c* it may seem natural to assume that a coalescence of occurrences can take place. For substituting in *Rabc* the object *a* for *c* gives the complex *Raba*, and substituting in *Rabc* the objects *b*, *a*, *b*, for the occurrences of *a*, *b*, *c* gives the complex *Rbab*. These complexes are obviously empirically indistinguishable, but if a coalescence of occurrences is allowed they can be identical (cf. Leo 2010, 147–148).

It should be noted that not always all complexes in a relation are empirically distinguishable. This is obvious for mathematical relations, but it is also the case for some other relations, like the conjunction of the binary love relation with the unary relation of loving *d*, where *d* is a fixed object.[7] For this relation, the conjunction of *a*'s loving *d* with *d* substitutable and *b*'s loving *d* with *d* fixed is a complex that is distinct from the conjunction of *b*'s loving *d* with *d* substitutable and *a*'s loving *d* with *d* fixed, but the two complexes are empirically indistinguishable (cf. Leo 2013, 364).

Under antipositionalism, different substitutions in a complex may result in the same complex, which is a defining characteristic of *strictly symmetric* relations. For the adjacency relation, for example, we have the complex of *a*'s being adjacent to *b*. Substituting in this complex *b* for (the occurrence of) *a* and *a* for (the occurrence of) *b* gives the same complex. This means that in the network of the relation we have a map from each complex to itself that switches the two objects involved.

One may worry that antipositionalism is less able to identify complexes than positionalism because in antipositionalism we don't have positions with meaningful names like *lover* and *beloved*. However, in antipositionalism we could give occurrences equally meaningful names like *lover in complex* ξ and *beloved in complex* ξ. Besides, names can be freely chosen; in both views on relations the meaning of names do not play a constitutive role.

There are alternative antipositionalist accounts possible. One could, for example, assume that any complex has for each object at most one occurrence.

7 The conjunction of two relations is a relation whose complexes are conjunctions of the complexes of the original two relations. See Leo (2013) for a detailed definition.

Then there is not really a need to talk about occurrences and one can simply substitute objects for objects in complexes.

2. Intertranslating the Views

In this section the translatability from positionalism to antipositionalism and vice versa will be examined. Particular attention will be given to the question whether the translations respect the COMPOSITION PRINCIPLE in appendix A. By examining the translations back and forth, we get a clear picture of the relative expressive power of positionalism and antipositionalism.

2.1. *From Positionalism to Antipositionalism*

Can a positionalist express himself in antipositional terms? We will describe what kind of networks of interrelated complexes a thick and a thin positionalist can construct, and discuss whether these networks are all acceptable for an antipositionalist as networks of relations.

2.1.1. From Thick Positionalism to Antipositionalism

Let us first assume you are a thick positionalist. Let R be a relation with positions $p_1, \dots, p_n$. Then you can simply create a network of interrelated complexes as follows. Let ξ be the complex obtained by assigning $a_1, \dots, a_n$ to $p_1, \dots, p_n$. Identify the pairs $\alpha_i = \langle \xi, p_i \rangle$ with occurrences of objects in ξ. If ξ' is the complex obtained by assigning $b_1, \dots, b_n$ to $p_1, \dots, p_n$, then define the assignment of $b_1, \dots, b_n$ to $\alpha_1, \dots, \alpha_n$ as a substitution in ξ resulting in ξ'.

By repeating the construction for every assignment of objects to the positions of R, you get a network of complexes interrelated by substitutions.

It is easy to verify that the resulting network of complexes satisfies the COMPOSITION PRINCIPLE OF ANTIPOSITIONALISM.

The construction is adequate for non-symmetric relations, but not for symmetric relations since in thick positionalism different assignments of objects to positions always result in different complexes.

A way out could be the use of equivalence classes of complexes to express strict symmetry of relations. The equivalence classes could be identified with what the antipositionalist regards as complexes. There is, however, a complication; not for every relation, occurrences of objects can be defined *non-arbitrarily* in set theory in terms of positions, complexes, and objects. This will be discussed in the last part of this section.

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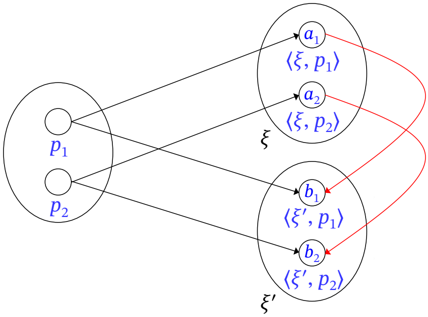

Figure 6: Translating thick positionalism to antipositionalism.

2.1.2. From Thin Positionalism to Antipositionalism

Now assume you are a thin positionalist. Let again R be a relation that comes with a set of positions. Without any adjustment, the complexes of the relation already form a network of complexes interrelated by substitutions—at least, if there are complexes. So, for the translation, we just retain the network of complexes.

The network of complexes satisfies the COMPOSITION PRINCIPLE OF ANTIPOSITIONALISM. But is it always acceptable as a relation for the antipositionalist?

If the relation R is not coalescence-free, then it might happen that not all the complexes are interrelated by substitutions. For example, let R be a ternary relation with only two assignments to its positions p_1, p_2, p_3 resulting in a complex, namely a, a, b and a, b, b, respectively. If the resulting complexes both have only two occurrences, then the complexes cannot be connected via a substitution.

It may be questionable whether an antipositionalist would regard such a network of complexes with unconnected parts as a relation. If not, then a thin positionalist who allows coalescence of occurrences could have relations for which an antipositionalist has no counterpart.

It is also possible that the thin positional relation has no complexes. So also in this case a thin positionalist has relations for which there is no antipositional counterpart.

In all other cases, the relations of the thin positionalist do have an antipositional counterpart.

2.1.3. Identifying Occurrences

As I said in section 1.2, a thick positional relation may have distinct complexes that intuitively should be the same. In translating such a relation to thin positionalism or antipositionalism, we may want to translate such similar complexes to the same complex. If so, then the question is how to define the occurrences of objects for the reconstructed complexes. In particular, we may ask whether the occurrences can be defined in a *non-arbitrary way* in terms of the positions, complexes, and objects of the original or the reconstructed relation.

If in the reconstructed complexes each object occurs at most once, then occurrences may simply be defined as ordered pairs $\langle \xi, a \rangle$, with ξ a reconstructed complex and a an object. But if we want the reconstructed relation to be coalescence-free, we have to distinguish different cases.

For coalescence-free relations without strict symmetry, we can define occurrences in a complex ξ as ordered pairs $\langle \xi, p_1 \rangle, \dots, \langle \xi, p_n \rangle$, with $p_1, \dots, p_n$ the positions of the relation. This is the translation depicted above in figure 6.

For coalescence-free relations with complete strict symmetry, we can define the occurrences of an object a in a complex ξ as triples $\langle \xi, a, 1 \rangle, \dots, \langle \xi, a, k \rangle$, where k is the number of positions to which a is assigned to obtain ξ.

However, for some other strictly symmetric coalescence-free relations, we cannot define occurrences for certain complexes in a non-arbitrary way in terms of positions, complexes and objects within the context of set theory. This is, for example, the case for a quaternary cyclic relation for which the complexes may be depicted as four objects equally spaced on a circle and such that rotating them over 90° gives the same complex.

The proof is given in appendix B.2.

2.2. *From Antipositionalism to Positionalism*

The name "antipositionalism" suggests that the view is against positions, but it is certainly not against a reconstruction of this notion within the confines of its theory.

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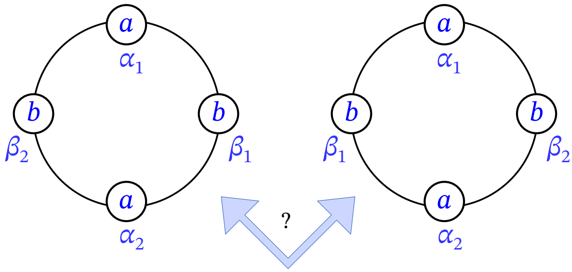

Figure 7: Occurrences cannot be reconstructed in a non-arbitrary way.

2.2.1. Reconstructing Positions

According to Fine (2000, 29) the antipositionalist can reconstruct positions as abstracts with respect to the equivalence relation *co-positionality*, where object a in state s is *co-positional* to object b in state t if s results from t by a substitution in which b goes into a (and vice versa). But this reconstruction is not satisfactory for cyclic relations, where the objects are arranged clockwise in a circle, because for such relations all objects in a state are co-positional with each other, and therefore we would get just one position (Leo 2008a, 357).

Here we will follow a different approach. Let R be an antipositional relation with an initial complex ξ_0 (i.e., a complex from which any complex of the relation can be obtained by a substitution). Then we could treat the occurrences of objects in ξ_0 as positions, but there are more elegant approaches; one makes use of abstraction and the other of subtraction.

Suppose that we may *abstract* from the nature of the objects of the occurrences. Then, by simultaneously abstracting in ξ_0 from the nature of the objects of all occurrences, we get a kind of skeleton complex.[8] What remains of the occurrences an antipositionalist may call the positions of the relation.

Instead of abstracting from the nature of the objects of the occurrences, we may perhaps also simultaneously *subtract* the objects from the occurrences. If so, then the result is again a skeleton complex with "empty" occurrences that can be taken as positions.

8 Abstracting from the nature of the objects may be understood as a Cantorian abstraction (cf. Fine 1998).

In my view the operation of abstraction and the operation of subtraction are both quite natural. It's hard to say what is the best choice. An advantage of abstraction is that it does not necessarily commit you to the existence of additional entities. It may be seen as just a way of speaking about a class of complexes (cf. Russell 2009, 33–34).[9] In favor of subtraction it may be argued that substitution is in fact a two-step operation, where in step one objects are subtracted and in step two objects are added. If so, then subtraction is an operation we implicitly already had.

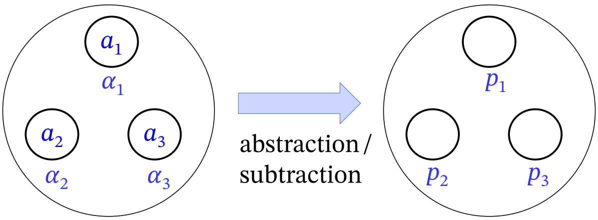

Figure 8: Translating antipositionalism to positionalism.

2.2.2. From Antipositionalism to Thick Positionalism

We start with an antipositional relation R with an initial complex ξ_0, and assume that the operation of abstraction or subtraction yields a skeleton complex ζ with reconstructed positions, each corresponding with exactly one occurrence of an object in ξ_0. Then there is a bijection π from the occurrences in ξ_0 to the positions in ζ.

For an assignment f of objects to the positions, we define as resulting complex (if it exists) the complex obtained by the substitution $\pi \cdot f$ in ξ_0 together with the positions being occupied by the assigned objects.

The translation may give more complexes than in the original relation. For example, if R is the adjacency relation, then the corresponding positional relation has two positions p_1, p_2, and for a's being adjacent to b it has two complexes, one with p_1, p_2 being occupied by a, b, and another with p_1, p_2 being occupied by b, a.

9 The occurrences of objects in an initial complex can collectively be used as a representation of the positions, and all such representations together form a non-arbitrary representation of the collection of positions. But it should be noted that, as a consequence of what is proved in appendix B.3, it is not always possible for an antipositionalist to identify the positions individually in a non-arbitrary way with an equivalence class.

2.2.3. From Antipositionalism to Thin Positionalism

For translating antipositionalism to thin positionalism, we follow the same route, except that we simply use the original complexes as the complexes for the positional relation. So we start again with an initial complex ξ_0, and we assume that by abstraction or subtraction we obtain reconstructed positions and a corresponding bijection π from the occurrences in ξ_0 to the positions. Then, for any assignment f of objects to the positions, define as resulting complex (if it exists) the complex obtained by substitution $\pi \cdot f$ in ξ_0.[10]

This completes the translation. To be acceptable for a thin positionalist, the reconstructed relation must satisfy the Composition Principle of Thin Positionalism.

This can be proved as follows. Let ξ_0, π be as in the translation, and let f be a substitution of objects for the reconstructed positions resulting in a complex ξ. Then substitution $\pi \cdot f$ in ξ_0 results in ξ as well. Let μ be a co-map of $\pi \cdot f$. Then, by the Composition Principle of Antipositionalism, for every substitution s' in ξ,

> s' results in an entity ξ' iff $\mu \cdot s'$ is a substitution in ξ_0 resulting in ξ'.

By the reconstruction of the positional relation, $\mu \cdot s'$ is a substitution in ξ_0 resulting in ξ' iff $\pi^{-1} \cdot (\mu \cdot s')$ is a substitution for the positions resulting in ξ'. So, because $\pi^{-1} \cdot (\mu \cdot s') = (\pi^{-1} \cdot \mu) \cdot s'$,

> s' results in ξ' iff $(\pi^{-1} \cdot \mu) \cdot s'$ is a substitution for the positions resulting in ξ'.

From this fact and the observation that $\pi^{-1} \cdot \mu$ is a surjective map from the positions to the occurrences of objects in ξ mapping each position p to an occurrence of the object substituted by f for p, it follows that $\pi^{-1} \cdot \mu$ is a co-map of f. This completes the proof.

If a relation has more complexes from which all of its complexes can be obtained by substitution, then any of them could be chosen for abstracting from the nature of the objects of the occurrences. As you might expect, the reconstruction of a positional relation is essentially independent of the choice of ξ_0. More specifically, the reconstructed sets of positions may perhaps be

10 Although $\pi \cdot f$ is just a map from the occurrences in ξ_0 to objects, I identify it here with a substitution in ξ_0.

different for different choices of ξ_0, but it is not difficult to show that the reconstructed relations are all the same up to isomorphism.

Nevertheless, there is a subtle complication; in set theory the positions cannot always be reconstructed "neutrally," i.e., without an arbitrary choice in terms of the basic ingredients of antipositionalism. This will be shortly discussed at the end of this section.

A serious restriction of the given reconstructions is that it only works for relations with an initial complex. But there might be more sophisticated reconstructions that also work for certain relations without initial complexes. However, for relations with a variable number of objects in different instantiations, like the relation of *forming a circle*, there may not be equivalent positional relations. This might mean that antipositionalism is a richer theory that offers more possibilities than positionalism.

2.2.4. Identifying Positions

An interesting question is whether for any antipositional relation with an initial complex a reconstruction of positions can be made with no arbitrary choices.

For relations without strict symmetry a non-arbitrary reconstruction of positions is possible. We can, for example, identify a position for such a relation with the equivalence class of occurrences of objects in initial complexes that can be mapped to each other by co-maps.

For strictly symmetric relations this reconstruction does not work. For some strictly symmetric relations there is simply no reconstruction of positions possible in set theory without an arbitrary choice. This is, for example, the case for a quaternary cyclic relation for which the complexes may be depicted as four objects equally spaced on a circle and such that rotating them over 180° gives the same complex, but rotating them over 90° gives a different complex when the objects are not all the same.

The proof that for this relation no non-arbitrary reconstruction of positions is possible is given in appendix B.3.

2.3. *Translations Back and Forth*

That positionalism and antipositionalism are translatable into each other is nice, but it doesn't say that much. With translations relevant information can in principle get lost. Therefore it is very interesting to investigate if translations

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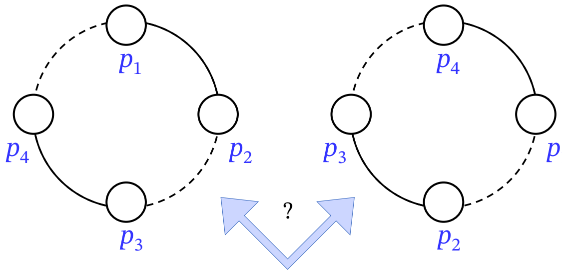

Figure 9: Positions cannot be reconstructed in a non-arbitrary way.

back and forth yield a structure that is isomorphic to the original relation. If this is the case, then the translation is really good.

First we translate back and forth starting from a positional relation, and then we translate back and forth starting from an antipositional relation.

2.3.1. From Positionalism to Antipositionalism and Back Again

We have the following results:

> Claim 1. For a thick positional relation, the translation to antipositionalism and back gives a reconstructed relation that is the same as the original relation, up to isomorphism.

This is easy to see. The translation to antipositionalism gives a coalescence-free network of reconstructed complexes without any strict symmetry, where the reconstructed complexes correspond one-to-one with the original complexes. By translating it back to thick positionalism we get a structure of reconstructed complexes and positions that matches the original relation, up to isomorphism.

> Claim 2. For a thin positional relation with at least one coalescence-free substitution for the positions, the translation to antipositionalism and back gives a reconstructed relation that is the same as the original relation, up to isomorphism.

We may prove this claim as follows. The translation to antipositionalism retains all complexes and the substitutions between them. Because the original relation has at least one coalescence-free substitution for the positions, it has an initial complex ξ_0. We use ξ_0 for the reconstruction of the positions. Then, for some bijection τ from the reconstructed positions to the original positions, any s with co-map μ is a substitution for the reconstructed positions iff $\tau \cdot s$ with co-map $\tau \cdot \mu$ is a substitution for the original positions. This proves the claim.

2.3.2. From Antipositionalism to Positionalism and Back Again

A minimal requirement for a translation of an antipositional relation R to positionalism and back again to result in essentially the same relation as the original one is that R has an *initial complex*, i.e., a complex from which any complex of the relation can be obtained by a substitution.

> Claim 3. For an antipositional relation with at least one initial complex, the translation to thick positionalism and back gives a reconstructed relation that is the same as the original relation, up to isomorphism if and only if the original relation is coalescence-free and without any strict symmetry.

We prove this as follows. Assume that R is an antipositional relation with an initial complex ξ_0. Furthermore assume that R is coalescence-free and without any strict symmetry. Translating R to thick positionalism gives complexes being a combination of the original complexes and positions being occupied by the assigned objects. Because R is without any strict symmetry, these reconstructed complexes correspond one-to-one with the original complexes. Because R is coalescence-free, translating back to antipositionalism gives a network of complexes in which the complexes have occurrences that correspond one-to-one to the occurrences in the complexes of R. From this it follows that the reconstructed relation is the same as the original relation, up to isomorphism.

The "only if" part of the claim follows because the translation of a thick positional relation to antipositionalism always gives a coalescence-free relation without any strict symmetry. This completes the proof.

> Claim 4. For an antipositional relation with at least one initial complex, the translation to thin positionalism and back gives a reconstructed relation that is the same as the original relation.

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The proof is straightforward. By translating from antipositionalism to thin positionalism, the original complexes and the substitutions between them are fully retained. Translating back to antipositionalism gives as a result again the original relation.

3. Conclusion

In this paper we compared positionalism and antipositionalism. The main conclusion is that, contrary to what the names suggest, the views are not really opposites of each other. In fact, a specific form of positionalism, which I called thin positionalism, is very similar to antipositionalism.

In thin positionalism as well as in antipositionalism substitution is taken as a primitive operation. In thin positionalism we have substitution of objects for positions of a relation, and in antipositionalism we have substitution of objects for occurrences of objects in relational complexes.[11] Substitution is in both cases used to characterize the structure of a relation.

As we have seen, the translations back and forth show that there is a very close relationship between thin positionalism and antipositionalism. The class of thin positional relations with at least one coalescence-free assignment of objects to its positions matches perfectly with the class of antipositional relations with at least one initial complex; they are translatable into each other without any loss of information.

In summary, the relationship between thin positionalism and antipositionalism may be expressed as follows:

1. both views rely upon the notion of substitution, which I regard as a fundamental operation for expressing relatedness between complexes;
2. the main difference between the views is that in positionalism the relatedness between complexes is expressed via positions and in antipositionalism it is expressed directly between complexes;
3. the views are for a significant range of relations translatable into each other in a natural way with complete preservation of structure.

What about the standard view? Relations of the same significant range could also be translated from the standard view back and forth to positionalism and antipositionalism. However, in this case the end result is not necessar-

11 In thin positionalism we also have substitutions between complexes, but, as we saw, a thin positional relation is completely determined by the substitutions for the positions.

ily isomorphic with the original relation. The reason is that in translating from the standard view to positionalism or antipositionalism some constitutive information—namely the order of the arguments—is lost. This puts the standard view apart from positionalism and antipositionalism.

It may go too far to say that thin positionalism and antipositionalism are essentially the same. In thin positionalism, a relation is seen as a universal and positions belong to the fundamental furniture of the world, whereas in antipositionalism no ontological commitment to relations as universals and to positions is needed.[12]

Because antipositionalism is apparently less demanding with respect to ontological commitments, I am inclined to regard it as the preferable view. Furthermore, a strong feature of antipositionalism is that it may accept relations with a variable number of objects involved in the complexes, as in the relation of *forming a group* and *forming a circle*, for which the positionalist may have no equivalent counterpart.

But there may perhaps be reason for not jumping to the conclusion that antipositionalism is in every way superior, because a positionalist may accept relations with no complexes and relations for which the translation to antipositionalism yields an unconnected network of complexes. Such relations may be unacceptable for an antipositionalist.

Despite the differences, I consider the agreement between positionalism and antipositionalism as fundamental. The analysis given in this paper shows that the views are essentially two sides of the same coin. Therefore I regard the name “antipositionalism” as misleading. A better name might be “apositionalism.”

A. Substitution Principles

In (1989, 235–238), Fine made a start for developing a *general theory of constituent structure*. The key notion of the theory is the operation of *substitution*. A substitution takes an entity ξ and a map from the *occurrences* of entities in ξ to entities as input, and gives an entity as a result (if any).

Fine gave the following example of a basic principle for the theory:

12 As Kit Fine pointed out to me, whether this means that the two views are genuinely distinct depends upon one’s willingness to draw a distinction between a kind of entity being basic or derivative within one’s ontology.

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> If F' is the result of substituting E' for the occurrence e of E within F, then there is an occurrence e' of E' within F' such that the result of substituting any expression E'' for e' within F' is identical to the result of substituting E'' directly for e in F. (1989, 236)

The notions of substitution and occurrence are taken as primitive.

Because we may simultaneously substitute entities for occurrences, I propose the following more general principle.

> Composition Principle. Let s be a substitution in an entity ξ resulting in an entity ξ'. Then there is a surjective map μ from the occurrences of entities in ξ to the occurrences of entities in ξ' such that

1. μ maps every occurrence α in ξ to an occurrence of the entity substituted by s for α,
2. for every substitution s' in ξ', s' results in an entity ξ'' iff $\mu \cdot s'$ is a substitution in ξ resulting in ξ'',

> where $\mu \cdot s'$ denotes the substitution that maps each occurrence α in ξ to the entity substituted by s' for $\mu(\alpha)$.

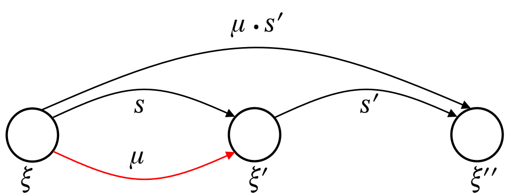

Figure 10: Composition Principle for Substitutions.

We call a map μ with this property a *co-map* of substitution s.

B. Neutral Reconstructions

In this appendix we show two things: (1) for some relations, occurrences of objects cannot be reconstructed set theoretically in a non-arbitrary way in terms of basic notions of thick positionalism, and (2) for some relations, positions cannot be reconstructed set theoretically in a non-arbitrary way in terms of basic notions of antipositionalism.

We will not give a precise definition of non-arbitrariness, but we will give a formal definition of *neutrality* that obviously any non-arbitrary construction in set theory should fulfill. This notion of neutrality, which was introduced in Leo (2008b), is interesting in its own right since it may be more generally applicable for showing that certain things cannot be modeled in set theory in a non-arbitrary way.

All reconstructions in this appendix are understood to be within the context of standard set theory with urelements. Other modeling media may provide more possibilities.

B.1. *The Notion of Neutrality*

I will define neutrality in the context of set theory with urelements A. The idea is as follows. Let X and Y be sets. Suppose that Y is constructed in a non-arbitrary way on the basis of X. Let π be a permutation of the urelements for which replacing in X each occurrence of each urelement a by $\pi(a)$ doesn't change the set. Then—since all urelements are set-theoretically indiscernible—replacing in Y each occurrence of each urelement a by $\pi(a)$ doesn't change this set either.

If Y has the property that each permutation of the urelements that keeps X unchanged also keeps Y unchanged, then we say that Y is *neutral* with respect to X.

The notion of neutrality may in principle be used to show that certain things cannot be constructed in a neutral way with respect to other things, and we will do that in the next sections, but first we give a formal definition of neutrality.

Let $\mathrm{V}[A]$ be the cumulative hierarchy with urelements A. Any function $u : A \to A$ can be lifted to a function $\tilde{u} : \mathrm{V}[A] \to \mathrm{V}[A]$ in an obvious way:

$$\tilde{u}(a) = u(a) \text{ for any } a \in A,$$

$$\tilde{u}(X) = \{\tilde{u}(x) \mid x \in X\}.$$

We may regard $\tilde{u}(X)$ as the result of replacing in X each occurrence of each urelement a by $u(a)$.

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Definition B.1. For $X, Y \in \mathsf{V}[A]$ we say that Y is *neutral with respect to* X if for any bijection $u : A \to A$,

$$\tilde{u}(X) = X \;\Rightarrow\; \tilde{u}(Y) = Y.$$

So if $A = \{a, b\}$, then any set in $\mathsf{V}[A]$ is neutral with respect to $\{a\}$, but $\{a\}$ is not neutral with respect to $\{a, b\}$.

I do not claim that the definition of neutrality completely characterises non-arbitrariness of a set-theoretic construction, but it should be clear that any non-arbitrary construction of Y on the basis of X will be neutral with respect to X.

B.2. *Reconstructing Occurrences*

We will show that not for every positional relation the occurrences of objects in the complexes can be neutrally reconstructed in terms of positions, complexes, states, and objects.

Let R be a positional relation for which the states may be depicted as four not necessarily distinct objects equally spaced on a circle and such that rotating them over 90° gives the same state.

A set-theoretical positional model for R is a tuple $\mathcal{M} = \langle C, S, O, P, \Gamma, \Omega\rangle$, with complexes C, states S, objects O, positions P, a map Γ from O^P to C, and a map Ω from C to S, where Γ maps assignments of objects to positions to complexes, and Ω maps complexes to their corresponding states.[13]

We assume that C, S, O, and P are mutually disjoint sets of urelements, and that O has at least four objects.

The symmetry of R can be expressed in terms of the model $\mathcal{M}$ as follows.

The set P can be written as $\{p_1, p_2, p_3, p_4\}$ such that for the permutation group G generated by the map taking p_1, p_2, p_3, p_4 to p_4, p_1, p_2, p_3, we have for every $f, g \in O^P$, $\Omega(\Gamma(f)) = \Omega(\Gamma(g))$ iff $g = f \circ \pi$ for some $\pi \in G$.

Let us now try to reconstruct a coalescence-free thin positional or antipositional model for R with the same states as in R and for each state just *one* corresponding reconstructed complex.

For every reconstructed complex ξ with four distinct objects a, b, c, d we may define its occurrences non-arbitrarily as pairs $\langle \xi, a\rangle, \langle \xi, b\rangle, \langle \xi, c\rangle, \langle \xi, d\rangle$, but, if each complex has four occurrences, then no neutral reconstruction of all occurrences is possible with respect to $\mathcal{M}$. This can be shown as follows.

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13 O^P denotes the set of functions from P to O.

Select two objects a and b. Let $\delta : O \to O$ switch the objects a and b and leave all other objects unchanged. Define $u : C \cup S \cup O \cup P \to C \cup S \cup O \cup P$ by:

$$u(x) = \begin{cases} \delta(x) & \text{if } x \in O, \\ \Gamma(\delta \circ f) & \text{if } x = \Gamma(f) \text{ for some } f \in O^P, \\ \Omega(\Gamma(\delta \circ f)) & \text{if } x = \Omega(\Gamma(f)) \text{ for some } f \in O^P, \\ x & \text{otherwise.} \end{cases}$$

It is not difficult to see that u is a bijection, $u \circ u = \mathrm{id}_{C \cup S \cup O \cup P}$, and $\tilde{u}(\mathcal{M}) = \mathcal{M}$.

Let *Rabab* be the state with objects a, b, a, b arranged in a circle (in that very order) and let $\alpha_1, \beta_1, \alpha_2, \beta_2$ be the occurrences of a, b, a, b in the corresponding reconstructed complex ξ_{abab}.

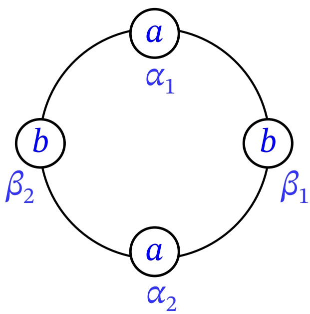

Figure 11: The occurrences cannot be reconstructed in a neutral way.

Now suppose that the occurrences are neutrally reconstructed with respect to $\mathcal{M}$. More specifically, suppose R has a coalescence-free thin positional or antipositional reconstruction $\mathcal{N}$ in $V[C \cup S \cup O \cup P]$ such that $\tilde{u}(\mathcal{N}) = \mathcal{N}$.

Then, because $u(Rabab) = Rabab$ and u switches a and b, $\tilde{u}(\alpha_1)$ must be an occurrence of b in the reconstructed complex ξ_{abab}. So, either $\tilde{u}(\alpha_1) = \beta_1$ or $\tilde{u}(\alpha_1) = \beta_2$.

Let c, d, e, f be distinct objects in O and let ξ_{cdef} be the complex obtained by substituting c, d, e, f for $\alpha_1, \beta_1, \alpha_2, \beta_2$ in ξ_{abab}. From $u(Rabab) = Rabab$ and

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$\tilde{u}(\mathcal{N}) = \mathcal{N}$ it follows that substituting c, d, e, f for $\tilde{u}(\alpha_1), \tilde{u}(\beta_1), \tilde{u}(\alpha_2), \tilde{u}(\beta_2)$ in ξ_{abab} results in ξ_{cdef} as well.

From this it follows that $\tilde{u}$ must preserve the relative order of the occurrences in ξ_{abab}. This means that either

$$\tilde{u} \text{ maps } \alpha_1, \beta_1, \alpha_2, \beta_2 \text{ to } \beta_1, \alpha_2, \beta_2, \alpha_1$$

or

$$\tilde{u} \text{ maps } \alpha_1, \beta_1, \alpha_2, \beta_2 \text{ to } \beta_2, \alpha_1, \beta_1, \alpha_2.$$

In both cases $\tilde{u}(\tilde{u}(\alpha_1)) \neq \alpha_1$. But, since $\tilde{u} \circ \tilde{u} = \widetilde{u \circ u}$,[14] this contradicts that $u \circ u = \mathrm{id}_{C \cup S \cup O \cup P}$.

So we conclude that if each state has just one reconstructed complex and each complex has four occurrences, then the occurrences cannot be neutrally reconstructed with respect to $\mathcal{M}$.

B.3. *Reconstructing Positions*

In a similar way as we did for occurrences, we can prove that not for every relation positions can be neutrally reconstructed in terms of the notions of antipositionalism.

I will show this again for a cyclic relation, but not for the same one. For an antipositional relation that holds of objects a, b, c, d when a, b, c, d are arranged in a circle (in that very order) it is possible to reconstruct the positions in a non-arbitrary way. I leave this as an exercise.[15]

Let R be an antipositional relation for which the states may be depicted as four distinct objects equally spaced on a circle and such that rotating them over 180° gives the same state, but rotating them over 90° does not give the same state.

We assume that each state of R has just one corresponding complex.

The symmetry of R can be expressed as follows.

For every complex ξ the occurrences of objects can be written as $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ such that for the permutation group G generated by the map

14 More generally, for functions $u, v : A \to A$, with A a set of urelements, $\tilde{u} \circ \tilde{v} = \widetilde{u \circ v}$. We prove this by $\in$-induction: (i) If $x \in A$, then $\widetilde{u \circ v}(x) = u \circ v(x) = u \circ \tilde{v}(x) = \tilde{u} \circ \tilde{v}(x)$. (ii) Let $x \in \mathrm{V}[A]$ and assume $\widetilde{u \circ v}(z) = \tilde{u} \circ \tilde{v}(z)$ for every $z \in x$. Then $\widetilde{u \circ v}(x) = \{\widetilde{u \circ v}(z) \mid z \in x\} = \{\tilde{u} \circ \tilde{v}(z) \mid z \in x\} = \tilde{u}(\{\tilde{v}(z) \mid z \in x\}) = \tilde{u} \circ \tilde{v}(x)$. So, by $\in$-induction, $\widetilde{u \circ v} = \tilde{u} \circ \tilde{v}$.

15 A clue to the solution can be found in Example 6.5 of Leo (2008b).

taking $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ to $\alpha_3, \alpha_4, \alpha_1, \alpha_2$, we have for every substitution s, t in ξ, if s results in a complex, then $\xi \cdot s = \xi \cdot t$ iff $s = \pi \cdot t$ for some $\pi \in G$.

In COROLLARY 7.8 of Leo (2008b) it is shown that for this relation positions cannot be neutrally reconstructed in terms of the notions of antipositionalism if substitution is directly done on objects. Here we show that it is also impossible when substitution is done on occurrences.

A set-theoretical antipositional model for R is a tuple $\mathcal{M} = \langle C, S, O, \mathrm{Oc}, \Pi, \Theta, \Omega \rangle$, with complexes C, states S, objects O, occurrences Oc, a map Π from Oc to O, a partial map Θ from $C \times O^{\mathrm{Oc}}$ to C, and a map Ω from C to S, where Π maps occurrences to their objects, Θ represents the substitutions in complexes of objects for occurrences, and Ω maps complexes to their corresponding states.

We assume that C, S, O, and Oc are mutually disjoint sets of urelements, and that each occurrence occurs in only one complex. Furthermore, we assume that O has at least four objects, and that R holds for any selection of four distinct objects in O in any order, but not for any other selection.

We call two states *siblings* if each can be obtained from the other by rotating the objects over 90°. Furthermore, we call two complexes *siblings* if their corresponding states are siblings, and we call two occurrences *siblings* if they are occurrences of the same object in complexes that are siblings. Note that by our assumptions each state, each complex, and each occurrence has exactly one sibling.

Define $u : C \cup S \cup O \cup \mathrm{Oc} \to C \cup S \cup O \cup \mathrm{Oc}$ by:

$$u(x) = \begin{cases} \text{sibling of } x & \text{if } x \in S, \\ \text{sibling of } x & \text{if } x \in C, \\ \text{sibling of } x & \text{if } x \in \mathrm{Oc}, \\ x & \text{otherwise.} \end{cases}$$

It is not difficult to see that u is a bijection, $u \circ u = \mathrm{id}_{C \cup S \cup O \cup P}$, and $\tilde{u}(\mathcal{M}) = \mathcal{M}$.

Let $P = \{p_1, p_2, p_3, p_4\}$ be reconstructed positions for R and let Ψ be a partial map from O^P to S that maps assignments of objects to positions to corresponding states. We may assume that the assignment of distinct objects a, b, c, d to p_1, p_2, p_3, p_4 results in the same state as the assignment of c, d, a, b to p_1, p_2, p_3, p_4. We denote this state as $Rabcd$.

Now suppose that P and Ψ are neutral with respect to $\mathcal{M}$.

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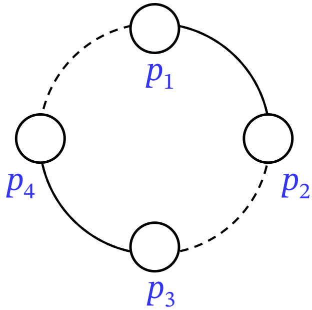

Figure 12: The occurrences cannot be reconstructed in a neutral way.

Let f be an assignment of a, b, c, d to p_1, p_2, p_3, p_4. Then $\tilde{u}(f)$ is the assignment of a, b, c, d to $\tilde{u}(p_1), \tilde{u}(p_2), \tilde{u}(p_3), \tilde{u}(p_4)$, and

$$\begin{aligned}
\Psi(\tilde{u}(f)) &= (\tilde{u}(\Psi))(\tilde{u}(f)) && \text{because } \tilde{u}(\Psi) = \Psi \\
&= \tilde{u}(\Psi(f)) && \text{because if } g: x \mapsto y, \text{ then } \tilde{u}(g): \tilde{u}(x) \mapsto \tilde{u}(y) \\
&= \text{sibling of } \Psi(f) && \text{by the definition of } u \\
&= Rdabc.
\end{aligned}$$

From this it follows that $\tilde{u}$ preserves the relative order of the positions. This means that either

$$\tilde{u} \text{ maps } p_1, p_2, p_3, p_4 \text{ to } p_2, p_3, p_4, p_1$$

or

$$\tilde{u} \text{ maps } p_1, p_2, p_3, p_4 \text{ to } p_4, p_1, p_2, p_3.$$

In both cases $\tilde{u}(\tilde{u}(p_1)) \neq p_1$. But, since $\tilde{u} \circ \tilde{u} = \widetilde{u \circ u}$, this contradicts that $u \circ u = \mathrm{id}_{C \cup S \cup O \cup O^c}$.

So we conclude that positions for R cannot be neutrally reconstructed with respect to $\mathcal{M}$.

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Converse Predicates and the Interpretation of Second Order Quantification

Fraser MacBride

In this paper I argue that we cannot interpret second-order quantification as quantification over an abundant supply of properties and relations conceived as the referents of predicates. My argument forges a hitherto unexplored connection between debates typically conducted independently, one about whether there are converse relations, the other about the interpretation of second-order quantifiers. I begin from the semantics of converse predicates. Either pairs of mutually converse predicates co-refer or they do not. If they do co-refer, I argue that we lack an understanding of the relevant class of higher-order predicates which are required for second-order quantification over a domain of relations. If they don't co-refer but pick out distinct converse relations, then I argue that whilst we may make some abstract sense of the higher-order predicates in question we do so only at the cost of having to impute implausible readings to lower-order constructions. Either way, I conclude that second-order quantification should not be interpreted as quantification over relations conceived as the referents of predicates.

How should we interpret second-order quantifiers? In this paper I argue that we cannot interpret second-order quantifiers as ranging over relations—not if second-order existential introduction is taken to be a straightforward generalization of first-order existential introduction.

My primary argument takes the form of a dilemma. Either pairs of mutually converse predicates, such as "ξ is on top of ζ" and "ξ is underneath ζ," refer to the same underlying relation or they refer to distinct converse relations. If they refer to the same relation, then we lack the supply of higher-order predicates required to interpret second-order quantifiers as ranging over a domain of relations. The higher-order predicates required for such an interpretation of second-order quantifiers are predicates true or false of the referents of lower-order predicates—that is, true or false independently of how the referents of those lower-order predicates are specified. But if mutually converse predicates co-refer, then we lack the supply of higher-order predicates required for such an interpretation. If, by contrast, mutually converse predicates refer to distinct converse relations, then whilst we can at least make abstract sense of the higher-order predicates required to interpret quantifiers as ranging over a domain of relations, the implausible consequences for the content of lower-order constructions render this interpretation of higher-order quantifiers a deeply implausible semantic hypothesis.

There has been a great deal of recent discussion both about whether or not there are converse relations and about whether we should interpret second-order quantification in terms of a range of properties and relations or otherwise. But these two debates have been conducted separately and independently of one another. Here I seek to show that there are important connections between them.

Some preliminaries. For brevity I state my argument in terms of binary relations but it is intended to generalize to relations of greater arity. By a second-order language I will mean one in which the second-order quantifier rules are a straightforward generalization of the first-order quantifiers rules, allowing for the introduction of the second-order existential quantifier into predicate position, and where these rules are supplemented with the Axiom Scheme of Comprehension according to which, roughly speaking, every predicate determines a relation (see Shapiro 1991, 66–67; Fine 2002, 103; and Williamson 2013, 227–229). What are mutually converse predicates? For present purposes, I take any two binary predicates U and V to be mutual converses iff, for any terms t, t', it is guaranteed by the rules of the language that $t\underline{U}t'$ is true iff $t'Vt$ is true.[1] Similarly, R and R^* are mutual converse relations iff, for any particulars x, y, xRy iff yR^*x, not as a matter of accident but metaphysical necessity.

Finally, what is a second-order predicate? A first-order predicate (say of the form "$F\xi$") results from the extraction of one or more names ("a") from a closed sentence ("Fa") in which it occurs and inserting a variable in the

1 Further refinements will be required to accommodate the phenomenon of inflected pronouns in English. For other natural and formal languages which place the predicate in prenex position and for natural languages, such as Latin and Hebrew, which rely more heavily upon case, prepositions and particles rather than the mere arrangement of terms, "mutual converses" will require different definitions accordingly.

resulting gap. A second-order predicate (say, of the form "$\exists x \Phi x$") results from the extraction of a first-order predicate ("$F\xi$") from a closed sentence ("$\exists x F x$") and inserting a variable into the resulting gap.[2] Our focus here will be binary first-order predicates ("$\xi R \zeta$") which result from the extraction of two names from a closed sentence ("aRb") and unary second-order predicates ("$a\Phi b$") which result from the extraction of a binary first-order predicate from a closed sentence.

1. Converse Predicates and Co-Reference

Whatever is true of an object picked out by a singular term is true of something. That's the primordial idea that justifies the operation of first-order existential introduction. But if converse predicates co-refer the operation of second-order existential introduction cannot be justified along such lines. To present my argument for this claim I begin by describing one semantic motivation for supposing that converse predicates co-refer.[3]

It may appear that we are up to our necks in ontological commitment to converse relations because in English, but not only in English, we have the active and passive voice for many verbs and an abundance of adjectives, adverbs and so on whose reciprocal behavior is readily modeled by converse relations: "above" and "below," "before" and "after," "greater" and "less," *et cetera*. But there's no need to posit converse relations to explain the reciprocal behavior of converse predicates. This is because the behavior of converse predicates can be explained more parsimoniously in terms of converse rules for their employment. The rules in question map the contexts in which pairs of mutually converse predicates occur onto the same configuration of things-in-relation, so there is no need to posit separate configurations of things-in-relation.

2 See Dummett (1981a, 38–39). Note that "predicates" as conceived here are interpreted signs or strings of signs which are true or false of the referents of the expressions to which they are applied and their variables are bindable. The class of predicates of a given type $n+1$ will include complex predicates or open sentences generated from closed sentences with n type terms replaced by variables, as well as including primitive signs of that type. Here I follow, for example, the usage of Shapiro (1991, 65) and Shapiro and Weir (2000).

3 The proposal that mutually converse predicates should be conceived as co-referring can be traced back (at least) to Russell (1984, 85). For alternative metaphysical and semantic motivations for so conceiving converse predicates see Evans (1959, 538), Sprigge (1970, 69–70), Armstrong (1978b, 42, 94), Williamson (1985, 256–257) and Fine (2000, 6–7).

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The matter can be considered from a more general perspective on representation. In order to systematically represent things-in-relation we use signs-in-relation; we encode information about how things are related by how we relate signs together.[4] Invariably there is more than one way of configuring signs to encode the same information about how things are related and we can switch between them so long as we keep track of the different means whereby different configurations of signs encode the relevant information.

Consider the worldly configuration of things-in-relation, famously depicted in the *Alexander Mosaic,* which consists of Alexander sitting astride Bucephalus at the Battle of Issus. The statement "Alexander is on top of Bucephalus" effectively encodes how Alexander is related to Bucephalus by one arrangement of signs along a horizontal line. The statement "Bucephalus is underneath Alexander" no less effectively encodes the same information by another arrangement of signs. Neither statement constitutes a privileged encoding of how Alexander and Bucephalus are related. One is as good as another because it is a matter of convention how we encode information about the vertical arrangement of Alexander and Bucephalus by placing their names along a horizontal line. There are two conventions one might employ: (a) placing the name of the thing which is on top to the left and the name of the thing underneath to the right; (b) placing the name of the thing underneath on the left and the name of the thing on top to the right. When we use the predicate "ξ is on top of ζ" we signal that we are exploiting convention (a) to encode information about how things are related by the spatial relation which "ξ is on top of ζ" stands for, whereas when we use "ξ is underneath ζ" we are exploiting convention (b) to encode information about the obtaining of the same relation. Grasping the rules for "ξ is on top of ζ" and "ξ is underneath ζ" we understand straightaway that "Alexander is on top of Bucephalus" represents the same worldly configuration as "Bucephalus is underneath Alexander." Accordingly, we also understand that what we represent concerning Alexander and Bucephalus using "ξ is on top of ζ" could have been equally well represented using only "ξ is underneath ζ" and vice versa—so we could have succeeded in describing how Alexander and Bucephalus are depicted by the *Alexander Mosaic* in relation to one another if we'd been provided with only one of this pair of converse predicates. But the primary argument here isn't that we only need one predicate and so only one relation. Nor is the argument

4 I take this to be the element of truth in Wittgenstein's picture theory, see his (1922, 3.1432, 4.012) and my (2018, 191–197).

that there is only one relation here because one predicate can be defined in terms of the other. It is rather that the contexts "Alexander is on top of Bucephalus" and "Bucephalus is underneath Alexander" are mapped by the correlated conventions of their respective predicates onto the same worldly configuration of Alexander and Bucephalus and so there is no need to posit different configurations involving different relations to correspond separately to them.

A similar story can be told about other pairs of mutually converse predicates in English—for example the active and passive forms of a verb ("ξ kissed ζ," "ξ was kissed by ζ"). They don't stand for different relations but the same relation, albeit relative to contrary conventions about how to exploit the arrangement of prefixed and appended signs to represent how the things for which the signs stand are related by whatever relation is picked out by the predicate the signs prefix and append.[5]

2. Converse Predicates and the Division of Semantic Labour

Objectual quantification involves quantification over a domain of entities (whether first-order or second-order). In this section I will argue that the intelligibility of objectual quantification presupposes a principle I will call "The Division of Semantic Labour." For singular constructions the principle can be stated in the following terms. It must be possible to distinguish between, on the one hand, an expression whose semantic role is exhausted by picking something out—which, so to speak, drops away once it has discharged this function—and, on the other hand, the rest of the sentence whose complementary role is to say such-and-such about what has been picked out—independently, that is, of how it was picked out.[6] It is only when the

5 The conventions invoked here apply to configurations of things-in-relations rather than merely individuals. Suppose we adopt the convention for the non-symmetric predicate "ξ loves ζ" that we are to place the name of the thing which is the lover to the left of the verb and the name of the thing which is beloved to the right of the verb. And suppose it is both the case that Romeo loves Juliet and Juliet loves Romeo. Then if we apply the convention to individuals we are left in the dark about how to apply the convention because neither Romeo nor Juliet is "the" lover. But this difficulty is avoided if the convention is applied separately to the configurations (1) Juliet's loving Romeo and (2) Romeo's loving Juliet—because with respect to (1), Juliet is the unique lover, whereas with respect to (2), Romeo is the unique lover.

6 The Division of Semantic Labour (in the singular case) was recognized by both Quine (1960, 141–142) and Strawson (1961, 102) who distinguish, on the one hand, expressions occurring in basic predications whose role is to specify or identify an object and, on the other hand, the rest

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Division of Semantic Labour applies to a context that an expression occurring in it may intelligibly give way to a bound variable.

This division is a prerequisite of objectual quantification for the following reason. If the capacity of the rest of the sentence to say such-and-such is nullified by the extraction of a referential expression—if, so to speak, the significance of the rest of the sentence evaporates when the referring expression is pulled out—then we cannot use the rest of the sentence to say such-and-such about the value of a variable upon an assignment of values to variables by replacing the referring expression with a bound variable. In that case the idea behind the rule of existential introduction will have been undone because we cannot intelligibly say that what is true of a certain item picked out by a given referring expression is true of something, i.e., true of it regardless of whether that expression picks it out.[7]

I will now argue that we cannot quantify into the positions occupied by converse predicates because the contexts in which they occur fail to exhibit the Division of Semantic Labour—assuming that mutually converse predicates co-refer.[8] To see this, first observe that it's a consequence of conceiving mutually converse predicates as co-referential that we also have to recognize that the substitution of co-referring predicates cannot be guaranteed to be

of the sentence whose role is to be true or false of that object however specified or identified. To cover statements in which plural definite descriptions or lists of names feature the principle would need to be augmented with plural quantifiers and pronouns—to distinguish between expressions whose semantic role is exhausted by picking out *somethings* and the rest of the sentence which says such-and-such about *them* independently of how *they* were picked out. (See Boolos 1984 for the need to recognize the irreducibility of plural forms.) Since the relevant issues surrounding substitution and quantification into the positions of first-order converse predicates already emerge in the singular case, I concentrate attention there.

7 Famously Quine (1961b, 145) provided "Giorgione was so-called because of his size" as an example of a context which is resistant to the substitution of co-referential expressions and to which the rule of existential introduction cannot intelligibly be applied. For a sustained treatment of this and other examples prima facie resistant to substitution and quantifying in, see Fine (1989) and Forbes (1996).

8 For present purposes I restrict the Division of Semantic Labour to atomic sentences. Whereas it is integral to the Fregean approach to quantification that complex predicates ("ξ is even and ξ is prime") as much as simple predicates ("ξ is even" and "ξ is prime") are true or false of the referent of a name, a Tarskian account explains away complex predicates in terms of simple predicates, i.e., atomic open sentences, and it is only they that are true or false of the referent of a name. See Dummett (1981b, 284–285). So Tarskians deny that "2 is even and 2 is prime" can be decomposed into "2" and a single predicate which is true or false of the referent of "2." But since Fregean and Tarskian accounts agree that simple predicates or atomic open sentences are true or false of the referents of names, their differences over complex predicates may be set aside.

truth preserving MacBride (2011). This is so even though such predicates occur in contexts like,

(1) Alexander is on top of Bucephalus

whose truth-value is functionally determined by the referents of its parts and how they are assembled, contexts, moreover, whose name positions are open to truth-preserving substitution by co-referring terms. Since, for example, "Sikandar" is the Persian name for Alexander, we can infer from (1) that,

(2) Sikandar is on top of Bucephalus.

Nonetheless, even if we conceive of "ξ is underneath ζ" and "ξ is on top of ζ" as co-referring, we cannot substitute the former for the latter in (1) whilst preserving truth, because the result is false,

(3) Alexander is underneath Bucephalus.

Why does substituting "ξ is underneath ζ" for "ξ is on top of ζ" take us from truth to falsehood even though, we are granting, "ξ is underneath ζ" and "ξ is on top of ζ" refer to the same relation? In order for this inference to have been valid what (3) says about the referent of "ξ is underneath ζ" would have had to be the same as what (1) says about the referent of "ξ is on top of ζ." But they don't and can't say the same about *it*. This is because what the rest of (1) says about the referent of "ξ is on top of ζ" cannot survive the extraction of "ξ is on top of ζ." The rest of (1), which is what results from extracting "ξ is on top of ζ" from (1), is the second-order predicate "Alexander Φ Bucephalus." When the variable "Φ" in "Alexander Φ Bucephalus" is replaced by "ξ is on top of ζ" the result is a sentence that says Alexander is on top of Bucephalus. But when "Φ" is replaced by "ξ is underneath ζ" the result is a sentence that says Bucephalus is on top of Alexander. Hence, so far from being the same, what (1) says about the referent of "ξ is on top of ζ" is incompatible with what (3) says about the referent of "ξ is underneath ζ" even though "ξ is underneath ζ" and "ξ is on top of ζ" refer to the same relation.

What emerges from this line of reflection is that the significance of "Alexander Φ Bucephalus" isn't freestanding but varies depending upon which first-order predicate is inserted in place of its variable. The failure of "Alexander Φ Bucephalus" to have a freestanding significance is a consequence of the fact that the rules which we understand when we grasp converse predicates rely upon different conventions about how to interpret the significance of the

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arrangement of corresponding signs ("Alexander," "Bucephalus"). What it means to prefix an occurrence of one predicate, say "ξ is on top of ζ," with a token of "Alexander" whilst appending a token of "Bucephalus" is different from what it means to prefix an occurrence of a mutually converse predicate, say "ξ is underneath ζ" with a token of "Alexander" whilst appending a token of "Bucephalus." Without a first level predicate to furnish the conventions required to interpret the significance of prefixing "Alexander" and appending "Bucephalus," the second-order predicate "Alexander Φ Bucephalus" means nothing at all—its significance evaporates as soon as a first-order predicate filling its argument place is extracted.

So the strategic situation is this—assuming that mutually converse predicates co-refer. In order for objectual quantification into the position occupied by "ξ is on top of ζ" in (1) to be intelligible, i.e., for

(4) (∃Φ)(Alexander Φ Bucephalus)

to be meaningful, (1) must admit of a semantic analysis into two discrete components, the first level predicate "ξ is on top of ζ" and the rest of the sentence, the second-level predicate "Alexander Φ Bucephalus" which, were (4) meaningful, (4) would affirm to be true of some relation in the domain. But (1) fails to satisfy the Division of Semantic Labour. The second level predicate left over once "ξ is on top of ζ" is extracted lacks self-standing significance. It isn't true or false of the referent of "ξ is on top of ζ" independently of how that relation is picked out. Since "Alexander Φ Bucephalus" lacks freestanding significance we cannot intelligibly affirm it of the value of a second-order variable, i.e., affirm it of a relation independently of how that relation is picked out by a first-level predicate.[9] Hence we cannot quantify into (1), and (4) is meaningless. The rule of existential introduction into the position of converse predicates, understood as a generalization of existential introduction into the positions of names, is thereby undone.

9 Whilst Williamson, in his (1985, 257), recognized that the substitution of co-referential converse predicates isn't guaranteed to be truth-preserving, he did not address the consequent difficulties, explained here, for quantifying into the positions of converse predicates. In his more recent *Modal Logic as Metaphysics* (2013) Williamson recommends higher-order-quantification into predicate position because of its theoretical virtues ("maximizing strong, simple generalizations consistently with what we know," 2013, 261) but he does not mention the issue of substitution failures for converse predicates. The line of argument I advance here shows that Williamson's views cannot be straightforwardly packaged together because there is an irreconcilable tension between conceiving of converse predicates as co-referential and quantifying into their positions (assuming the quantification to be objectual).

3. Another Meaning for "Alexander Φ Bucephalus"?

I have argued against the intelligibility of higher-order quantification (objectually conceived) on the grounds that "Alexander Φ Bucephalus" lacks meaning in isolation, i.e., independently of the insertion of a first-level predicate into its argument position—because otherwise there's nothing to settle how to interpret the significance of the prefixed and appended terms. It may be thought that this is going too far. One can envisage an objector granting that "Alexander Φ Bucephalus" lacks a *determinate* significance—because what the significance will be of prefixing "Alexander" and appending "Bucephalus" to a given occurrence of a predicate depends upon the rules governing the predicate that happens to occur between them. Nevertheless, this objector continues, this doesn't rule out "Alexander Φ Bucephalus" having a *determinable* significance, i.e., its being a second-level predicate which is true of the referent *R* of a first level predicate (when inserted into its argument position) just in case *R* relates Alexander to Bucephalus in *some manner or other* but without settling any determinate arrangement for them.

The immediate difficulty with this objection is that if "Alexander Φ Bucephalus" is granted the kind of determinable significance proposed, then other sentences get assigned the wrong truth conditions. From (1) follows,

(5) ¬(Alexander is underneath Bucephalus).

Now according to the semantic hypothesis under consideration, (5) has a higher-order parsing according to which (5) says that it's not the case that the relation which is the referent of "ξ is underneath ζ" satisfies "Alexander Φ Bucephalus," i.e., it's not the case that that relation has the determinable property of relating Alexander to Bucephalus in some manner or other. But this makes (5) incompatible with (1) which says that the same relation, i.e., the referent of "ξ is on top of ζ," *does* relate Alexander to Bucephalus in some manner or other, specifically relating Alexander to Bucephalus so that Alexander is on top of Bucephalus. But (5) isn't incompatible with (1) but entailed by it. So the semantic hypothesis that "Alexander Φ Bucephalus" has self-standing but determinable significance results in faulty assignments of truth-conditions.

Denying that "Alexander Φ Bucephalus" has the self-standing significance required for quantifying into the position of converse predicates is consistent with allowing that "Alexander Φ Bucephalus" has some weaker kind of significance. After all, when "Alexander Φ Bucephalus" is completed with

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a given first level predicate, the result is a statement with a certain content, or, to speak more generally, a certain semantic value. So, prima facie, we can assign it the derived syntactic category S/(S/NN) (see Ajdukiewicz's categorial grammar 1967). But we cannot interpret "Alexander Φ Bucephalus" as having as a semantic value a function from the referents of binary predicates to the semantic values of sentences. Since, we are supposing, mutually converse predicates have the same referent, such a function will map the semantic value of "ξ is underneath ζ" to the same semantic value (of the kind appropriate to a sentence), as it maps the semantic value of "ξ is on top of ζ." But the result of substituting a co-referential but converse predicate, "ξ is underneath ζ" for "ξ is on top of ζ" in a sentence in which "Alexander Φ Bucephalus" occurs, is not guaranteed to preserve the semantic value of the sentence upon which the substitution is performed. Nor does it follow even if it is conceded that "Alexander Φ Bucephalus" belongs to a derived syntactic category, that "Alexander Φ Bucephalus" has content in the sense relevant to sustaining the intelligibility of second-order quantification, i.e., has content in the sense of itself having the capacity to be true or false of a relation independently of how that relation is specified by a first level predicate.

4. Relations and the Axiom Scheme of Comprehension

I have taken us along a route from point (a) supposing that converse predicates co-refer to point (b) the unintelligibility of second-order quantification conceived as quantification over the referents of binary predicates, the connecting link being that if converse predicates co-refer then there's a lack of extractable higher-order predicates capable of being true or false of their referents independently of how they are picked out. But are there other routes between these two points?

To suppose that mutually converse predicates co-refer is to adopt a (relatively) sparse view of our ontological commitments. The view is (relatively) sparse insofar as "ξ is on top of ζ" and "ξ is underneath ζ" are conceived as equally good predicates for referring to one and the same relation—so less abundant than a view according to which our use of "ξ is on top of ζ" and "ξ is underneath ζ" commit us to distinct converse relations. But the Axiom Scheme of Comprehension for second order logic,

COMP. $\exists R^n \forall x_1 \ldots x_n (R^n x_1 \ldots x_n \leftrightarrow \Phi x_1 \ldots x_n)$

where R^n is an n-ary relation variable which does not occur free in Φ, is typically conceived as embodying an abundant conception of relations because, taken together, the instances of (COMP) tell us that every formula determines a relation. Doesn't this already establish that embracing second order logic is incompatible—because COMP is abundant—with the sparseness of supposing converse predicates to co-refer?

Now it is certainly true that COMP is straight out incompatible with certain sparse conceptions of relations. COMP says that every formula determines a relation even if the formula in question isn't satisfied by anything. So embracing COMP forces the admission of uninstantiated relations where the corresponding formulae are unsatisfied. This means that if we admit only instantiated relations, what's often called an "Aristotelian" conception of relations, or universals more generally, then we must reject COMP.[10] To bring second-order logic in line with this "Aristotelian" stricture, COMP needs to be restricted to recognize only relations that correspond to formulas that are true of something:

ARISTOTELIAN COMP. $\exists x_1 \ldots \exists x_n \Phi x_1 \ldots x_n \rightarrow \exists R^n \forall x_1 \ldots \forall x_n (R^n x_1 \ldots x_n \leftrightarrow \Phi x_1 \ldots x_n)$.

Further restrictions along these lines can be envisaged. ARISTOTELIAN COMP still requires a relation for every polyadic predicate that's satisfied. But this won't be sparse enough for us if, for example, we're doubtful that there are relations corresponding to disjunctive predicates even if they're satisfied.

By contrast to an Aristotelian approach which requires relations to be instantiated, the (relatively) sparse doctrine that mutually converse predicates are vehicles for referring to one and the same relation does not conflict with the existential requirements of COMP. This is because (a), unlike the Aristotelian approach, the doctrine that mutually converse predicates co-refer does not require that the relations to which they refer are instantiated. Moreover, (b) COMP does not require that each formula determines a unique relation but only that each formula determines a relation—which is consistent with different formulas having the same referent. So whilst COMP requires that "ξ is on top of ζ" and "ξ is underneath ζ" both pick something out, this requirement does not by itself force us towards a more abundant conception of

10 See Armstrong (1978a, 126) for "Aristotelian realism." See Shapiro and Weir (2000, 265–266) for the suggestion of an Aristotelian second order logic and the proposed restriction on COMP.

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relations according to which "ξ is on top of ζ" and "ξ is underneath ζ" pick out distinct converses.

Williamson, in his treatment of higher-order logic, argues against the restriction of COMP to natural properties and relations, e.g., the universals which, according to Armstrong, are only to be recognized a posteriori on the basis of total science. Rather, according to Williamson, COMP is the "most obvious example of a logical principle of higher-order logic that depends on unnatural properties and relations" (Williamson 2013, 227). Williamson advances his case on the grounds that the extensive literature on naturalness has failed to supply a fruitful logic of natural properties and relations. By contrast, Williamson maintains, COMP is an informative logical principle which depends "on the absence of any naturalness restriction" (Williamson 2013, 227) because it allows us quantify into the position of formulae, however unnatural the conditions they define, e.g., not smoking or being everything bad. But this line of reflection doesn't establish that the existence of converse relations can be settled by appeal to COMP alone. COMP only tells us that to every formula there corresponds a property or relation. COMP taken by itself does not tell us that there is a 1-1 correspondence between formulas on the one hand and properties and relations on the other, however unnatural.

Nonetheless, it can be shown in short order that supposing mutually converse predicates to co-refer conflicts with the application of second-order generalization to atomic formulae—even without relying upon the full strength of COMP which applies to formulae of arbitrary complexity. From

(1) Alexander is on top of Bucephalus

it follows that

(5) ¬(Alexander is underneath Bucephalus).

Applying the operation of existential generalization to (1) and (5) it follows that

(6) $(\exists\Phi)$(Alexander Φ Bucephalus)

and

(7) $(\exists\Phi)\neg$(Alexander Φ Bucephalus).

There's no formal contradiction here because the variables in (6) and (7) aren't bound by the same initial quantifier. But we cannot coherently suppose that the open sentences which occur in (6) and (7) are both satisfied under the same assignment of a relation to "Φ" because the higher-order predicates "Alexander Φ Bucephalus" and "¬(Alexander Φ Bucephalus)" express contradictory properties of relations. But if both (1) and (5) are interpreted as saying something about the same relation, picked out by "ξ is on top of ζ" and "ξ is underneath ζ" respectively, then their existential entailments should be compatible with the open sentences which occur in (6) and (7) both being satisfied on the same assignment of a relation to "Φ."

It's important to appreciate how the fact that the open sentences which occur in (6) and (7) cannot be true upon the same assignment of values to variables conflicts with supposing both that converse predicates co-refer and that second-order existential generalization is the analogue of first-order generalization. Why? Because it's mysterious how, if (1) and (5) incorporate reference to only *one* relation, applying the operation of second-order existential generalization to them can result in statements, (6) and (7), which taken together are ontologically committed to *two* relations. The idea behind the operations of second-order existential generalization—conceived as an analogue of the operation of first-order quantification—is that whatever is true of the referent of a first-order predicate is true of (second-order) something. But this inference loses its justification if whatever is said to be true of something cannot be true of the referent of the first-order predicate. Since the open sentences which occur in (6) and (7) cannot be true upon the same assignment of values to variables, the application of existential generalization to (1) and (5), assuming their first-level predicates co-refer, must take us from saying things true of one and the same relation to saying things which can only be true of at least one other relation. But then it is unclear how existential generalization is guaranteed to preserve truth—because we have undertaken a passage from talking about one relation to committing ourselves to at least two. So we have an unstable package of commitments: (a) that the predicates of (1) and (5) refer to one and the same relation, (b) that (6) and (7) taken together are committed to the existence of two relations, and, (c) the rule of second-order existential introduction is guaranteed to preserve truth when understood as an analogue of first-order existential generalization.

In light of preceding sections, we can appreciate how the failure of sentences like (1) and (5) to exhibit the requisite Division of Semantic Labour (assuming their first-order predicates co-refer) contributes to this unstable package of views. What (1) affirms of the referent of "ξ is on top of ζ" isn't the negation of what (5) denies of "ξ is underneath ζ" because the respective rules governing

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the use of "ξ is on top of ζ" and "ξ is underneath ζ" reverse the semantic significance of their prefixed and appended terms. But because quantifying into (1) and (5) extrudes these rules about how to interpret the significance of their flanking terms—by replacing the first-order predicates which carry these rules with bound variables which don't—we are left with the bare statements (6) and (7), whose constituent open sentences cannot be true upon the same assignment of values to variables.

5. Converse Relations

What have we learnt about the possible interpretation of second-order quantifiers? Earlier I argued that if mutually converse predicates co-refer, then we cannot intelligibly objectually quantify into the positions they occupy for lack of the requisite higher-order predicates. I have also argued that the operation of second order existential generalization cannot be intelligibly combined with such commonplace truths about mutual converses as (1) and (5) whilst supposing that mutually converse predicates co-refer. This was the first horn of the dilemma envisaged in the introduction.

Prima facie it would not be unreasonable to conclude that second-order languages are committed to converse relations after all—because these problems can be made to go away by assuming that mutually converse predicates pick out distinct converse relations. But even if pairs of mutually converse relations are admitted, thus avoiding the difficulties that arose from dispensing with them, higher-order predicates of the form "$a\Phi b$" are still required for the intelligibility of quantification into the positions of converse predicates, i.e., higher-order predicates capable of being true or false of a relation belonging to the domain independently of how that relation is specified. So the question still remains even if it is granted that mutually converse predicates pick out distinct converse relations: do we have an understanding of higher-order predicates of the form "$a\Phi b$" which will enable us to interpret second-order quantification as quantification over a domain of relations? I will argue that we don't. This is the second horn of the dilemma envisaged in the introduction.

We have already considered the proposal that predicates of the form "$a\Phi b$" have a purely determinable significance—so that, for example, "Alexander Φ Bucephalus" stands for a property of a relation, viz., the property of holding between Alexander and Bucephalus in some manner or other, a property which is indifferent to the order in which Alexander and

Bucephalus are related by whatever relation has the property. The problem identified earlier with this proposal was that it gets the truth-conditions of (1) and (5) wrong if mutually converse predicates co-refer. But the problem of conceiving "Alexander Φ Bucephalus" as having this kind of determinable significance is a problem for non-symmetric relations per se regardless of whether they are accompanied by converses. Consider,

(1) Alexander is on top of Bucephalus

and one of its consequences,

(8) ¬(Bucephalus is on top of Alexander).

If "Alexander Φ Bucephalus" has purely determinable significance, then "Bucephalus Φ Alexander" does too, but they will mean the same. The latter will stand for a property that a relation has if it relates Bucephalus and Alexander in some manner or other. But a relation has the property of relating Bucephalus and Alexander in some manner or other iff it has the property of relating Alexander and Bucephalus in some manner or other—because the property of relating some things in some manner or other is order-indifferent. Then (8) will have a higher-order parsing according to which (8) says that it's not the case that the non-symmetric relation that "ξ is on top of ζ" picks out has the order-indifferent property of relating Alexander and Bucephalus in some manner or other. But (1) will have a corresponding parsing according to which (1) says that the relation "ξ is on top of ζ" picks out does have that property and (8) follows from (1). This problem doesn't go away if the relation that "ξ is on top of ζ" has a converse because it's a problem that arises solely by reflection upon that relation without consideration of its converse—the relation that "ξ is underneath ζ" picks out doesn't feature.

We can avoid this problem by interpreting "Alexander Φ Bucephalus" as standing for a property sensitive to the order in which Alexander and Bucephalus are related by whatever relation has this property.[11]

11 One way to sidestep all these problems would be to restrict the rule of second-order existential introduction to the positions of symmetric predicates, i.e., contexts where it makes no semantic difference which left-right flanking arrangement of names are used, or, more radically, to quantification over monadic predicates. But this restriction is unappealing because a second-order language without quantification into the positions of non-symmetric predicates would be unable to codify categorical versions of key mathematical principles, one of the key attractions of higher-order languages. Consider Cantor's Theorem construed as the claim that no binary relation can represent the collection of all subsets of its domain ($\forall R \exists X \forall x \exists y[(Rxy \wedge \neg Xy) \vee (\neg Rxy \wedge Xy)]$).

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But unless the order in question is explicable independently of how "Alexander Φ Bucephalus" is completed by the insertion into its argument position of a first level predicate standing for a relation, we will still have failed to secure the Division of Semantic Labour which I have argued is required for second-order quantification objectually conceived.

In order for a predicate of the form "$a\Phi b$" to have the required self-standing significance it must stand for a higher-order property which relations have independently of how they are picked out. This requirement is fulfilled if relations hold between the things they relate in an order, where the notion of order in play is absolute in the following sense: for any relation R which holds between any two things a and b, either R applies to a first and b second or b first and a second. If that is how relations apply to the things they relate, then there is a higher-order property any relation has if it applies to a first and b second, another higher-order property any relation has if it applies to b first and a second—properties which relations have independently of how they are picked out by first-level predicates because they are properties relations have solely in virtue of how they apply rather than how they are depicted. If that is indeed the case, then a higher-order predicate of the form "$a\Phi b$" meeting our requirement may be understood as standing for the property that any relation has if it applies to a first, b second.

6. The Untoward Semantic Consequences for Atomic Statements

What is important for present purposes is to appreciate the untoward consequences of so interpreting higher-order predicates of the form "$a\Phi b$." These include consequences for our understanding of atomic statements which entail second-order generalizations. Why so? Applying existential generalization to a statement of the form "aRb" whose first-order predicate picks out a relation yields a statement of the quantified form "$\exists\Phi\, a\Phi b$." If a higher-order predicate of the form "$a\Phi b$" expresses the higher-order property that a relation has when it applies to a first and b second, then what a statement of the form "$\exists\Phi\, a\Phi b$" says is that some relation has that property. But in order for existential generalization to have its usual justification this is a property the entailing statement of the form "aRb" must already have affirmed of the

For further examples, including the Continuum Hypothesis and the Well-Ordering Principle, see Shapiro (1991, 97–108).

relation picked out by its first-order predicate. In other words, it's a consequence of the proposed interpretation of higher-order predicates of the form "$a\Phi b$" that a statement of the form "aRb" already says that the referent of a first-order predicate has the property of applying to a first and b second.

It follows that we can test the proposed interpretation of predicates of the form "$a\Phi b$" by checking whether atomic constructions which entail existential generalizations of the form "$\exists\Phi\, a\Phi b$" can be interpreted as saying that a relation has the property of applying to a first and b second. I will argue that the proposed interpretation fails this test for both symmetric and non-symmetric atomic constructions.

Since second-order logic permits existential quantification into the positions of symmetric predicates, it follows—assuming the proposed interpretation of higher-order predicates—that atomic statements in which symmetric predicates occur attribute to symmetric relations the property of applying to the things they relate in an order. But it is far from plausible that they do. Consider, for example,

(9) Darius differs from Alexander

and

(10) Alexander differs from Darius.

If predicates of the form "$a\Phi b$" mean what they're proposed to mean, then (9) says that the relation picked out by "ξ differs from ζ" applies to Darius first and Alexander second, whereas (10) says that it applies to Alexander first and Darius second. But, as both linguists and philosophers have reflected, prima facie statements like (9) and (10) don't say different things but are distinguished solely by the linguistic arrangement of their terms.[12] So prima facie interpreting higher-order predicates of the form "$a\Phi b$" as standing for

12 See, for example, Langacker (1990, 223), Dowty (1991, 556) and Rappaport Hovav and Levin (2015, 600) where it is suggested that (9) and (10) have the same content or have arguments whose roles cannot be distinguished. In the *Principles of Mathematics*, Russell famously advocated the view that statements like (9) and (10) express distinct propositions (1903, sec. 94). For a more recent endorsement of this view about symmetric relations, see Hochberg (1980, 40–41). But Russell had earlier maintained, in his "Fundamental Ideas and Axioms of Mathematics" (1899, 278), that statements like (9) and (10) say the same and would later revert to this view in his *Theory of Knowledge* (1984). See MacBride (2012b, 141–144) and (2018, 153–182) for discussion of Russell's evolving views and MacBride (2012a) for an examination of Hochberg's treatment of relations.

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a property that a relation has if it applies to *a* first and *b* second imports ordinal notions—first, second—into the content of atomic constructions expressing symmetric relations, ordinal notions which are alien to our ordinary understanding of statements like (9) and (10).[13]

Second-order logic also permits existential quantification into the positions of non-symmetric predicates. Is it at all realistic to interpret a statement in which a non-symmetric predicate occurs as saying of a non-symmetric relation that it has the property of applying to things it relates in an order? Certainly there is a significant class of non-symmetric constructions, paradigmatically action sentences, in which the arrangement of terms may be felt to depict an order imposed upon the things they pick out. Consider, for example,

(11) Bucephalus kicks Oxyathres

which might be conceived as representing a kind of "energy flow" from the agent (Bucephalus) to the patient (Oxyathres) (see Langacker 1990, 221–222). In this kind of case it is perhaps relatively natural to say that the relation which "ξ kicks ζ" stands for is represented as applying to Bucephalus first and Oxyathres second. But there are what linguists sometimes describe as "static" cases which aren't comfortably described in such terms, for example,

(12) Alexander has lighter hair than Darius,

and,

(13) Alexander is to the left of Darius.

With regard to neither statement does there seem to be a sense in which one participant is described as the "agent" rather the "patient"; neither is identified as the "energetic partner." So there's nothing corresponding to "energy flow" between Alexander and Darius here. Indeed there seems nothing to distinguish Alexander and Darius in how they are described except that they are the things that stand in the relation identified by the predicate—as one thing lighter haired than another, as one thing to the left of another.[14]

13 A similar point applies to constructions incorporating partially symmetric predicates like "ξ is between ζ and η" where, for example, "Oxyathres is between Alexander and Darius" and "Oxyathres is between Darius and Alexander" prima facie differ only by the linguistic arrangement of the terms "Darius" and "Alexander" rather than differing because of the way the relation is described as applying to them.

14 See Huddleston (1970, 510) and MacBride (2014, 6). Of course, it may be that comprehending these statements a language speaker alights attention upon Alexander and Darius in a given

Of course, the term "Alexander" occurs first in (12) in the sense that it is the first term that we encounter as readers of English when we scan the sentence from left to right. But it's only an accidental feature of English that we read left to right and it's a further accidental feature that we describe something as being lighter haired than something else by writing its name to the left of the verb. There are actual languages, such as Hebrew or Arabic, as well as possible ones, which don't have these accidental features but different ones.

What is nonetheless essential for depicting states that result from the application of non-symmetric relations, hence common to different languages whose features may otherwise vary, is that for each *n*-ary predicate in a language there be some rule for assigning a distinguished significance to each occurrence of a term in a closed sentence that results from completing the predicate with terms. In English we employ, for example, the rule that a term which occurs to the left of the predicate "is lighter haired than" in a statement like (12) has the significance of standing for something that is lighter haired than something else which it is the significance of the right-flanking term to stand for. This rule suffices to interpret what (12) says but it doesn't invoke the ordinal notions of "first" and "second" to do so. This shows that it isn't essential for depicting a state that results from the application of a non-symmetric relation that we conceive of the relation as applying to the things it relates to something first and something second—because all that is required to interpret (12) is a rule that settles a distinguished significance for the occurrence of each term and the rule provided does so without invoking "first" and "second." What the rule does is co-ordinate the arrangement of terms in a sentence with the way that the objects corresponding to the terms must be arranged for the sentence to be true. But neither the arrangement of terms, right and left of the verb, nor the arrangement of corresponding objects, lighter-haired to darker-haired, is fundamentally ordinal in character.

Isn't there a straightforward counter to be made to these claims? Surely it is the *raison d'être* of relations to relate things "in an order"—a feature which, for example, distinguishes non-symmetric relations from monadic properties? Constructions like (12) and (13) describe Alexander and Darius as being related by certain non-symmetric relations. Since non-symmetric relations have the distinguishing feature of relating things "in an order," it

order. But this psychologistic notion of content is evidently different from the objective notion of content at stake which pertains to the content of what is said rather the manner of its grasping.

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follows that (12) and (13) describe Alexander and Darius as being related "in an order." So (12) and (13) must presuppose ordinal notions after all!

This counter trades upon the ambiguity of the phrase "in an order," which admits of a weaker and a stronger reading. Once the ambiguity is taken into account it's evident that the conclusion doesn't follow from its premises. The weaker reading of "in that order" is simply that of relating things so that they are arranged one way rather than another—so, for example, that one thing is above another. The stronger reading is that of relating things so that one thing occurs first, the other second. The weaker reading does not imply the stronger reading. From the fact that one thing is above another it doesn't follow that one thing is first, the other second. Note that the weaker reading is consonant with one grammatical use of "order" in ordinary English. When, for example, we describe placing chess pieces in their proper order before the start of a game, we don't mean that one piece is placed first, another second. Similarly, when a historian describes how Alexander arranged his men in a certain order before the Battle of Issus, this doesn't mean describing which men Alexander put first, second and so on, but rather how he placed the Thessalonian cavalry on the left flank, the Macedonian cavalry on the right flank and so forth.[15] Now we can readily acknowledge that it is the *raison d'être* of non-symmetric relations to relate "in an order" in the weak sense without having to suppose that they do so in the strong sense. We don't thereby compromise our capacity to distinguish non-symmetric relations from properties because properties don't relate the things that bear them in any sense. But if non-symmetric relations only relate "in an order" in a weak sense, then it doesn't follow from (12) and (13) describing Alexander and Darius as being related by non-symmetric relations that they must also be describing Alexander and Darius as being related first and second, i.e., "in an order" in the strong sense.

Acknowledging order in the weak sense does allow us to admit talk of coming "first" and "second" but only as an eliminable *façon de parler*. So, for example, we can say that Alexander comes first, Darius second in the relation "ξ is to the left of ζ" stands for, meaning by that just that Alexander is to the left of Darius. And we can say that Darius comes first, Alexander second in the relation that "ξ is to the right of ζ" stands for, meaning by that just that Darius is to the right of Alexander. But the notions of "first" and "second" are only defined here relative to the specification of a relation—"first" and

15 When Defoe described Robinson Crusoe as putting up shelves "to order my Victuals upon," he didn't mean that Crusoe wanted somewhere to arrange coconuts first, ship's biscuits second, mangoes third or anything of the sort (Defoe 1719, 7).

"second" relative to the relation that "ξ is to the left of ζ" stands for, "first" and "second" relative to the relation that "ξ is to the right of ζ," and so on. Indeed we might have introduced a different *façon de parler* whereby saying that Darius comes first, Alexander second in the relation "ξ is to the left of ζ" stands for, also just means that Alexander is to the left of Darius. So it doesn't follow from granting order in this weak sense that one thing's being to the left of another makes one thing first or second in some sense that can be expressed without specifying a given relation. So acknowledging order in the weak sense doesn't provide a basis for making sense of one thing coming first, another second in a relation regardless of whether or how the relation is specified, i.e., coming first or second in the absolute sense. And it's order in the strong sense, I've argued, which is required to make sense of objectual quantification into predicate position.

So far we have tested the proposed interpretation of higher-order predicates of the form "$a\Phi b$" by checking whether atomic constructions which entail second-order generalizations of the form "$\exists\Phi\ a\Phi b$" can be read as saying that a relation has the property of applying to a first and b second (in the strong sense). I've argued that the proposed interpretation fares poorly because neither symmetric constructions ((9) and (10)) nor some non-symmetric atomic constructions ((12) and (13)) plausibly admit of such a reading. Consider now a further consequence of the proposed interpretation of predicates of the form "$a\Phi b$" that if there is a higher-order property of applying to a first and b second (in the strong sense), then any relation can be compared to another with respect to this property MacBride (2015). Why should the intelligibility of such comparisons be a consequence of the proposed interpretation? Because if there is such a higher-order property then for any binary relation and two things it relates to one another, there's a fact of the matter about which of them it applies to first, which second. Hence, if any two relations R and S relate any two things a and b, then there is a fact of the matter about whether (i) R and S both apply to a first, b second, or whether (ii) both apply to a second and b first, or whether (iii) R applies to a first and b second and S applies to a second and b first, or whether (iv) R applies to b second and a first and S applies to a first and b second. But, as I have argued, it isn't part of what we ordinarily mean when we say that one thing has lighter hair than another or that one thing is to the left of another that anything comes first or second (in the absolute sense) in the relations "ξ has lighter hair than ζ" or "ξ is to the left of ζ" stand for. Since coming first or second (in the absolute sense) isn't part of what we ordinarily mean when we use these predicates, it

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cannot be a further part of what we ordinarily mean that there is a fact of the matter about whether the relations they stand for apply to the same pair of things in the same or a different order.

Accordingly the proposed interpretation of higher-order predicates of the form "$a\Phi b$" fails to mesh with what we mean by what we say using lower-order predicates that serve as arguments to higher-order predicates of this form. If that were what predicates of the form "$a\Phi b$" meant, then their application would impose an order (in the strong sense) on the relata of relations. But we have no idea of what the relevance of such an order could be to our ordinary classificatory practices—because our facility with such constructions as (9) and (10) in which "ξ differs from ζ" occurs, or (12) and (13) in which "ξ has lighter hair than ζ" and "ξ is to the left of ζ" occur, don't give a semblance of our relying upon it at all.

This point has significance for the justification of second-order logic itself. Introducing second-order quantifiers brings about a sea change in the expressive capacities of language, so we cannot expect to explain second-order quantifiers before introducing them. So how can we hope to justify the introduction of second-order quantifiers? Williamson maintains that we can account for second-order quantifiers retrospectively by seeking to explain how our understanding of those quantifiers is "rooted in our understanding" of constant predicative expressions of the same category as the quantified variables (2013, 258). But since we don't understand the predicative expressions in question as standing for relations which apply to the things they relate in an order (in the strong sense), our understanding of second-order quantifiers as ranging over a domain of relations which apply to the things they relate in an order (in the strong sense) can hardly be rooted in our understanding of constant predicative expressions. So we cannot justify the introduction of second-order quantifiers even "retrospectively" if they are interpreted this way.

Might there be an alternative interpretation of higher-order predicates of the form "$a\Phi b$" over which we have more control and which will facilitate an interpretation of second-order quantifiers as ranging over a domain of relations? The ordinary language construction "—*bears*---*to*___," as it figures in

(14) Alexander bears a great resemblance to Philip,

might appear to be a promising candidate for a construction in which our understanding of a predicate of the form "*a*Φ*b*" might be rooted. Roughly speaking, the idea is that a relation *R* satisfies the predicate "*a*Φ*b*" just in case *a* bears *R* to *b*, whereas *R* satisfies "*b*Φ*a*" just in case *b* bears *R* to *a*. Nevertheless, the natural language construction "—*bears*---*to*___" is unsuited to this role.[16]

One obstacle is that "*a*Φ*b*" and "*a* bears---*to b*" have different logical forms—hence it is problematic to suppose that our understanding of the one is rooted in the other. The key difference is that whilst "*a*Φ*b*" takes a first level predicate as its argument, "*a* bears---*to b*" takes noun phrases rather than predicates in its argument position, for example, the indefinite description "a great resemblance" which occurs in (14). Because they take noun phrases, rather than predicates, constructions like (14) are more naturally formalised in first-order terms as expressing a ternary relation between three first-order entities, one of them a relation. Another difference is that whereas "*a* bears---*to b*" has a converse, viz., the passive form "--- is borne by *a* to *b*," "*a*Φ*b*" does not. Because "*a*Φ*b*" and "*a* bears---*to b*" are so logically different, it doesn't follow from the fact that we understand constructions of the form "*a* bears---*to b*" that we also understand predicates of the form "*a*Φ*b*." Nor does it follow that if we don't understand "*a*Φ*b*," that we don't understand "*a* bears---*to b*" either.

A further consideration against this proposal is that for a wide range of cases, constructions of the form "*a* bears---*to b*" admit of a deflationary reading in first-order terms (see MacBride 2015, 188). According to this reading, what it means for *a* to bear a relation *R* to *b* is simply that *aRb*. So "*a* bears---*to b*" doesn't furnish a means of understanding how a relation applies independently of the lower order construction to which it reduces when its argument

16 Fine has made the suggestion that a converse relation be conceived as an ordered pair of an underlying neutral relation and an ordering of its argument positions, albeit without endorsing the suggestion because he eschews argument positions (Fine 2000, 11). In that case "*a*Φ*b*" might be interpreted as standing for a property had by a relation when *a* figures in its first argument position and *b* in its second. (Thanks to Jan Plate for pointing out the relevance of Fine's suggestion to the present discussion). But I doubt this proposal fares any better than the interpretation we have been considering. We no more have a grasp of which argument position of, e.g., the relation picked out by "ξ is to the left of ζ" come first and which second than we have a grasp of which thing the relation applies to first and which second. Moreover, it is just as questionable to suppose that when we understand an atomic construction like (13), we grasp that one of the argument positions, e.g., *right* figures first in the sequence which constitutes the converse relation in question whilst *left* figures second. Of course there are further, more familiar objections to be raised to invoking argument positions as pieces of our ontology. See Fine (2000, 17–18; 2007, 58–59) and MacBride (2007, 36–44; 2014, 10–12; and MacBride 2020, sec. 4).

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position is completed. In support of this reading, witness the equivalence of (14) and

(15) Alexander greatly resembles Philip.

It's not just that (14) entails (15), but the fact that (14) appears to be just a longwinded way of saying what (15) says.

Now it may be acknowledged that there are a limited number of cases in natural language resistant to this deflationary reading, cases where the "bears" construction appears to take quantifier phrases in its argument position, notably

(16) The text bears some relation to the facts

and

(17) The text bears no relation to the facts.

It is arguable that grammatical appearances are misleading here, that in fact there is no genuine quantification over relations going on and really (16) and (17) are more transparently rendered as saying that some of the text is true and none of it is (respectively). Nonetheless, even if there is quantification over relations in play in (16) and (17), these statements don't correspond in any straightforward sense with second-order quantificational claims. This is because anything of the form

(18) $(\exists\Phi)(a\Phi b)$

is a higher-order logical truth, and anything of the form

(19) $\neg(\exists\Phi)(a\Phi b)$

is a higher-order logical falsehood, whereas (16) and (17) are contingent claims. Accordingly, *if* (16) and (17) involve genuine quantification, it is more natural to read the constituent quantifiers as first-order. For these reasons, the natural language construction of the form "*a* bears---*to b*" appears unsuited as a basis for understanding what the genuinely higher-order predicate "$a\Phi b$" really means.

7. Conclusion

I have argued that whether mutually converse predicates co-refer or they don't, difficulties arise for the interpretation of higher-order quantifiers as ranging over a domain of relations. If, on the one hand, mutually converse predicates co-refer, then the attempt to quantify into the positions they occupy conflicts with the Division of Semantic Labour. If, on the other hand, they pick out distinct relations, then we lack a grasp of the higher-order predicates required to characterize relations in a higher-order setting, a grasp that is appropriately rooted in our understanding of atomic statements. We may have other theoretical reasons to hold the metaphysical doctrine that relations apply in an order (in the strong sense), but I have argued that that doctrine isn't credible as a presupposition of higher-order logic.

These arguments don't tell us that second-order quantification per se is unintelligible because it remains open that second-order quantifiers may be interpreted along other lines, i.e., other than ranging over a domain in mimicry of the manner in which first-order quantifiers are typically understood to do so. Nevertheless, we now have novel and independent reasons to favour alternative interpretations that don't treat second-order existential introduction as a straightforward generalization of first-order existential introduction—whether in terms of quantification over the extensions of predicates, rather than properties and relations conceived as the referents of predicates, or in terms of quantifiers that aren't conceived as having ranges at all.[17] And we now have strong reasons to doubt that second-order logic has a distinguished claim to be the logic of relations because of the difficulties that attend quantifying into the positions of converse predicates.[*]

Fraser MacBride

17 Alternative interpretations vary from Shapiro's (1991) relatively conservative proposal that second-order quantifiers range over extensions of predicates conceived as sets to the more radical interpretations inspired by Prior's idea that non-nominal quantifiers lack a range altogether (Prior 1971, 31–33; MacBride 2006, 442–447; Wright 2007; and Sainsbury 2018, 28–61). The conclusion of this paper can also be seen as support for Leo's more radical proposal that we require a logic that eschews any kind of artificial ordering altogether (Leo 2014, 2016).

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Converse Relations and the Sparse-Abundant Distinction

Francesco Orilia

Traditionally, we distinguish between relations and their converses, e.g., *above* and *below* or *before* and *after*. This distinction poses a dilemma. Is a relation really distinct from its converse or are they one and the same? There are contrasting arguments that favor one or the other reply, both of them in Russell, who first opted for the former (in *Principles of Mathematics*) and then for the latter (in *Theory of Knowledge*). Since then, accounts of relations that side with one or the other option have flourished. A hybrid approach to properties and relations (attributes), according to which there are both sparse and abundant attributes, is here offered as a way out of the dilemma: distinct converses are acknowledged at the semantic or propositional level of abundant attributes, and rejected at the truthmaker or ontological level of sparse attributes. A positionalist account of relations is also adopted, *role positionalism*, according to which positions are understood as *roles*, which are ontological or semantic counterparts of the thematic roles invoked in linguistics. In this way, distinct abundant converses differ because of the different roles involved in them, but they are intimately connected in that they correspond to a single sparse relation.

Traditionally, we distinguish between relations and their converses, e.g., *above* and *below*, *before* and *after*, *giving* and *receiving*. This poses a dilemma. Is a relation really distinct from its converse or are they one and the same? To put it otherwise: should we admit, *pro-converses option*, that relations have distinct converses, or should we rather, *anti-converses option*, deny that? There are two contrasting arguments that favor one or the other alternative. Both of them can be found in Russell's *Principles of Mathematics* (*POM*, 1903). One is a *semantic* argument; in a nutshell, pairs of *converse predicates* such as "is above" and "is below," appear to have different meanings and thus must stand for distinct relations. The other is an *ontic* argument; if, e.g., an airplane flies over a bird, even though at some point we can describe how they are mutually situated with two different converse predicates, “the airplane is above the bird” or “the bird is below the airplane,” surely there is just one relational state of affairs or fact that we are describing, which suggests that only one relation is involved.

In *POM*, Russell privileged the semantic argument and thus opted for the pro-converses option. He did this by buying a *directionalist* approach to relations, the *standard view*, according to Fine (2000). Later on, however, in the 1913 manuscript *Theory of Knowledge* (*TK*, 1984), he came to privilege the ontic argument and shifted to the anti-converses option. He thus endorsed a *positionalist* account of relations. Since then, many philosophers have opted for one of the options while rejecting the other. Followers of the pro-converses option include Grossmann (1983), Wilson (1995), van Inwagen (2006). Moreover, this route seems implicit in first-order logic with its standard model-theoretic semantics, where relational predicates are interpreted as sets of ordered sets. Among the supporters of the anti-converses option, there are Castañeda (1975), Williamson (1985), Hochberg (1987), Fine (2000), Dorr (2004), MacBride (2014), Paolini Paoletti (2021b). I myself have defended an account that seems to leave no room for converse relations (Orilia 2008, 2011, 2014).

However, both the semantic and the ontic arguments make reasonable demands on a theory of relations, and thus these “exclusivist” approaches do not fully release the tension that the dilemma generates. I shall thus offer a way out that tries to do justice to both of its horns. Following Bealer (1982) and Lewis (1983, 1986), it is common to distinguish between a *sparse* and an *abundant* conception of properties and relations (in short, *attributes*) (see Orilia and Paolini Paoletti 2020, sec. 3.2). The way out takes advantage of a *dualist* view, which admits both sparse and abundant attributes. In essence, at the ontological, or truthmaker, level, where attributes are sparse, there are no distinct converses, whereas at the semantic, or propositional, level, where attributes are abundant, there are distinct converses. At both levels the proposed approach is *role positionalist*, that is, it takes positions to be *roles*, such as *agent*, *patient*, *source*, *destination*, *location*, etc. [whether *o-roles* or *c-roles*, as we shall see; Orilia (2010), 6]. The motivations for, and the implications of, this move will be clarified in the following.

Here is a preview of the paper. In section 1, I consider the two arguments offered by Russell in *POM* and briefly illustrate the directionalist approach of *POM* and the positionalist approach of *TK*. In section 2, I focus on the ontic argument and show how it can be accommodated at the truthmaker level by a role positionalism that buys the anti-converses option. In section 3, I elaborate on the semantic argument and show how we can do justice to it by invoking abundant relations with a role positionalism that makes room for the pro-converses option. In section 4, I discuss how sparse and abundant attributes can co-exist in a dualist view of attributes that reconciles the pro-converses and the anti-converses options. In section 5, I briefly consider some possible objections and close the paper.

1. Russell's Two Arguments, Directionalism and Positionalism

In *POM*, Russell hints at the ontic argument that later will lead him to positionalism, but he sets it aside, while giving greater weight to the semantic argument, based on the different meanings of pairs of converse predicates such as "greater" and "less." Here is the relevant passage (in 1903, sec. 219):

> It may be said that, owing to the exigencies of speech and writing, we are compelled to mention either *a* or *b* first, and that this gives a seeming difference between "*a* is greater than *b*" and "*b* is less than *a*"; but that, [*ontic argument*] in reality, these two propositions are identical. But [*semantic argument*] if we take this view we shall find it hard to explain the indubitable distinction between *greater* and *less*. These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and are certainly relations.

In an effort to accommodate the semantic argument, in *POM* Russell develops an approach according to which relations have an intrinsic *sense* or *direction*.[1] It can thus be aptly called *directionalism*. Russell (*POM*, 1903, sec. 94) puts it thus: "it is characteristic of a relation of two terms that it proceeds, so to speak, *from* one *to* the other. This is what may be called the *sense* of the relation [...]." The idea is that, since relations are endowed with a sense or direction, they are exemplified by relata as given in an appropriate order. And there can be relations that differ from one another merely in their direction and otherwise have, one might suggest, an identical *content* (Fine 2000, 11); such relations are mutual converses. In this way, Russell makes room for the

1 Russell uses the term "direction" in *TK* but not in *POM*, as far as I can tell.

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pro-converses option. For example, *above* and *below* differ merely in their respective directions, say d_1 and d_2, and otherwise have the same content, say C. Hence, we could represent them as "C_{d_1}" and "C_{d_2}," respectively. They are such that, necessarily, if C_{d_1} is exemplified by two objects in a certain order, then C_{d_2} is exemplified by the same objects in the opposite order. In effect, the approach is telling us that a relation is exemplified not simply by some objects but by an ordered set of objects (Castañeda 1975, 239).

To illustrate, suppose the airplane, a, is flying over the bird, b, so that the following is true:

(1) a is above b.

In this case, there is a fact consisting of the relation *above* proceeding from the airplane to the bird, i.e., the relation C_{d_1} exemplified by an ordered set with the airplane and the bird, in that order, as members:

(1#) $\mathbf{C_{d_1}}\langle \mathbf{a}, \mathbf{b}\rangle$,

and there is also another fact, conveyable by

(1′) b is below a,

consisting of the relation *below* proceeding from the bird to the airplane, i.e., the relation C_{d_2} exemplified by a different ordered set, with the airplane and the bird in the opposite order as members:

(1′#) $\mathbf{C_{d_2}}\langle \mathbf{b}, \mathbf{a}\rangle$.

Here I have used boldface fonts to highlight the intention to represent a state of affairs, or more generally, an entity at the ontological level of truthmakers.[2] When deemed useful, I shall follow this convention in the following as well.

Directionalism presents an *ontic* hurdle, we may say, for it is of course very hard to make sense of the idea that objects are exemplified in an order (van Inwagen 2006; MacBride 2020, sec. 1). In *TK*, however, Russell abandons directionalism not so much for this hurdle but because he comes to privilege

2 I am assuming there are both propositions and states of affairs (or facts), with true propositions made true by states of affairs and false propositions lacking a corresponding state of affairs. In *POM*, Russell does not distinguish between states of affairs and propositions and takes the distinction between true and false propositions as indefinable. Hence, from his *POM* perspective, we should say that (1#) and (1′#) are two true propositions rather than two states of affairs. However, we can neglect this for present purposes.

the ontic argument while downplaying the semantic argument (1984, 84). As we can see from the above example, by distinguishing converses via directions, directionalism invites us to assume that there are two distinct facts, (1#) and (1′#), where we should think there is only one fact. To avoid this multiplication of facts, Russell comes to favor positionalism, in which relations have no intrinsic directions, thereby leaving no room for distinguishing converses in the way directionalism does. In Fine's (2000, 10–11) terminology, positionalist relations are "neutral or unbiased" and, correlatively, the relations with sense of directionalism are "biased." However, such neutral relations are exemplified in different ways by relata, depending on the different "positions," or "argument-places," that the relata have with respect to the relation. For example, (1) and (1′) are different representations of one and the same fact consisting of a neutral relation, N, jointly exemplified by the airplane and the bird, in such a way that the former has one position, say P_1, with respect to the relation, and the latter has another position, say P_2. In contrast, if it were the bird to be above the airplane, N would be exemplified by the airplane and the bird in such a way that the former would have position P_2 and the latter position P_1. Fine (2000, 11) puts it as follows: "Exemplification must be understood to be relative to an assignment of objects to argument-places," and also suggests that we can view positions as holes of different shapes and exemplification with respect to positions, or assignment to argument-places, as the filling of such holes by relata; in *TK*, Russell proposes a different picture in terms of the hooks and eyes of goods-trucks (1984, 86). Useful as these metaphors may be for illustrative purposes, they must be ultimately set aside in favor of a more precise characterization of what exemplification of a relation with respect to a position amounts to. We shall deal with this in the next section. For the time being, let us follow the hole metaphor and assume that in our case the holes are [ ] and ( ), with the airplane filling the former and the bird the latter. Then, the unique fact represented by both (1) and (1′) can be represented thus:

(1c) **N**(**a**)[**b**].

This fact exists if (1) and (1′) are true. The writing order in this approach should not be taken to convey any information. Thus, (1c) and

(1d) **N**[**b**](**a**)

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are one and the same fact.[3]

Even though, as noted, first-order logic and its set-theoretical semantics may be viewed as implicitly embodying directionalism, the current scenario seems to be more favorable to the anti-converses option, as the recent works cited above testify. This may be due to the fact that the focus has been on the ontological level, while the semantic level has been neglected. However, both levels deserve consideration. I shall now turn to the ontological level and then move to the semantic level.

2. Positionalist Relations as Sparse Attributes

As traditionally understood, sparse attributes account for the objective resemblances of things and for their causal powers, and with empirical science we try to individuate them a posteriori. They have coarse-grained identity conditions based on necessary equivalence. To illustrate, among sparse attributes we admit there are properties accepted by current science such as *negative charge* or *spin up*, but we now rule out that there is *caloric* or *unicorn*. We also admit a property such as (*made of molecules of*) H_2O, but we do not see this as a property over and above the property *water*. They are one and the same property on account of the fact that, necessarily, whatever is water is made up of molecules of $\mathrm{H_2O}$. I am taking for granted here what Schaffer (2004) calls the "scientific conception" of sparse attributes, according to which they include not only the fundamental attributes of microphysical reality, but also attributes from all layers of reality: macro-physical, chemical, biological, psychological. Hence, H_2O counts as a sparse property. And, as this example shows, sparse attributes need not be simple, for H_2O is a complex property involving, *inter alia*, the further properties *hydrogen* and *oxygen*.

As Schaffer (2004, 99) notes, sparse attributes should be invoked when we look at reality as a source of truthmakers for true sentences or propositions. Following Armstrong (1997), we may view truthmakers as states of affairs consisting of the exemplification of attributes by objects, where the attributes

3 As Fine (2000) makes it clear, both directionalism and positionalism can be seen as different explanations of *differential application* (or *relational order*, in Hochberg's (1987, 443) terminology), i.e., that relations can be exemplified by the same relata in different ways; e.g., *loving* is exemplified in one way by Romeo and Juliet insofar as Romeo loves Juliet and in another way, insofar as Juliet loves Romeo. Beside considering the problems posed by converses, current approaches to relations are quite sensitive to those raised by relational order (MacBride 2020, sec. 4). It seems to me that directionalism is not fully successful in accounting for it (see Orilia 2008, sec. 6), but we need not insist on this for present purposes.

in question are sparse attributes. In *monadic* states of affairs, there is simply an object exemplifying a sparse property, whereas in *relational* states of affairs, there are objects jointly exemplifying a sparse relation. Let us consider some examples. Suppose we focus on c, a certain amount of water in a glass before us, and make the following claims:

(2) c is water;
(2′) c is made of H_2O molecules.

They are both true, since c is in fact water and thus also a liquid made of H_2O molecules. However, as noted above, there is just one sparse property, call it ***W***, somehow characterizable as both water and H_2O. Accordingly, there is just one fact making (2) and (2′) true, namely:

(2*) **W**(**c**).

Imagine we now focus on a triangularly-shaped object, d, and assert:

(3) d is trilateral;
(3′) d is triangular.

They are both true, but there is only one sparse property that can be invoked to account for their truth, i.e., a certain shape, call it *T*, which d exemplifies, somehow characterizable as both triangular and trilateral. And thus, there is just one fact that makes both of them true:

(3*) **T**(**d**).

Let us now go back to (1) and (1′). Just as for the pairs (2)-(2′) and (3)-(3′), it is natural to assume that there is just one truthmaker, and thus, one should think, only one relation should be invoked in putting forward such a truthmaker. Directionalism offers us two distinct relations, whereas positionalism is content with just one. Clearly, the latter is favored at the ontological level that we are now considering. It is an approach that offers us just one relation when different ways of thinking and speaking might suggest there are two relations, pretty much as in each of the above examples, we get one property instead of two.

However, as we saw, positionalism calls for a clarification of what the exemplification of a neutral relation with respect to positions amounts to. This can hardly be done without dwelling in turn on the nature of positions.

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Fine (2000, 10) tells us that they are *specific entities*; what sort of entities? I think the best course is to take positions to be properties that are exemplified by the relata of a relational state of affairs inasmuch as, or insofar as, such relata jointly exemplify the relation: when the relata jointly exemplify the relation, by the same token they also exemplify the positions in question (Orilia 2011, 2014).[4] Which properties work as positions and which relation is the neutral relation in our case?

What we find in reality is a certain spatial configuration with two items vertically aligned with respect to the earth's surface, and the configuration is such that one of the two items is closer to such a surface and the other is further away from it, so that one's location is higher than the other's. Thus, the neutral relation is a relation of vertical alignment (cf. MacBride 2007, 34) with respect to the earth's surface, call it ***V***, and the positions could be characterized as *superior* and *inferior*. Hence, the single truthmaker for (1) and (1′) postulated by positionalism turns out to be as follows:

(1*) $\mathbf{V}(\mathbf{superior}(\mathbf{a}), \mathbf{inferior}(\mathbf{b}))$.

Again, the writing order should not be taken to convey any information: (1*) is the same fact as

(1**) $\mathbf{V}(\mathbf{inferior}(\mathbf{b}), \mathbf{superior}(\mathbf{a}))$.

This notation is meant to highlight that the exemplification of the neutral relation **V** by the two relata, **a** and **b**, goes hand in hand with the exemplification of the properties **superior** and **inferior** by the relata in question, so that the existence of (1*) involves the existence of two further facts consisting of the exemplification of the two positions by the relata, namely **superior**(**a**) and **inferior**(**b**). It is important here not to be misled by the fact that we are used to read formulas of first-order logic of the form "$R(x, y)$" as telling us that the relation R holds between entities x and y; for (1*) and (1**) do *not* tell us that the relation of vertical alignment, **V**, holds between the two entities **superior**(**a**) and **inferior**(**b**). It rather tells us that this relation holds between **a** and **b** *insofar as* there are also the facts **superior**(**a**) and **inferior**(**b**).[5]

4 Expressions such as "insofar as" or "by the same token" are counterparts of Latin expressions such as "quatenus" or "et eo ipso" used by Leibniz in his analyses of relations Orilia (2008).

5 More generally, a *relational* formula of the type "$\boldsymbol{R}(\boldsymbol{p}_1(\boldsymbol{a}_1), \ldots, \boldsymbol{p}_n(\boldsymbol{a}_n))$," where "$\boldsymbol{R}$" stands for a neutral relation, each "$\boldsymbol{p}_i$" stands for a position and each "$\boldsymbol{a}_i$" stands for a relatum, tells us that the relation $\boldsymbol{R}$ holds between the relata insofar as each relatum $\boldsymbol{a}_i$ exemplifies the correspond-

Starting from Russell himself, positions have typically been considered entities somehow rigidly associated with one specific relation (Russell 1984; Hochberg 1987; Fine 2000; Gilmore 2013; Dixon 2018). For example, there are positions *lover* and *beloved* associated with *loving* and to no other relation; *hater* and *hated* associated only with *hating*; *giver*, *given*, and *givee* associated only with *giving*; and so on. Positions as so conceived are, we may say, *idiosyncratic*. In contrast with this, I have argued (2011, 2014) that positions had better be considered as *inter-repeatable*, i.e., multiply associated with different relations, for this may reflect objective resemblances in the real world, "similarities in arrangement" (2011, 5), which we should want to capture in our conceptualization. For example, there is something in common in the nice situation of someone loving someone else and in the nasty situation of someone hating someone else, namely that in both cases we can distinguish an active role, exemplified by the lover or by the hater, and a passive role, exemplified by the beloved or the hated. This can be captured by associating the same positions, *agent* and *patient*, to the different relations *loving* and *hating*. Similarly, e.g., the same positions, *source*, *theme*, and *destination*, can be associated with both *walking* and *running*, as triadic relations involving an item moving from one place to another. I have called positions as so conceived *onto-thematic roles*, in short, *o-roles* (2011), as they could be seen as ontological counterparts of the thematic roles postulated in linguistics, which I shall briefly discuss in the following.[6] Thus, for example, the state of af-

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ing position $\boldsymbol{p_i}$. Each "$\boldsymbol{p_i(a_i)}$" in this formula could be called a *positional term*. The structure **…(…, …, …)** of this notation, where the first gap is meant to be filled by a term for a neutral relation, and the gaps within the parentheses by positional terms, could be taken to correspond to the Leibnizian notion *insofar as*, which I have invoked to explain how the exemplification of a neutral relation should be understood. The irrelevance of the writing order can be made explicit by a general identity law. Given a formula A of the type "$\boldsymbol{R}(\boldsymbol{p}_1(\boldsymbol{a}_1), \dots, \boldsymbol{p_n}(\boldsymbol{a_n}))$," call *positional permutation* of A either A itself or any formula that results from A by writing in a different order the positional terms in A. (Clearly, if there are n positional terms in A, there are n! positional permutations of A.) Then the identity law is:

(IS) For any two positional permutations P_1 and P_2 of $\boldsymbol{R}(\boldsymbol{p}_1(\boldsymbol{a}_1), \dots, \boldsymbol{p_n}(\boldsymbol{a_n}))$, $P_1 = P_2$.

For example, "$\mathbf{V}(\mathbf{superior}(\mathbf{a}), \mathbf{inferior}(\mathbf{b}))$" and "$\mathbf{V}(\mathbf{inferior}(\mathbf{b}), \mathbf{superior}(\mathbf{a}))$" are positional permutations of each other, and thus (IS) certifies that this identity holds: $\mathbf{V}(\mathbf{superior}(\mathbf{a}), \mathbf{inferior}(\mathbf{b})) = \mathbf{V}(\mathbf{inferior}(\mathbf{b}), \mathbf{superior}(\mathbf{a}))$.

6 Positions had better be conceived of, not only as inter-repeatable, but also as *intra-repeatable*, i.e., as capable of being associated more than once with the same relation in a given state of affairs (Orilia 2014, sec. 3). I take it for granted that o-roles, as well as the c-roles to be discussed in the next section, are not only inter-repeatable but also intra-repeatable.

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fairs of Romeo's loving Juliet is *loving* exemplified by Romeo insofar as he exemplifies *agent* and by Juliet insofar as she exemplifies *patient*, which more formally could be put as **L**(**agent**(**r**), **patient**(**j**)). Similarly, the state of affairs of Romeo's father, Montague, hating Juliet's father, Capulet, is *hating* exemplified by Montague insofar as he exemplifies *agent* and by Capulet insofar as he exemplifies *patient*, or **H**(**agent**(**m**), **patient**(**c**)). We may call this approach *role positionalism*.[7]

Going back to our airplane and bird example, from a role-positionalist perspective, we should view the *superior* and *inferior* positions as o-roles, and thus we should see whether there are similarities in arrangement that they capture. If we look at directions in a sufficiently general way, not confined to spatial directions, there is room for noting a generality that is relevant here. There is a direction from lower to higher locations as we move in space away from earth, but similarly, there is a direction from earlier times to later times or from lower to higher magnitudes. We may thus see *superior* and *inferior* as o-roles that can be associated not only with spatial relations such as *vertical alignment* but also with relations of degrees of magnitudes, **D**, and of temporal succession, **T**. For example, we could acknowledge that the fact that makes it true that the height of Peter, h_1, is more than that of Mary, h_2, is something like $\mathbf{D}(\mathbf{superior}(\mathbf{h_1}), \mathbf{inferior}(\mathbf{h_2}))$, and that the fact that makes it true that the battle of Waterloo, b_1, is before the battle of Stalingrad, b_2, is something like $\mathbf{T}(\mathbf{inferior}(\mathbf{b_1}), \mathbf{superior}(\mathbf{b_2}))$ (since the time that has already elapsed when the former battle has taken place is more than the time that has already elapsed when the latter battle has taken place).[8]

To the extent that role positionalism distinguishes neutral relations and o-roles that can be associated with different neutral relations, it should similarly distinguish between a neutral relation as such, the bare neutral relation, so to

7 Since Castañeda (1967) commented on Davidson's theory of events, o-roles have been typically viewed as relations linking events, states of affairs, or the like to participants in them (see, e.g., Parsons 1990—I speak simply in terms of states of affairs, as for present purposes nothing hinges on this). I prefer my line in which o-roles are properties, since it grants a positionalist account of differential application (see Orilia 2011, sec. 5). Role positionalism has been endorsed by Paolini Paoletti (2016, 2021b), who, however, takes o-roles to be modes rather than properties understood as universals, as in my approach.

8 Alternatively, instead of invoking *superior* and *inferior*, we could appeal to the o-roles *source* and *destination*, respectively, as suggested in Orilia (2014, sec. 8). The corresponding thematic roles are, in fact, commonly used to indicate a directionality. However, this directionality is always taken to involve an object (typically classified as *theme*) moving (possibly in a metaphorical sense) from the source to the destination. In contrast, in the cases discussed above, there is no moving object.

speak, and a neutral relation as endowed with o-roles, which could be called an *embellished* relation.[9] We can conveniently represent embellished relations by allowing for blank spaces after the symbols corresponding to o-roles. To illustrate, the state of affairs **L**(**agent**(**r**), **patient**(**j**)) involves, on the one hand, the neutral loving relation, **L**, and, on the other hand, the following embellished relation: **L**(**agent**( ), **patient**( )). Similarly, **H**(**agent**(**m**), **patient**(**c**)) involves, on the one hand, the neutral hating relation, **H**, and, on the other hand, the following embellished relation: **H**(**agent**( ), **patient**( )). In appealing to this notation, it is important to emphasize once more that writing order is not significant in this context, so that, e.g., **L**(**agent**( ), **patient**( )) and **L**(**patient**( ), **agent**( )) are the same relation.[10,11]

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9 Fine (2000, 11) implicitly makes a similar distinction within positionalism between neutral relations as considered independently of positions and neutral relations as endowed with positions and points out the analogous difference in directionalism between biased relations, involving a content and a direction, and the pure contents somehow implicit in biased relations.

10 We can convey this point in a general fashion with this identity law for sparse, embellished relations:

(IR) For any two role permutations P_1 and P_2 of $\boldsymbol{R}(\boldsymbol{r}_1(\,), \dots, \boldsymbol{r}_n(\,))$, $P_1 = P_2$,

where a role permutation in a formula A of the kind "$\boldsymbol{R}(\boldsymbol{r}_1(\,), \dots, \boldsymbol{r}_n(\,))$" is either A itself or any formula that results from A by writing in a different order the *role terms*, "$\boldsymbol{r}_i(\,)$," in A. For instance, "**L**(**agent**( ), **patient**( ))" and "**L**(**patient**( ), **agent**( ))" are role permutations of each other, and thus, by (IR), **L**(**agent**( ), **patient**( )) = **L**(**patient**( ), **agent**( )). (IR) is analogous to the identity law for states of affairs (IS) (see footnote 5). When considering the latter, however, I had not yet dwelled on viewing positions as o-roles, and thus (IS) was presented in terms of positions rather than o-roles.

11 Partially symmetric relations such as *arranged clockwise in a circle* (Fine 2000, n. 10) and *playing tug-of-war* (MacBride 2007, 42) may appear to be problematic for positionalism. As a response, Donnelly (2016) has developed *relative positionalism*, according to which positions are understood as *relative*, i.e., as properties possessed by relata relative to other relata. Dixon (2019) defends this approach and notes that in order to handle similarities in arrangement, it could be turned into a form of relative role positionalism, which adopts relative inter-repeatable o-roles, rather than relative idiosyncratic positions (see his 2019, n. 11). I am using here my terminology (Dixon does not refer to my view in this context). However, if positions, whether idiosyncratic or inter-repeatable, are understood as relative, they appear to presuppose relatedness, which is what positionalism tries to explain in terms of positions (MacBride 2020, sec. 4). It thus seems to me a better course to tackle these problematic partially symmetric relations on a case-by-case basis, so as to show that they reduce to more primitive relations that can be understood in terms of o-roles without recourse to relative positions (Orilia 2011, 9, n.11).

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3. Distinct Converses as Abundant Attributes

Abundant attributes are assumed a priori as meanings of predicates and contributors to mental contents, i.e., accusatives of intentional attitudes such as beliefs. They exist, even if unexemplified. For example, *unicorn* can still be acknowledged among the abundant properties as meaning of the predicate "is a unicorn," even though it has turned out that nothing exemplifies such a property. And we can have a mental content involving it; e.g., someone may correctly believe that nothing is a unicorn and someone else may incorrectly believe that something is a unicorn. Abundant properties have very fine-grained identity conditions, not reducible to necessary equivalence. For example, despite their necessary equivalence, *water* and H_2O are distinct abundant properties working as meanings of two distinct predicates such as "is water" and "is H_2O," respectively. One of these properties requires ordinary, commonsensical knowledge to be grasped, whereas the other requires some grasp of chemistry. And in fact, someone may have a mental content involving the former without thereby having a mental content involving the latter; e.g., someone could believe that c, the liquid in the glass, is water without believing that c is H_2O. Thus, sentences (2) and (2′) express two different propositions, i.e.,

(2a) water(c)

and

(2′a) H_2O(c).

And someone could believe the former without believing the latter.

Similarly, despite their necessary equivalence, *triangular* and *trilateral* are distinct abundant properties working as meanings of two distinct predicates such as "is triangular" and "is trilateral," respectively, and in principle, someone could believe that the triangularly-shaped object, d, is triangular without thereby believing that d is trilateral, so that (3) and (3′) express different propositions, namely,

(3a) trilateral(d)

and

(3′a) triangular(d).

In the former case, the necessary equivalence in question can be known a posteriori via empirical investigation, whereas in the latter case, it can be known a priori via conceptual analysis. When the required conceptual analysis is simple and trivial, it may be hard to imagine that someone could have a belief involving a certain property *P* without having a corresponding belief involving a property *Q* that, by conceptual analysis, is equivalent to *P*. However, it becomes easier to see once we focus on cases in which the analysis is non-trivial and a fair amount of inferential effort is indispensable.

Now, just as "trilateral" and "triangular" appear to have distinct meanings and thus are taken to stand for different abundant properties, similarly, as Russell urges in his semantic argument, converse predicates such as "greater" and "less," or "is above" and "is below," appear to have distinct meanings and thus should be taken to stand, from this abundantist perspective, for distinct mutual converses. And in fact, we should acknowledge that someone might have a belief involving a certain abundant relation without thereby having a corresponding belief involving a converse of the relation in question.

Consider (1) and (1′), as well as these other pairs of sentences:

(P1) (i) 4 is greater than 2;
(ii) 2 is less than 4;
(P2) (i) Romeo loves Juliet;
(ii) Juliet is loved by Romeo;
(P3) (i) Milan is north of Rome;
(ii) Rome is south of Milan;
(P4) (i) the year 2019 is before the year 2020;
(ii) the year 2020 is after the year 2019;
(P5) (i) Tom owns the car;
(ii) the car belongs to Tom;
(P6) (i) John gives the ball to Richard;
(ii) Richard receives the ball from John.

It might be hard to imagine that someone could believe the proposition expressed by one member of one of these pairs without believing the proposition expressed by the other member of the pair. And yet, it should be granted that some amount of inferential effort, modest as it may be, is necessary to convince oneself that the sentences in each pair express necessarily equivalent propositions. So that, before this inferential effort, one could believe any of these propositions without believing their necessarily equivalent mates. In sum,

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we should make room for the pro-converses option so as to allow converse predicates to have different meanings.

One way to do this is by buying role positionalism. Arguably, it is a peculiarly interesting and plausible way, since the appeal to roles appears to be fruitful in linguistics in accounting for a wide range of phenomena (see, e.g., Davis 2011, 400), and, as noted, it aims at capturing existing generalities. It is then worth seeing how the pro-converses option can be accommodated at the abundant level from a role-positionalist perspective. Before doing it, some clarifications are in order.

The thematic roles invoked in linguistics, in short, *t-roles*, can be seen as properties that noun or prepositional phrases implicitly have in the context of the sentences in which they occur. Phrases can come to have the t-roles they happen to have in a variety of ways, depending on different languages, and to understand which t-roles are in play is crucial to understanding a sentence and translating it into a language that exploits different conventions in assigning t-roles. Consider, for example, these equivalent English and Latin sentences:

(E) Mark kills Antony with the sword;
(L) Marcus Antonium gladio interficit.

According to a typical analysis, in (E) "Mark," "Antony," and "with the sword" have the t-roles *Agent*, *Patient*, and *Instrument* (following Davis 2011, I use an initial uppercase letter to indicate t-roles—this helps us to distinguish them from o-roles and from the c-roles to be considered in a moment). The expressions in question gain such t-roles, respectively, as follows: by preceding the verb, by following the verb, by containing the preposition "with." Similarly, in (L), "Marcus," "Antonium," and "gladio" have the t-roles *Agent*, *Patient*, and *Instrument*. However, in this case, they acquire these t-roles by having appropriate case endings, namely, "-us," "-um" and "-o," respectively. It is essential to realize that, despite these different conventions, the same t-roles are involved in both sentences in order to understand them and see that they translate each other. Clearly, we grasp which t-roles phrases may have because we associate them with roles or functions that objects can play: objects can indeed act, undergo the effects of actions, or be used as tools.[12] There are then

12 This may seem to conflict with taking t-roles to be properties of both noun phrases, e.g., "Mark," *and* prepositional phrases, e.g., "with the sword" (as I have done). For whereas we typically take noun phrases to correspond to individuals that play roles in situations, there is not such a

meanings and mental contents corresponding to the t-roles. Since abundant properties are posited as meanings and mental contents, it is then natural to say that there are abundant properties such as *agent*, *patient*, *instrument*, and the like, which we grasp as concepts in recognizing the t-roles involved in the sentences we use and which occur as constituents of the propositions expressed by such sentences. We may call them *cognitive-thematic roles*, or, in brief, *c-roles* (Orilia 2011, 6). Thus, in order to understand a *relational* sentence expressing a *relational* proposition, we must grasp not only which neutral relation is expressed by the verb in the sentence but also the c-roles in question, and thus which embellished relation is expressed by the verb taken together with the t-roles. Grasping what such roles are, and which arguments they are associated to, goes hand in hand with grasping the embellished relation.

To illustrate all this and how c-roles occur as constituents of propositions, let us consider the proposition expressed by both (E) and (L), which I represent as follows:

(E/L) kill(agent(m), patient(a), instrument(s)).

It should be clear from this notation that, just as I viewed o-roles as sparse properties that are exemplified by relata inasmuch as such relata exemplify a certain neutral relation, I similarly assume that c-roles occur as abundant properties attributed to arguments of an abundant relation. In this case, *killing* is the abundant neutral relation, and m, a, and s are the arguments. In general, from a role-positionalist standpoint, a relational proposition, which attributes a relation to some arguments, involves, by the same token, the attribution of the relevant c-roles to the arguments in question. Thus, (E/L) is taken to entail these further propositions: agent(m), patient(a), instrument(s).

It is useful to note here that there are two senses in which we can identify a predicate in a basic sentence, such as (E) or (L). On the one hand, we can say that the predicate is the verb, "kills" in (E) and "interficit" in (L); we may call this the *verbal predicate*. The verbal predicate typically expresses a neutral relation, which can be seen as a constituent of the proposition expressed by the sentence in which the verb occurs. For example, both "kills" and "interficit" express the neutral relation *killing*, which is a constituent of the proposition (E/L). On the other hand, there is the predicate constituted

direct correspondence in the case of prepositional phrases: "with the sword" as such is not taken to correspond to an individual that plays a role in a situation. However, prepositional phrases typically contain noun phrases that correspond to individuals that play roles in situations, e.g., "the sword." Hence, there is really no conflict.

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by the verb and the t-roles implicitly present in the sentence, which we may call the *phrasal predicate*. We can make the phrasal predicate explicit by appealing to variables. For example, in (E) we have the phrasal predicate "x kills y with z," and in (L) we have the phrasal predicate "x-us y-um z-o interficit."[13] The phrasal predicate expresses an embellished relation of the abundant level, which can also be seen as a constituent of the proposition expressed by the sentence in which the phrasal predicate occurs. We can appropriately represent the embellished relations of the abundant level by resorting to the lambda notation. Thus, for example, the embellished relation expressed by both the English and the Latin phrasal predicate that we are considering is λxyz kill(agent(x), patient(y), instrument(z)), which can be seen as a constituent of the proposition (E/L).

We are now ready to see how we can distinguish converses from this role-positionalist point of view. The idea is that converse *phrasal* predicates express distinct embellished relations, typically involving different c-roles. Let us go back to (1) and (1′) to illustrate this. In the first place, it is important to understand which propositions they express and, thus, in particular, which neutral relation is expressed by the verbal predicate and which c-roles are in play. It seems clear that the verbal predicate, "is," expresses, in this case, a neutral relation such as *situated*. This suggests that a *theme* c-role is in play since the t-role *Theme* is typically attributed to the noun phrase working as subject in sentences with a verbal predicate of this sort, a noun phrase intuitively corresponding to an object situated in a location (see, e.g., Jackendoff 1983, chap. 9). Moreover, it appears that the "above" of (1) and the "below" of (1′) correspond to two distinct c-roles. In keeping with the idea that c-roles are properties, we may say that the former corresponds to the property of being a boundary of a place extending upward (away from the earth's surface), the *abover* property, whereas the latter corresponds to the property of being a boundary of a place extending downward (toward the earth's surface), the *belower* property. In sum, an object that exemplifies *abover* is the lower boundary of some space, which counts as a place that some other object occupies, and similarly, an

13 Phrasal predicates sensitive to case endings must, of course, be managed with care because attention must be paid to the distinction between a case ending and the word root to which the case ending is attached; variables are taken to correspond to the latter. For example, in "Maria Antonium amat" ("Mary loves Antony"), there are word roots "Mari-" and "Antoni-" with nominative and accusative case endings, "a" and "um," respectively. Accordingly, we get the phrasal predicate "x-a amat y-um." Alternatively, one may invoke here traditional names of case endings and rather convey the phrasal predicate as follows (with obvious abbreviations): x-nom amat y-acc."

object that exemplifies *belower* is the upper boundary of some space, which counts as a place that some other object occupies (it should be noted that the abover object is the object that is below, the bird in our example, and the belower object is the object that is above, the airplane in our example; this may sound counterintuitive, but it is in line with the fact that the preposition "above" precedes the noun phrase standing for the object that is below, and "below" precedes the noun phrase standing for the object that is above). Hence, the propositions *a is above b* and *b is below a*, expressed respectively by (1) and (1′), can be represented as follows:

(1a) situated(theme(a), abover(b));
(1′a) situated(theme(b), belower(a)).

What (1a) conveys is that the airplane occupies a place by being situated within the space extending upward from the bird, whereas (1′a) tells us that the bird occupies a place by being situated within the space extending downward from the airplane. We know by conceptual analysis that these propositions are equivalent, indeed necessarily equivalent, as they simply offer different ways of conceptualizing the same spatial configuration; when two objects are vertically aligned, we can see one as placed in a spatial region delineated in the upward direction by the other object, or we can see the latter object as placed in a spatial region delineated in the downward direction by the former object. Thus, in general, we know that, necessarily, $\forall x \forall y$(situated(theme(x), abover(y)) $\leftrightarrow$ situated(theme(y), belower(x))).

We can now identify the converses *above* and *below* with the two embellished relations λxy situated(theme(x), abover(y)) and λxy situated(theme(x), belower(y)). They have a common neutral relation, *situated*, and also a c-role in common, namely *theme*, but they crucially differ in that one involves the *abover* role and the other the *belower* role. As the above discussion shows, we know that they are mutual converses by conceptual analysis, just as we know that the propositions (1) and (1′a) are necessarily equivalent.[14]

As we saw, when Russell, in *POM*, accepted the pro-converses option, he did this by endorsing directionalism. It should be clear at this point that this choice is in the way of a full understanding of how converse predicates may differ

14 It is worth noting that we need not take these c-roles as rigidly associated with the spatial relation *situated*. Just as with the **superior** and **inferior** o-roles discussed in the previous section, the c-roles *theme*, *location*, *abover*, and *belower* could be seen as inter-repeatable and associated with relations of temporal succession and of degrees of magnitude (Jackendoff 1983, chap. 10).

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in meaning. For directionalism makes it seem as if the difference between two converse relations has simply to do with the order in which the relata are given.[15] This leads to the typical way in which, following *POM* (1903, sec. 28, §94), the distinction between a relation and a corresponding converse is introduced (Fine 2000, 3; MacBride 2020, sec. 1): a *converse* of a binary relation R is a relation R^* such that, necessarily, R holds between x and y whenever R^* holds between y and x. For example, *above* has *below* as its converse since the former holds between x and y, *in that order*, whenever the latter holds between y and x, *in that other order*. More generally, a converse of an n-ary relation R is a relation R^* such that, necessarily, R holds between $x_1, \ldots, x_n$, just in case R^* holds between a permutation of $x_1, \ldots, x_n$, e.g., $x_2, x_1, x_3 \ldots, x_n$. For example, *giving* holds between x, y, and z (i.e., x gives y to z) whenever *receiving* holds of the permutation z, y, x (i.e., z receives y from x). More formally, in the familiar language of quantificational logic, one simply says "$Rx_1 \ldots x_n$," or "$R(x_1, \ldots, x_n)$," instead of "R holds between $x_1, \ldots, x_n$."

In contrast with what directionalism suggests, thinking of the relata in a certain order seems neither necessary nor sufficient to capture the perceived meaning difference in members of pairs of converse predicates such as "is above"/"is below." Turning again to Latin, wherein word order is less rigid than in English, allows us to bring this easily to the fore. For example, in Latin, we can say both "Maria supra equo est," which we can literally translate in standard English as "Mary is above the horse," and equivalently "sub Maria equus est," which we can literally translate in not quite standard yet intelligible English as "below Mary, the horse is." In both cases, we think first of Mary and then of the horse, and yet, in one case, we are thinking of them as related by *above* and in the other case as related by *below*. Hence, it does not seem that thinking order is sufficient to tell us which of these pairs of relations is involved. On the other hand, in Latin, beside "Maria supra equo est," we can equivalently say "supra equo Maria est," which we can literally translate into intelligible English as "above the horse, Mary is." In one case we think first of Mary and then of the horse, and in the other case we think first of the horse and then of Mary, and yet it seems in both cases we think of them as related by *above*, not first by *above* and then by *below*. Thus, it seems that thinking order is not necessary to switch from one relation to its converse.[16]

15 This shortcoming of directionalism adds up to its problem with Russell's ontic argument and its inadequacy in explicating differential application, mentioned in footnote 3.

16 In this discussion of directionalism, and perhaps elsewhere in the paper, I may give the impression that I take prepositions such as "below" and "above" as straightforwardly standing for relations.

Fortunately, as we have seen, we need not bind the pro-converses option to directionalism. By buying role positionalism, converses can be distinguished via different c-roles, independently of the sequential order by which we think of relata, as illustrated by the analysis of (1) and (1′) provided above. However, from the fine-grained standpoint of the abundant conception, the sequential order emphasized by directionalism may well be significant, and if this is taken into account, we can somehow recover the standard way of distinguishing between a relation and its converse and find a grain of truth in directionalism. The point is that thinking is sequential, at least as far as it is exercised with natural language, which works sequentially (Castañeda 1975, 243): we think via propositions that we express with natural language sentences, which are constructed by concatenating words in a sequential order, and this order could be relevant in determining which propositions are expressed. Consider, for example, "John is nice and Mary is beautiful" and "Mary is beautiful and John is nice." These two sentences differ merely in the order in which their sub-sentences are conjoined, and yet they could be taken to express two distinct, albeit necessarily equivalent, propositions that differ from each other in the order in which the conjuncts flank the conjunction (Bealer 1982, 54). After all, even in this case, some inferential effort is required to see the equivalence in question. Similarly, e.g., "a is above b" and "above b, a is" can be taken to express different, albeit necessarily equivalent, propositions: situated(theme(a), abover(b)) and situated(abover(b), theme(a)), which differ from each other merely in the order in which the subconstituents, theme(a) and abover(b), somehow occur in them. And accordingly, we should then also admit that there are two *above* embellished relations: a *theme first above*, namely $\lambda xy\,\text{situated}(\text{theme}(x), \text{abover}(y))$, and a *theme second above*, namely $\lambda xy\,\text{situated}(\text{abover}(x), \text{theme}(y))$. Clearly, the former holds between a and b just in case the latter holds between b and a, or, more formally, $\lambda xy\,\text{situated}(\text{theme}(x), \text{abover}(y))(\text{a}, \text{b}) \leftrightarrow \lambda xy\,\text{situated}(\text{abover}(x), \text{theme}(y))(\text{b}, \text{a})$.[17] This is in line with the stan-

In fact, as we have seen, I view them as standing for c-roles. Turning away from these prepositions and from Latin, a good example to illustrate how distinct converses may be evoked independently of thinking order is provided by the following pair of sentences: "the airplane is longer than the bird," "the airplane is less short than the bird."

17 The formulas on the two sides of the biconditional are respectively equivalent, by lambda conversion, to two other formulas, namely, "situated(theme(a), abover(b))" and "situated(abover(b), theme(a))," which should in turn be regarded as equivalent. The law of

dard way of presenting the distinction between a relation and its converse, and thus we could view λxy situated(theme(x), abover(y)) and λxy situated(abover(x), theme(y)) as converses. Their difference is however trivial, since it has to do simply with the order in which the c-roles involved in these relations occur. We should thus distinguish between *serious* converses, such as λxy situated(theme(x), abover(y)) and λxy situated(theme(x), belower(y)), which differ in some c-role, and *trivial* converses, such as λxy situated(theme(x), abover(y)) and λxy situated(abover(x), theme(y)), which differ merely in the order of the c-roles involved in them.[18] Directionalism is, at best, fit to capture the distinction between trivial converses. However, since it is silent about roles, it cannot tell us anything about the more intriguing differences between serious converses.[19]

4. Relations in the Dualist View of Attributes

The sparse and abundant conceptions of attributes are typically viewed as rival (see Orilia and Paolini Paoletti 2020, sec. 3.2), and if one looks at them in this fashion, not much is gained by noting that the former favors the anti-

lambda conversion is typically assumed once one resorts to the lambda notation and goes as follows: $\lambda x_1 \dots x_n A(t_1, \dots, t_n) \leftrightarrow A(x_1/t_1, \dots, x_n/t_n)$, where $A(x_1/t_1, \dots, x_n/t_n)$ is the wff resulting from simultaneously replacing each x_i in A with t_i (for $1 \leq i \leq n$), provided t_i is free for x_i in A.

18 Once we freely appeal to variables and the lambda notation, we can generate different terms for relations by simply changing the order of the variables we choose. And given the importance attributed to order at the abundant level, one may think that these terms may well stand, at least in some cases, for further distinct converses. For example, in addition to "λxy situated(theme(x), abover(y))," there is "λxy situated(theme(y), abover(x))," and one may think that the latter stand for a converse of the relation expressed by the former term; after all, we should grant, by lambda conversion, that λxy situated(theme(x), abover(y))(a, b) $\leftrightarrow$ λxy situated(theme(y), abover(x))(b, a). However, it does not seem wise to admit that distinct relations can be generated simply because we grant all this freedom in the choice of variables. We can avoid this result by using variables in a more regimented way in an effort to appropriately represent embellished relations. That is, we could conventionally assume that both the lambda variables (the ones following the lambda operator) and the variables in the open formula bounded by the lambda variables must always be used in alphabetical order (Orilia 2019, sec. 4). This rules out, as ill-formed, terms such as "λyx situated(theme(x), abover(y))," in which the lambda variables are not in alphabetical order, and terms such as "λxy situated(theme(y), abover(x))," in which the variables in the open formula are not in alphabetical order.

19 In Orilia (2019), I had already made room for the idea that there are distinct converses at the level of abundant attributes, but there I focused only on trivial converses without appealing to c-roles in order to investigate serious converses.

converses option and the latter the pro-converses option. We would still not know which option to pick. However, the two conceptions need not be viewed as rivals. Indeed, they should be considered as complementary, and in fact, the very promoters of the distinction accepted a hybrid view with both sparse and abundant attributes in order to account at the same time for the objective resemblances in the physical world and for matters of meaning and mental content. Following this line, we can accept both the anti-converses and the pro-converses options. Let us see how.

Abundant attributes can be taken to *correspond* to sparse attributes pretty much as the two Fregean senses of "Hesperus" and "Phosphorus" correspond to one and the same planet, or as the two Fregean senses of "the square root of 4" and "the even prime number" correspond to the number two, so that identity statements about properties can be taken to express the fact that two different abundant attributes correspond to the same sparse attribute (Orilia 1999).

The water/H_2O and triangular/trilateral examples can illustrate how this works. Let us start with the former and go back to sentences (2) and (2′). We saw that there are good reasons to think there is only one state of affairs, (2*), which involves a certain sparse property, **W**, and makes (2) and (2′) true. But we also saw that there are good reasons to think there are two distinct propositions, (2a) and (2′a), expressed by these two sentences, one involving the abundant chemical property H_2O and another involving the abundant commonsensical property *water*. Empirical investigation reveals that both properties correspond to one sparse property in the physical world, **W**. This correspondence may be expressed by an identity statement such as *H_2O is water* (or *to be H_2O is to be water*). However, in this perspective, the "is" of statements such as this should not be taken to express identity but the correspondence in question. It may be noted here that we shouldn't simply assert that water is H_2O, but that water is *reduced* to H_2O. This can and should be granted, of course, but it is quite compatible with the idea that we have two abundant properties corresponding to a single sparse property; we can grant that there is a reduction because the abundant property H_2O, by being embedded in a successful scientific theory with great explanatory and predictive power, reveals the hidden nature of the sparse property in question more perspicuously than the commonsensical abundant property *water*.[20]

20 We can then also say that the proposition that c is H_2O grounds the proposition that c is water, even though both have the same truthmaker.

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Consider now the trilateral/triangular example and turn to sentences (3) and (3′). Again, we granted a single truthmaker, (3*), involving a certain sparse property, **T**, and also granted two different propositions, (3a) and (3′a), expressed by these sentences, involving the different properties *triangular* and *trilateral*. As in the water/H_2O case, there are two abundant properties that correspond to the single sparse property **T**. There are, however, important differences: in this case, it is conceptual analysis that reveals that the two abundant properties must correspond to one sparse property, and we have no reason to think that one of these abundant properties reveals more perspicuously than the other the real nature of the sparse property.[21]

Let us finally move to converse relations and thus to our paradigmatic above/below example and to sentences (1) and (1′). It seems to me that the difference between *above* and *below* is analogous to the difference between *triangular* and *trilateral*. We acknowledged that there is only one state of affairs that makes both (1) and (1′) true, and hence we put forward a sparse neutral relation of vertical alignment, **V**, and the sparse o-roles **superior** and **inferior**, so that the state of affairs in question turns out to be (1*). We also admitted there are two propositions expressed by (1) and (1′) and accordingly put forward the propositions (1a) and (1′a), involving two different embellished abundant relations: λxy situated(theme(x), abover(y)) and λxy situated(theme(x), belower(y)). These two relations can be taken to correspond to the same sparse embellished relation, **V**(**superior**( ), **inferior**( )), just as *triangular* and *trilateral* correspond to the same sparse property, **T**. In both cases, we know a priori by conceptual analysis that there is such a correspondence, and we have no reason to think that one of the abundant attributes in question reveals more perspicuously than the other the

21 Once we distinguish two abundant properties corresponding to one sparse property, as is the case with *water* and *H_2O*, or *triangular* and *trilateral*, then the following results: on the one hand, all sorts of distinct abundant attributes can be constructed from the abundant properties in question, and, on the other hand, the relevant sparse property is involved at the truthmaker level. Consider, for example, the two abundant relations *contains more water than* and *contains more H_2O than* (I take such relations to be embellished relations, thus involving c-roles, but for the sake of making this point, it does not matter which they are). The former should be taken to contain *water* as a constituent, whereas the latter should be taken to contain *H_2O* as a constituent, and accordingly, they are distinct just as *water* and *H_2O* are distinct. However, the true propositions involving them will have truthmakers that involve the same sparse property, **W**. Suppose, for example, that *a contains more water than b* and *a contains more H_2O than b* are true. Then, there will be a truthmaker for both involving **W**, a state of affairs such as **a contains more W than b** (which I take to involve appropriate o-roles, which is not important to specify for the sake of making this point).

real nature of the sparse attribute. It should be noted here, however, that we can conceive of an abundant embellished relation that corresponds to the sparse relation in a more revelatory way. We could express this with a predicate such as "x and y are vertically aligned with x as superior and y as inferior" and take it to be λxy vertical-alignment(superior(x), inferior(y)). This abundant embellished relation has a distinct trivial converse, namely λxy vertical-alignment(inferior(x), superior(y)), which of course reveals the nature of the sparse relation **V**(**superior**( ), **inferior**( )) just as well. In contrast, there is no converse for the sparse relation: **V**(**superior**( ), **inferior**( )) and **V**(**inferior**( ), **superior**( )) are one and the same, as emphasized in section 2.[22] This sparse relation is involved in the truthmaker of (1) and (1′), namely (1*), which is the same as (1**).

Bealer (1982, 186) assumes there are primitive simple attributes, which are both sparse and abundant, wherefrom complex sparse attributes and complex abundant attributes are differently constructed: *condition-building operations* generate coarse-grained sparse attributes, and *thought-building operations* generate fine-grained abundant attributes. To illustrate, suppose P and Q are two primitive simple attributes, and & and $\wedge$ are, respectively, a thought-building conjunction operation and a condition-building conjunction operation; then P and Q are both abundant and sparse attributes, and $P \,\&\, Q$ and $P \wedge Q$ are, respectively, an abundant attribute and a sparse attribute. Similarly, $Q \,\&\, P$ and $Q \wedge P$ are, respectively, an abundant attribute and a sparse attribute. However, abundant attributes are extremely fine-grained, and thus $P \,\&\, Q$ and $Q \,\&\, P$ are distinct. In contrast, sparse properties are coarse-grained, and thus $P \wedge Q$ and $Q \wedge P$ are one and the same attribute. If we followed this line, we could similarly say that abundant c-roles and neutral relations, at least to the extent that they are primitive and simple, could be identified with sparse neutral relations and sparse o-roles, respectively. We could say, for example, that the abundant *vertical-alignment*, *superior*, and *inferior* are identical to the sparse **V**, **superior**, and **inferior**. Alternatively, we could say that even

22 Of course, in our boldface notation conventionally adopted to represent sparse relations, we can distinguish the two terms "V(superior( ), inferior( ))" and "V(inferior( ), superior( ))," which differ by the order in which the role terms are written. However, since there is no reason to think that in the realm of sparse attributes these two terms correspond to two distinct relations, we assume that **V**(**superior**( ), **inferior**( )) = **V**(**inferior**( ), **superior**( )), so as to neutralize the wealth of options offered by writing order, and more generally, we assume the identity law (IR) of footnote 10. In contrast, we saw that thinking order makes a difference at the level of abundant attributes, and thus no law analogous to (IR) is assumed for the lambda terms that represent abundant embellished relations.

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at the level of primitive simple attributes, we have correspondences between abundant and sparse attributes that fall short of identity, so that, e.g., the abundant *vertical-alignment*, *superior*, and *inferior* correspond, respectively, to the sparse **V**, **superior**, and **inferior** but are not identical to them. We may leave this open for present purposes, and similarly, we could leave it open whether there are complex sparse attributes built up from condition-building operations in the manner proposed by Bealer.

5. Conclusion

I considered in detail only one example of converses, but I expect it suffices to illustrate the general strategy and to indicate how other converses can be treated in an analogous manner. The role positionalism put forward here accommodates both Russell's ontic and semantic arguments and provides a way out of the dilemma they raise by rejecting converses at the level of sparse attributes and accepting them at the level of abundant attributes. It might seem, however, that it pays too high a price for this, since this strategy involves an ontological commitment to both sparse and abundant attributes. One might worry that lovers of desert landscapes would prefer only sparse attributes and lovers of jungles only abundant attributes, and that the combination of sparse and abundant attributes might be indigestible to both. However, the recourse to this dualism of attributes is independently motivated by the need to account simultaneously for matter and mind, or referents and meanings, and it is only by neglecting one or the other aspect that we can have the illusion of dispensing with either sparse or abundant attributes. And thus, it is quite legitimate to avail oneself of attribute dualism to resolve the dilemma about converses.

Even so, one could suspect that role positionalism has too many ontological commitments, for it is committed not simply to relations but to both neutral and embellished relations. In contrast, one could perhaps do with simply relations, as in the *primitivism* put forward by MacBride (2014) or in Fine's *anti-positionalism* (2000, sec. 4), further developed by Leo (2008, 2014), or even without relations, as in approaches that take all relations to be internal and do not consider internal relations as a real addition to being (Simons 2010; Lowe 2016). However, the distinction between neutral and embellished relations results from the appeal to roles, and roles, as we have seen, are needed to explicate how relations are exemplified by relata in ways that give rise to similarities in arrangements. Hence, having both neutral relations and embellished relations is not a burden but a theoretical advantage, as it helps us to account for the relatedness we find in the world and in our thinking about the world. This relatedness, it seems to me, is simply not fully appreciated by those who deny that there are relations. On the one hand, external relations appear to be ubiquitous; for instance, the very existence of mechanisms and structures presupposes them (Paolini Paoletti 2021a, 2021b), and, on the other hand, it is far from obvious that internal relations are not additions to being (MacBride 2020, sec. 3).

Of course, to do full justice to these objections would take us too far afield. I trust, however, that I have done enough to motivate this *dualist role positionalism*, as we may call it. It is a view that needs much further research, for its full development requires an appropriate inventory of o-roles and c-roles. I hope that this paper may contribute to stimulate research in this direction.*

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Non-Symmetric Relation Names

FRASER MACBRIDE & FRANCESCO ORILIA

Is it possible to name non-symmetric relations? If non-symmetric relations had distinct converses, then the difficulty of picking out and distinguishing a non-symmetric relation from its converses would plausibly present an insuperable obstacle to introducing names for them. But we argue that if non-symmetric relations lack converses, then the aforementioned difficulty does not arise. Moreover, we argue, at the semantic level, that English or modest extensions of English have the expressive resources to name non-symmetric relations whose adicity is greater than 2. Van Inwagen's case that it is impossible to name non-symmetric relations serves as our foil.

Can we name non-symmetric relations? If we cannot name them but only express relations with predicates, then we end up in an awkward predicament akin to Frege's paradox of the concept *horse*. Suppose the predicate "*x* loves *y*" expresses a dyadic non-symmetric relation. What relation does this predicate express? If we cannot name non-symmetric relations, then we cannot answer that question. The grammar of the question requires a name, or a definite description capable of figuring in the grammatical position of a name, to answer it—for example, "the relation of loving." If so, we are left in the awkward predicament of being unable to make the non-symmetric relation "*x* loves *y*" express the literal subject of our discourse, even though it is right under our noses and expressed by a familiar predicate. Frege's paradox of the concept *horse* is similar in the following respect. Predicates refer to concepts, according to Frege. But if we try to say what concept the predicate "*x* is a horse" refers to, we must use a name or definite description—for example, "the concept *horse*." But, by Frege's lights, names and definite descriptions pick out complete things, whilst the referents of predicates are incomplete. So "the concept *horse*" cannot pick out the referent of "*x* is a horse" (Frege 1892). Our inability to say what non-symmetric relation or Fregean concept a given predicate expresses speaks in favour of nominalism. Why believe in things as semantically awkward as non-symmetric relations or Fregean concepts, things that are resistant to being named?

But suppose we can neither name nor readily give up non-symmetric relations because their existence follows from other things we say. Then our predicament is both awkward and apparently inescapable. According to van Inwagen, this is indeed the predicament in which we find ourselves with respect to object-language reference to non-symmetric relations—although he does not draw our parallel with Frege.

In this paper, we argue that we need not succumb to van Inwagen's predicament.[1] At the ontological level, we are not committed to distinct converses of non-symmetric relations by our use of converse predicates. At the semantic level, we do have the resources in English, or modest extensions of English, to name relations within the object language. Many other natural languages have equal resources of this kind, or even better resources than English. The resulting perspective at which we arrive is one that vindicates the realist tradition not only because it recognizes that we can quantify over universals (relations) and employ predicates to express them, but also because it allows singular statements about them. We have reason to believe in the existence of universals (relations) because, inter alia, we are able to make statements in which a name is used to pick out a universal (relation) and the rest of the statement in question is used to characterise *it*.

1. The Case against Relation Names

Distinguish two classes of assertions: (a) assertions we make in order to describe how things are qualified, what they are doing, or the kinds of things they are; (b) assertions we make to describe how things are arranged or related. In English, we employ adjectives, nouns, and intransitive verbs to make assertions of the first class, whereas we also call upon transitive verbs, prepositions, and the paraphernalia of grammatical case to make assertions of the second. According to van Inwagen (2004, 2006), we have reason to believe in properties and relations because their existence follows, respectively, from the fact that assertions of the first class are said of only one thing, whilst assertions of the second class can only be said of two or more things. Suppose we assert that Delphi is north of Thebes. Then there is *something* asserted of Delphi

1 See MacBride (2011) for a related argument to the effect that we need not succumb to Frege's predicament either.

and Thebes, something we can't assert of them separately but only relative to one another. The thing asserted is a dyadic non-symmetric relation—van Inwagen calls it a "doubly unsaturated assertible." So we have the same reason for believing in non-symmetric relations as for properties. Properties and relations are both asserted of things, albeit different numbers of things. Our commitment to them is inescapable because the existence of properties and relations follows from the assertions we make. But, van Inwagen argues, we cannot give a name to the relation we assert of Delphi and Thebes when we assert that Delphi is north of Thebes, or to any other non-symmetric relation. By contrast, van Inwagen maintains, the "singly unsaturated assertibles" we assert of someone when we declare she/he is wise or loves honour more than life, i.e., properties, have names even in natural language—"wisdom" and "loving honour more than life." There's a further awkwardness here we haven't mentioned before. According to van Inwagen, properties are monadic relations, i.e., a limiting case of relations. So it's an embarrassment for realists like him that monadic relations can be named but $n > 2$-adic non-symmetric relations cannot.

Why does van Inwagen take non-symmetric relations to be such troublesome creatures? He claims we have good reason to believe in such relations because they are expressed by ubiquitously employed vehicles of assertion, viz., open sentences with two or more free variables. Grant him this. Then a closed term resulting from the application of an operator to an open sentence of two or more variables would be an exemplary name of the relation expressed by that open sentence—provided that the diversity and arrangement of the variables be respected in the binding of them. Such a closed term would be exemplary in the sense that if there were such an operator, then the open sentence expressing the relation could be retrieved from the closed term in which its two or more variables are bound by the aforementioned operator. Van Inwagen calls such closed terms "formal names" of relations because they would reveal or make manifest the relations they purport to denote. But, he argues, we lack any understanding in English, or even philosophers' English, or any extension of our language, of such an operator, so there are no formal names for relations.

Van Inwagen argues for this conclusion by eliminating one after another of what he takes to be all the plausible candidates for an operator that would yield formal names of relations. Key to his argument is what he describes as a metaphysical assumption that applies to all $n > 1$-adic relations. He states this assumption for the case $n = 2$ as follows: "Every dyadic relation has

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at least one converse; there are non-symmetrical dyadic relations; no non-symmetrical dyadic relation is identical with any of its converses" (2006, 453). Call it "(*MetaA*)," short for "Metaphysical Assumption." Van Inwagen refers to (*MetaA*) as a single assumption, but we note that it really is a conjunction of three separate assumptions. (*MetaA*) will be critical for our case against van Inwagen. (*MetaA*) entails that every non-symmetric relation has a distinct converse. This raises the bar for a closed term succeeding in being a formal name of a non-symmetric relation; to pick out a non-symmetric relation, a formal name must enable us to discriminate the relation in question from its converse(s). Van Inwagen argues that we have no inkling of an expression we understand that reaches that bar.

Focusing initially upon the $n = 2$ case, he takes the binary lambda abstraction operator as an example of an operator that appears to fulfil the brief of yielding formal names for dyadic non-symmetric relations because it's a device that binds the variables in an open sentence to yield a closed expression. Consider, for example,

(1) λxy x is north of y.

Is there a reading of (1) in English or philosophers' English that confirms it to be a formal name of a non-symmetric relation? The kinds of constructions that philosophers typically draw upon to talk about relations are "r holds between x and y" and "x bears r to y." So the two most obvious readings of (1) are:

(2) The relation that holds between x and y if and only if x is north of y,

and

(3) The relation that x bears to y if and only if x is north of y.

Van Inwagen objects to both.

The problem he finds with (2) is that it is an improper description if (*MetaA*) is granted and the predicate "holds between x and y" is understood as an order-insensitive construction, so that, for example, "holds between Denmark and Italy" is synonymous with "holds between Italy and Denmark." Take a relation $R1$ that holds between two things whenever one is north of another. Then, by (*MetaA*), $R1$ has at least one converse, $R2$. But if $R1$ holds between two given things, then $R\mathbf{2}$ holds between those same things too. Think of $R1$ and $R2$ provisionally as the relations *being north of* and *being south of*—provisionally because van Inwagen's aim is to undermine any confidence that we can pick out non-symmetric relations and distinguish them well enough to give them names or definite descriptions. *R*1 and *R*2 apply to the things they relate in different orders. But "*R* holds between *x* and *y*" is order-insensitive, so it cannot capture the information that distinguishes a relation from its converse. So (2) doesn't distinguish *R*1 from *R*2. But if (1) is to be a formal name of a relation, it must be read as a definite description proper.

The obvious fix to (2) is to augment the "*R* holds between *x* and *y*" construction to make it order-sensitive:

(2.1) The relation that holds between *x* and *y* in that order if and only if *x* is north of *y*.

The thinking behind (2.1) is that adding "in that order" to (2) makes it semantically sensitive to the syntactic order in which the terms occur, so we can exploit that order to encode information about how the relation applies to the things those terms pick out. But van Inwagen argues that "in that order" introduces an unwanted lapse of extensionality. He considers the "*x* and *y*" in (2.1) a plural term. Replacing the variables with names (e.g., "Denmark and Italy"), he claims, will yield an expression that co-refers with any plural term that results from a permutation of those names (like "Italy and Denmark"). But the former plural term is not substitutable *salva veritate* for the latter in (2.1), even though (van Inwagen maintains) the plural terms in question are co-referring.

To avoid this lapse of extensionality, van Inwagen envisages augmenting (2) by explicitly specifying the order in which the relation in question relates the things named,

(2.2) The relation that holds between *x* and *y* in the order "*x* first, *y* second," if and only if *x* is north of *y*.

But van Inwagen dismisses (2.2) because he cannot make any sense of this absolute, metaphysical notion of order. He raises the rhetorical question, "But what is it for a relation to hold between—for example—Italy and Denmark in the order 'Denmark first, Italy second'? You may well ask" (2006, 460). Having raised the rhetorical question, van Inwagen moves along.

Unable to envisage another way of converting (2) into a proper description that distinguishes a relation from its converse(s), van Inwagen gives up on (2) and turns to (3), which is not vulnerable to the objections above. By contrast

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to "the relation that holds between x and y," definite descriptions of the form, "the relation that x bears to y" are order-sensitive. If a non-symmetric relation $R1$ is borne by one given thing to another, then its converse $R2$ isn't.

So far as (3) is concerned, so good: van Inwagen has no objection to using (3) as an English or at least philosophers' English reading of (1) itself. But we don't just need to understand (1), the formal name of a dyadic non-symmetric relation. We also need to understand all the closed expressions that result from the application of n-ary abstraction operators where $n > 2$, in order to provide formal names of $n > 2$-adic non-symmetric relations. Van Inwagen's objection to (3) is then that the construction "the relation that x bears to y" is expressively inadequate to this more general task. It has only two argument positions. So it lacks the logical multiplicity to provide, for example, an interpretation of a closed expression resulting from binding an open sentence of three free variables with a ternary abstraction operator, like "λxyz x gives y to z."

Van Inwagen considers augmenting the expressive power of "the relation that x bears to y" by inserting plural terms (such as "Denmark and Italy") into one of its argument positions. Using this augmentation, we can form the following two definite descriptions of a non-symmetric triadic relation: (i) "the relation that x bears to y and z" and (ii) "the relation that x and y bear to z". But because the plural term-forming operator "and" is order-insensitive, (i) is equivalent to (iii) "the relation that x bears to z and y", whilst (ii) is equivalent to (iv) "the relation that y and x bear to z". So there are only two ways of so describing a triadic non-symmetric relation. Van Inwagen doesn't make his objection explicit, but presumably the upshot is that (i) and (ii) are only suited to describe triadic relations that are indifferent to the permutation of two of the things they relate (like *x is between y and z*) but unsuited to the description of fully non-symmetric relations, which are sensitive to the permutation of any of their terms (like *x gives y to z*).

Having thus dispensed with what he thinks are the only plausible candidates for providing informal readings of (1), van Inwagen turns to what he deems to be the last resort of believers in formal names for relations. The last resort is taking (1) as a primitive name for a non-symmetric relation without needing to translate it into English or philosophers' English. Van Inwagen acknowledges that we understand lambda-abstracts like (1) and his favoured "canonical relation names", which are a variation on lambda abstracts, well enough to calculate the truth-values of the sentences in which they occur but not well enough to settle a unique reference for such lambda-abstracts: "[W]e know how, using the semantics, to calculate the truth-values of relation sentences with two relational terms. But—it seems to me—we have no idea what these sentences mean or what the relational terms refer to" (2006, 468). This is because our grasp of a lambda abstract or a canonical relation name does not proceed via an identification of its referent but only via a determination of the truth conditions of the contexts in which it occurs. So we don't know which relation the lambda abstract picks out, but only that the entire context in which it features, a relation sentence, is equivalent to a context in which it doesn't, a non-relational counterpart. Ipso facto, the semantics doesn't tell us which out of a range of mutually converse relations a lambda abstract or a canonical relation-name for a non-symmetric relation denotes. So if (*MetaA*) holds, (1) can't be a formal name of a non-symmetric relation after all.

Let's sum up. Van Inwagen has argued that we cannot provide an English or philosophers' English reading of lambda abstracts like (1) in terms of constructions like (2) or (3) or their emendations, nor can we understand lambda abstracts like (1), or his favoured canonical relation names, in the absence of a translation into English or philosophers' English. Van Inwagen's case against names for non-symmetric relations relies upon the metaphysical assumption that non-symmetric relations have distinct converses, a consequence of (*MetaA*). Because we inhabit a metaphysical environment abundant with converse relations, singling out a given non-symmetric relation requires distinguishing it from its converse(s). Because, he claims, we cannot single out a non-symmetric relation from its converse(s), he concludes that we cannot understand or introduce a name for the (purported) relation in question.

2. Relation Names and the Metaphysics of Non-symmetric Relations

The master assumption behind van Inwagen's arguments is that non-symmetric relations have distinct converses. We present two independently attractive conceptions of non-symmetric relations, according to which they don't have distinct converses. So, from their points of view, there's no need to distinguish a non-symmetric relation from its converse in order to understand its name. For present purposes, we don't decide between these different conceptions because van Inwagen's case that we cannot name relations presupposes both are false, but he doesn't provide arguments that rule out either.

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Consider the statements (a) "WWI is before WWII" and (b) "WWII is after WWI". Evidently, they are mutually entailing in the sense that it's not possible for one to be true and the other false. Now distinguish "abundant" from "sparse" semantics for these sentences—in virtue of the contrasting number of non-symmetric relations to which accounts of these kinds are committed. According to accounts of the abundant kind, under which van Inwagen's view falls, the binary predicates "*x* is before *y*" and "*x* is after *y*" are used to ascribe two distinct relations—two distinct but mutually converse non-symmetric relations. So whilst (a) reports upon the obtaining of one non-symmetric relation, (b) reports upon another, the converse of the first. Nevertheless, (a) and (b) are mutually entailing because it is in the nature of this pair of relations that in any possible circumstance where one holds between *x* and *y*, the other holds between *y* and *x* (for any *x* and *y*). The mutual entailment of the statements (a) and (b) thus has a distinctively ontological source in the "metaphysical entanglement" of the converse relations expressed by their respective predicates—that, as a matter of metaphysical necessity, whenever one relation holds one way, its converse holds the other way.

By contrast, according to accounts of the sparse kind, "*x* is before *y*" and "*x* is after *y*" express one and the same non-symmetric relation. So (a) and (b) report upon the obtaining of one and the same relation; they differ because their constituent predicates invoke converse rules for evaluating the significance of the statements in which they occur. The mutual entailment of (a) and (b) is a consequence of the semantic entanglement of their constituent predicates—that, as a matter of the rules of our language, what we say when we make use of one of these predicates flanked by singular terms in one arrangement is the same as what we say when we use the converse predicate flanked by the same singular terms in the reverse arrangement. Whether we choose to use (a) or (b) depends upon pragmatic factors, i.e., which event it suits our conversational purposes to mention first, i.e., left-most, in the sentence we use to make the report. In the same way, we consider the mutual entailment of statements whose terms have been permuted but respectively involve the active and passive forms of a verb, e.g., (c) "Antony loves Cleopatra" and (d) "Cleopatra is loved by Antony", to be explained in terms of the contrasting rules governing active and passive forms rather than a necessary connection between the diverse relations they introduce. It's not a choice of subject matter but conversational pragmatics, if not simply a stylistic predilection, that makes us prefer one form rather than another to describe how Antony and Cleopatra are related.

We distinguish between *iconic* and *role-theoretic* versions of the sparse account and present a thumb-nail sketch of each.

(ICONIC) By the iconic version, we mean the view that language users succeed in representing how things stand in relation to one another by exploiting the fact that linguistic signs stand in relation to one another too.[2] We succeed in representing how things stand by using arrangements of signs to model the arrangement of things, the things in question being the things the signs stand for. Different arrangements of signs may serve equally well to model the same arrangement of things. We can exploit the fact that a given occurrence of a name, say "WWI", stands in a relation of left-flanking to an occurrence of a predicate, which is right-flanked by an occurrence of "WWII", to model WWI's preceding WWII. But we can equally well model WWI's preceding WWII by using an arrangement of signs in which an occurrence of "WII" stands in a relation of left-flanking an occurrence of a predicate that is right-flanked by an occurrence of "WWI". When we use "*x* is before *y*" to frame a token sentence, we understand as a matter of convention that it is the former modelling technique that is being exploited to represent which event precedes another, whereas when we use "*x* is after *y*", we understand as a matter of convention that it is the latter technique in play. *Eo ipso*, we understand that (a) and (b) say the same thing because, whilst they consist of different arrangements of signs, the different modelling conventions associated with their predicates co-ordinate them with the same worldly arrangement of events. We also understand that (e) "WWII is before WWI" isn't entailed by (a) because (e), consisting of a different arrangement of signs, models a different arrangement of events.

(ROLE) By the role-theoretic version, sometimes called positionalism, we mean the view that relations apply to things in virtue of their having "roles" or "positions" which are filled by their relata, where roles or positions are conceived as *bona fide* entities—by contrast to the iconic view, which treats role and position-talk along deflationary lines, so, roughly speaking, "*a* occupies

2 Called "iconic" after Peirce (1903, 273–274), who conceived of iconic diagrams as representing "relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts." Wittgenstein's "picture theory" is similar: "That the elements of the picture are combined with one another in a definite way, represents that the things [in the world] are so combined with one another" (1922, 2.15). See MacBride (2018, 191–197; 2024a, sec. 1) for further historical and philosophical development of the iconic view.

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the *before* role whilst *b* occupies the *after* role" reduces to "*a* is before *b*".[3] Roles or positions may be understood as somehow corresponding to the thematic roles widely appealed to in linguistics, such as *agent*, *patient*, *instrument*, *beneficiary*, *goal*, *location*, *source*, *destination*, etc. (See Davis 2011.) Or as rigidly associated with specific predicates, so that we can speak, e.g., with respect to "*x* loves *y*" of *lover* and *beloved*, with respect to "*x* gives *y* to *z*" of *giver*, *givee*, and *given* positions. We favour the former view (so far as role-theoretic views are concerned) because it enables us to capture generalisations about what different relations have in common, e.g., agent/patient structure, but do not press the point here. Our grasp of predicates, such as "*x* loves *y*" and "*x* is loved by *y*", that express the same relation relies upon an understanding of converse conventions about how to represent the manner in which roles or positions in the relation are filled. We understand that an occurrence of a name left-flanking "*x* loves *y*" denotes what fills the *agent* role or *lover* position of the relation the predicate picks out, and the corresponding occurrence of a right-flanking name what fills its *patient* role or *beloved* position, whereas the occurrence of a name left-flanking "*x* is loved by *y*" denotes what fills the *patient* role or *beloved* position, and the corresponding occurrence of a right-flanking name what fills the *agent* role or *lover* position. The upshot is that (c) and (d) say the same thing because they co-ordinate the same items to the same role or position of the same relation. We also understand that (c) doesn't entail (c′) "Antony is loved by Cleopatra", because (c′) represents a different assignment of items to roles or positions.

Such sparse accounts, according to which "*x* is before *y*" and "*x* is after *y*" co-refer, appear to be open to a knock-down objection: members of a pair or family of mutually converse predicates cannot co-refer because one cannot be substituted for another whilst preserving truth-value in extensional contexts. For example, if we substitute "*x* is after *y*" for "*x* is before *y*" in (a) "WWI is before WWII", the result is (h) "WWI is after WWII", so we pass from truth to falsity. Similarly, moving from (i) "Obama is a former president" to (j) "Biden is a former president", we pass from truth to falsity—this is enough to settle that "Obama" and "Biden" don't co-refer (see Quine 1960, 142–143). But this objection isn't knock-down because substitution failure amongst converse predicates doesn't have to mean that the predicates in question don't

3 See Williamson (1985, 257–258) and Orilia (2011) for different developments of the role-theoretic view.

co-refer (see MacBride 2011, 307–309; 2024b). It need only mean that converse predicates don't just refer but refer relative to the aforementioned converse rules, whether spelled out in terms of the iconicity of our representations or rules involving roles/positions. We conclude that it's not because converse predicates don't refer to the same relation that substituting one of them for another may fail to preserve truth-value. It's because such a substitution forces a reinterpretation of the linguistic context in which the predicate occurs, i.e., the semantic significance of the left and right flanking singular terms.

Van Inwagen (recall) dismissed with incredulity the hypothesis that non-symmetric relations hold of their relata in an order—where the notion of order is an absolute and abstract metaphysical notion, as Russell once maintained (1903, sec. 94). We note that neither the iconic nor role-theoretic accounts are committed to relations holding of their relata in an order (in the metaphysical sense of which van Inwagen disapproves). The iconic account exploits case-by-case conventions, depending upon the operative predicate, co-ordinating the manner in which the terms of a sentence are arranged with the manner in which a relation holds amongst the things for which the terms stand if the sentence is true. Here, the notion of "manner" isn't elliptical for some general notion of order. It's schematic, to be filled out in particular cases with reference to the relevant conventions. There is no more need, we maintain, to expect there to be a single rule governing the use of predicates than there is a need for a single rule governing adjectives—because we have to learn piecemeal, for example, whether adjectives are intersective, subsective, or non-subsective (see Lassiter 2015). For example, with regard to (a), we exploit the convention that the left-flanking term stands for something that precedes the event for which the right-flanking term stands if (a) is true.[4] So there's no appeal to one event coming first, the other second, in some absolute, metaphysical sense of order (although in this case, one is first and the other second in temporal order). The role-theoretic account also exploits case-by-case conventions. Which convention we use depends upon the operative predicate in a sentence and the syntactic arrangement of the terms in the sentence. The convention in play co-ordinates the things for which the terms stand with the roles or positions of the relation that the operative predicate denotes. This obviates the need to appeal to one thing coming first, another second in an absolute metaphysical sense in favour of a co-ordination of things picked out with roles or positions.

4 We assume, but do not argue here, that "precedes" denotes a dyadic relation.

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We do not adjudicate here between the iconic and the role-theoretic views. What is important for present purposes is that both avoid converse relations. Van Inwagen's case that we fail to grasp relation names depends upon the existence of converse relations, but he provides no argument to rule out either view. So he fails to establish his conclusion.

Van Inwagen does acknowledge the possibility that his conclusion, that we have no grasp of relation names, might be taken as a *reductio ad absurdum* of the hypothesis that non-symmetric relations have distinct converses. Nevertheless, he declares (*MetaA*) "an assumption I refuse to forego" and accordingly offers "some intuitive considerations in favor of the existence of non-symmetrical dyadic relations" (2006, 453–454). He argues that there are things that can be said of two people in two different ways and may be true of them said one way but not the other—things that aren't predicates or any other kind of linguistic item but dyadic non-symmetric relations. But even if van Inwagen succeeds thereby in establishing (I) that there are non-symmetric relations, it doesn't follow (II) that every non-symmetric relation has at least one converse, nor (III) that no non-symmetric relation is identical with any of its converses.

In other words, the intuitive considerations that van Inwagen adduces speak in favour of one component of (*MetaA*) but not the other two. Hence, such considerations don't entitle him to refuse to forgo (*MetaA*) in all its parts. But the metaphysical hypothesis upon which van Inwagen relies to establish that we lack a grasp of relation names, viz., that every non-symmetric relation has at least a distinct converse, doesn't rely upon just one component of (*MetaA*) but all three; the hypothesis in question doesn't follow from (I) alone but only from (I) taken together with (II) and (III). Because van Inwagen fails to provide support for (II) or (III), he fails to rule out the legitimacy of others taking a modus tollens where he has taken a modus ponens. Meanwhile, we have argued in this section that (II) is false upon an iconical or role-theoretic conception.

3. Relation Names in English and Extended Versions of English

Let us turn to the question of the expressive adequacy of English with respect to non-symmetric relations—the extent to which English as it is, or an extended version of English, allows us to form names or definite descriptions for non-symmetric relations. We argue that, suitably augmented, both the "holds between" and the "bears" constructions provide us with a supply of definite descriptions for non-symmetric relations with the requisite logical-grammatical multiplicity to express n-ary relations where $n > 2$, definite descriptions we really do understand.

We agree with van Inwagen that to be adequate for framing names for non-symmetric relations, the "holds between" construction requires to be supplemented with the "... in that order" operator. Van Inwagen (recall) maintains that this requirement cannot be fulfilled because, either, the notion of order invoked is syntactic, in which case there is a violation of extensionality, or this notion is metaphysical, but this is hardly acceptable. We also agree with van Inwagen that an absolute, metaphysical notion is hardly acceptable. But we deny that conceiving the "... in that order" operator in syntactic terms as sensitive to the syntactic order of the terms of the contexts in which it occurs results in a violation of extensionality.

Certainly the phrases (A) "Denmark and Italy in that order" and (B) "Italy and Denmark in that order" have different semantic significance—when "... in that order" is understood in the syntactic terms we favour. But there's only reason to think there's been a violation of extensionality if we go along with (at least) the further assumption upon which van Inwagen relies, viz., that the plural terms "Denmark and Italy" and "Italy and Denmark" occur as semantically significant ingredients of these phrases. But we don't grant this assumption because it isn't an independently plausible assumption to make.

Why so? The operator "... in that order" is responsive to the order in which the preceding singular terms occur. It isn't responsive to the singular terms *en bloc* as one plural term. So there's no reason to think that this operator has just a single argument position for one plural term; the plural term is an idle wheel in the semantics because what counts is the order of the singular terms—from a mid-20th century failure to take plural terms seriously, we shouldn't leap to seeing plural terms wherever there's a list. For this reason, we think that it is more reasonable to take "... in that order" as a multigrade, order-sensitive operator—multigrade because the number of occurrences of singular terms preceding it may vary depending upon the polyadicity of the relation described in the sentences in which it occurs (MacBride 2005). But if (A) and (B) don't have semantically significant occurrences of plural terms, then there is no ostensible violation of extensionality because each occurrence of a name is open to substitution by a co-referring expression. We can even substitute definite descriptions, for example, "the European country shaped

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like a boot" for "Italy". We conclude that (2.1) serves perfectly well as an informal reading of (1) translated into philosophers' English. So, we conclude, it is already possible to form names for non-symmetric relations in English, or at least philosophers' English, using the "holds between" construction.

We also hold that the "*x* bears *R* to *y*" construction, that occurs in (3) above, can be augmented with enough grammatical-logical multiplicity to cover $n > 2$-adic relations. It's the grammatical articulation of the verb "bears", as it is used in current English or philosophers' English, that suits it to describing the manner in which dyadic non-symmetric relations hold of the things they relate. The grammatical articulation of the construction as it is currently used could be displayed thus: "[subject] bears [direct object] to [indirect object]." The position of a direct object is taken by a relation name, whilst the term that denotes the thing that is said to bear the relation in question and the term that denotes the thing to which the relation is borne take subject and indirect object positions, respectively. We are able to express the two different ways that a dyadic non-symmetric relation is capable of applying to two given things by permuting the terms that stand for them between the subject and indirect object positions of the verb ("*a* bears the relation *R*1 to *b*", "*b* bears the relation *R*1 to *a*"). But the grammatical categories that we exploit to express the manner in which dyadic non-symmetric relations apply are inadequate to triadic cases. This is because, as van Inwagen reflects, "subject and indirect object are *two* grammatical categories, and there is no third category that can be used to create a form of words that stands to triadic relations as '…bears…to…' stands to dyadic relations. (The category 'direct object' is already taken: the relation is the direct object of 'bears')" (2006, 477, n.28, italics in original).

We agree that the English verb "bears" lacks the requisite number of associated grammatical categories to describe the holding of a triadic non-symmetric relation. But we disagree that English or philosophers' English need be this way. This is because we think it is only a contingent fact about English that the verb "bears" has only three grammatical categories associated with it, so only the wherewithal to describe the holding of a dyadic relation. And if this is only a contingent fact about English, we see no barrier to enriching English or philosophers' English to include a novel grammatical category to be associated with the "bears" construction to encode information about the occurrence of the third term of a non-symmetric triadic relation, a further novel category to encode information about the occurrence of the fourth term of a tetradic non-symmetric relation, and so on as the need arises.

Perhaps you are doubtful that it is a contingent fact that "bears" has only three grammatical categories associated with it. Or perhaps you think we should be cautious about the question of whether we could really understand a version of English enriched with additional grammatical categories. Or perhaps you think we can only really understand such enrichments insofar as they can be elucidated in terms of the English we already understand—as Strawson argued against Carnap's tolerant employment of novel linguistic systems (see Carnap 1934, sec. 17; and Strawson 1963, 518). But our own estimation is that there is a narrow but traversable path to tread between outright scepticism, thinking that we just don't understand novel forms, and wishful thinking that we invariably do understand novel forms.

Outright scepticism can't be right because languages have been expressively enriched and are being expressively enriched for scientific and other theoretical purposes all the time—something that philosophers are often keen to point out to license the introduction of their own novel technical vocabulary. What is no less significant for the present discussion, but not to our knowledge pointed out by philosophers anywhere else, is that there are certain respects in which many natural languages have become expressively *impoverished* over time. For example, many Indo-European languages had more grammatical categories in the past than they do now. It would seem perverse to think that what was possible for our forebears to understand isn't possible for us. But, we also grant, it is important to beware of wishful thinking too because the marks we scratch on the page don't mean what we want just because that's what we want them to mean—even if meaning is use, not every use is meaningful. We suggest avoiding the extremes, wanton scepticism on the one hand, naive credulity on the other, by showing how novel grammatical categories may be introduced whilst still being related or analogous to familiar categories we already understand.

We already have an understanding in English of the thematic roles (agent, patient, goal, instrument, etc.) associated with verbs and their markers, roles that are widely invoked in linguistics. As ordinary language users, we exploit these roles to describe the obtaining of relations expressed by verbs. So when we understand, for example, "David kicked Peter", we do so by distinguishing two roles: the kicker, or more generally, agent role, associated with the subject of the verb "kick", and the kicked or patient role associated with its object. Another of these roles, location, is typically expressed in English using prepositions, as in "Daphne ran in the park." Now this is a distinctive feature of English. Neither Sanskrit nor archaic Latin require the use of preposi-

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tions for this purpose but allow for an associated grammatical category, the locative case. The locative case has disappeared from most contemporary Indo-European languages. Nonetheless, we can readily imagine an historical scenario in which the locative case was still available in English and that this case might be exploited to augment the use we make of "bears", i.e., to add a location term so that we can say that a relation is borne by something to something else relative to a location, i.e., a three place relation. And if we can imagine English augmented in this way using an archaic grammatical case, it would seem unduly reactionary to refuse to envisage English enhanced with novel grammatical cases corresponding to the other thematic roles we associate with verbs.

Alternatively, to the same end, we might allow "bears" to be followed by any number of indirect objects of the type "x as R", where R is a thematic role, which we already understand because of their association with verbs: "the relation that x as agent bears to y as theme to z as goal", etc. Similarly, we can imagine utilizing English prepositions, such as "via", "through", "for", etc., to augment "bears" to handle triadic non-symmetry relations. For example, we might use descriptions of the following form: "the relation that x bears to y via z".

We conclude that even if we don't have names for $n > 2$-adic non-symmetric relations, we might have had them, and we can still invent them. It is more wayward scepticism than the conscientious exercise of theoretical caution to refuse to admit the possibility of extending the expressive resources of present-day English to enable us to name non-symmetric relations by so enriching the logico-grammatical multiplicity of the "bears" construction. Whilst natural languages, like English, weren't designed and didn't evolve for the purpose of enabling us to reflect explicitly upon the significance of relation words, our mastery of prepositions, the thematic roles associated with verbs, etc., provide us with the wherewithal to work our way up. We're not forced to choose between sticking with what's currently expressible in natural language or starting over again—having to decide whether, as natural language speakers, we have been truly wise in how we presently restrict ourselves or whether we have just been too timid to take flight.*

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In Defense of Relations

Edward N. Zalta

Two recent arguments draw startling and puzzling conclusions about relations and 2nd-order logic (2OL). The first argument concludes that 2nd-order quantifiers can't be interpreted as ranging over relations. This conclusion is puzzling because it calls into question the traditional understanding of 2OL as a formalism for quantifying over relations. The second argument, which concludes that unwelcome consequences arise if relations and relatedness are *analyzed* rather than taken as *primitive*, utilizes premises that imply that 2OL faces the very same consequences. This is puzzling because relations and predication are taken as primitive in 2OL, and so the latter should be immune to the problems raised for the analysis of relations. I consider these two arguments in light of a precise theory of relations. In particular, I show that object theory (Zalta 1983, 1988), which is an extension of 2OL, provides systematic existence and identity conditions for relations, properties, and states of affairs that forestall the two arguments.

1. Setting Up the Problems

I take relations to be a fundamental kind of entity, and in this paper I investigate some of the principles needed to characterize them. Recently, philosophers have raised puzzling questions about converse and non-symmetric relations and about the states of affairs in which they play a role (Williamson 1985; Dorr 2004). In addressing these and other questions, some philosophers and philosophical logicians have attempted to *analyze* relations and the manner in which they relate. Such analyses, which sometimes appeal to other fundamental notions, raise questions of their own, such as whether or not there are positions (argument places, slots, or thematic roles) in a relation (Fine 2000; Gilmore 2013; Dixon 2018; and Orilia 2014, 2019); what it is for the relata to bear or stand in a relation; and whether there is an order of application or a manner of completion that connects relations and their relata.

In this paper, however, I take the notions of *relation* and *relation application* (i.e., *predication*) to be so fundamental that they can't be further analyzed and so must instead be axiomatized. This starting point is analogous to that of the mathematics of set theory—the notions of *set* and *set membership* are considered so fundamental that the best we can do is axiomatize them. As with set theory, an axiomatic theory of relations has to state, at the very least, conditions under which the entities being axiomatized exist and conditions under which they are identical. In what follows, I'll reprise just such a theory. It was first proposed in 1983 and was couched in a relatively simple extension of second-order logic ("2OL"). The resulting system gives us the framework we need to address the most important questions that have been raised about relations, including some of the questions that arise when relations are analyzed.

My *defense* of relations is focused on two recent arguments that draw rather puzzling conclusions for relations considered as primitive, axiomatized entities. The first argument appears in a recent paper by MacBride (2022, 1), where he concludes, by way of a dilemma, that "we cannot interpret second-order quantifiers as ranging over relations." MacBride is not claiming that relations don't exist or that some other (e.g., ontologically more neutral) interpretation of 2nd-order quantifiers is to be preferred, but rather that 2nd-order quantifiers *can't* be interpreted unproblematically as ranging over relations.[1] This conclusion is startling because it calls into question the traditional understanding of 2OL as a formalism for quantifying over relations. Philosophers and logicians since Russell have supposed that relational statements of natural language of the form "a loves b," "a gives b to c," etc., can be uniformly rendered in the predicate calculus as statements of the form $Ra_1 \ldots a_n$, where $Ra_1 \ldots a_n$ expresses the claim that $a_1, \ldots, a_n$ exemplify (or stand in or instantiate) R. For example, in his description of 2OL, Väänänen (2019, sec. 2) notes that "[t]he intuitive meaning of $X(t_1, \ldots, t_n)$ is that the elements $t_1, \ldots, t_n$ are in the relation X or are *predicated* by X." So it is puzzling to be informed that when we existentially generalize on the statement "$Ra_1 \ldots a_n$" to derive the claim "$\exists F(Fa_1 \ldots a_n)$," we can't regard this latter claim as quantifying over relations.

1 Thus, I am not objecting to other interpretations of the second-order quantifiers, either in plural terms (Boolos 1984, 1985), denominalized terms (Rayo and Yablo 2001), or neutral terms (Wright 2007). Rather, I'm confronting an argument that concludes such quantifiers can't be successfully interpreted as ranging over relations.

The second argument and puzzling conclusion appear in MacBride (2014). On the one hand, MacBride argues that relations, predication (relation application), and relatedness should be taken as primitive (2014, 1, 2, 15), on the grounds that any analysis leads to unwelcome consequences. On the other hand, the unwelcome consequences he describes for the analysis of relations are already present in 2OL with identity ($2OL^{=}$), where relations and predication are primitive. He endorses the primitive nature of relatedness when he writes:

> I will argue that the capacity of a non-symmetric relation *R* to apply to the objects *a* and *b* it relates so that *aRb* rather than *bRa* must be taken as ultimate and irreducible. [...] It's a familiar thought that we cannot account for the fact that one thing bears a relation *R* to another by appealing to a further relation relating *R* to them—that way Bradley's regress beckons. To avoid the regress we must recognize that a relation is not related to the things it relates, however language may mislead us to think otherwise. We simply have to accept as primitive, in the sense that it cannot be further explained, the fact that one thing bears a relation to another [citations omitted]. But it is not only the fact *that* one thing bears a (non-symmetric) relation *R* to another that needs to be recognized as ultimate and irreducible. *How R* applies—whether the *aRb* way or the *bRa* way—needs to be taken as primitive too. (MacBride 2014, 2, italics in original)

While this seems correct, the argument that MacBride gives for this conclusion ensnares $2OL^{=}$, where relatedness is primitive. His argument revolves around the following claim (Russell 1903, sec. 218–219):[2]

(1) Every (binary) non-symmetric relation *R* has a converse R^* that is distinct from *R*.

MacBride argues that any analysis of relations and relation application that endorses (1) gives rise to "unwelcome consequences," namely (a) a multiplicity of converse relations[3] and (b) "the profusion of states that arise from the

2 Russell actually talked about "asymmetric" relations, but we'll discuss the differences below, where we formally define non-symmetric relations. I don't think anything hangs on the difference.

3 For example, ternary non-symmetric relations have 5 converses, and quarternary non-symmetric relations have 23.

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application of these relations" (2014, 4). Consequence (a) is puzzling because 2OL$^=$, in which relations, predication, and relatedness are primitive, has a formal representation of (1) as a theorem. So it seems we face a multiplicity of relations no matter whether we endorse (1) by way of an analysis or by way of 2OL$^=$. As part of our investigation, we'll also examine consequence (b) and MacBride's conclusion that there is no good analysis of the identity and distinctness of states of affairs. He says:

> What vexes the understanding is [...] an *analysis* of the fundamental fact that $aRb \neq bRa$ for non-symmetric R. [...] Anyone who wishes to give an analysis of the fact that $aRb \neq bRa$ faces a dilemma. [...] Since neither [...] [of the] analyses are satisfactory, this recommends our taking the fact that $aRb \neq bRa$ to be primitive. (MacBride 2014, 8, italics in original)

[The full quote is provided later in the paper.] When we examine this (second) dilemma, we'll see that there is an analysis that is immune to the dilemma and that MacBride doesn't consider. One can unproblematically analyze the *identity* of states of affairs within a theory on which the fact that a state of affairs *obtains* is primitive.

My plan is as follows. In section 2, I lay out the first puzzling argument and conclusion, i.e., the dilemma used to establish that the 2nd-order quantifiers don't range over relations. The argument begins by suggesting that if they do, then pairs of converse predicates either refer to the same relation or they don't. Each disjunct leads to a horn of the dilemma. I then spend the remainder of section 2 showing that the first disjunct fails, so that we need not worry about the first horn. In section 3, I examine the argument that leads from the second disjunct to the second horn and narrow our focus to an issue on which the conclusion rests, namely, a question about the identity of certain states of affairs. In section 4, I examine the second puzzling argument and conclusion from MacBride's (2014) paper and connect the argument there with the issue on which we focused in section 3. Then in section 5, I review a theory of relations and states of affairs that MacBride doesn't consider but which has consequences for the issues we've developed. In section 6 and section 7, I use the theory in section 5 to develop two alternative analyses of the issue (about the identity of states of affairs) on which both of MacBride's puzzling conclusions rest. I show that these answers undermine the main lines of argument that MacBride uses to establish his conclusions.

From this overview, it should be clear that in sections 2–4, we'll extend 2OL in known ways that systematize the language that MacBride uses in his arguments. However, starting in section 5, I'll appeal to the theory of abstract objects developed in Zalta (1983, 1988, 1993), which I henceforth refer to as "object theory" ("OT").[4] OT extends 2nd-order logic in a way that allows us to state unproblematic identity conditions for relations and states of affairs. So my goal throughout will be to show that 2OL has been deployed and extended to formulate a theory of relations, predication, and states of affairs that forestalls the puzzling conclusions.

Before we begin, however, it is important to review some terminology and notation. "2OL" refers only to the formal, axiomatic system of second-order logic under an objectual interpretation (i.e., where the quantifiers range over domains of entities). My arguments don't require that we interpret 2OL in terms of *full* models (where the domain of properties has to be as large as the full power set of the domain of individuals); instead, *general* models (where the domain of properties is only as large as some proper subset of the power set of the domain of individuals) suffice. The only requirement is that the models validate the axioms of 2OL. In what follows, I'll represent a binary atomic predication as "Rab" instead of "aRb," except when we're discussing *identity*, in which case I'll use "$a = b$" (i.e., infix notation). As noted earlier, the atomic formulas of 2OL have the form "$F^n x_1 \dots x_n$" and can be read as "x_1, ..., and x_n exemplify (or instantiate) F^n," and we'll often drop the superscript on F indicating arity since this can be inferred.

No explicit notion of *order* is required here; we only require that "Rab" and "Rba" *say* different things; to say a and b exemplify R is not to say b and a exemplify R; to say x, y, and z exemplify F is not to say x, z, and y exemplify F; and so on (more about this later). In these examples, the predicate can be replaced by any nominalized relation term of the right arity. Finally, I'll use $F, G, H, \dots$ as 2nd-order variables; Greek letters will be used as metavariables instead. So when MacBride talks about the 2nd-order quantified sentence "$\exists\Phi(a\Phi b)$," I'll represent this sentence as "$\exists F(Fab)$."

In the next few sections, we shall extend 2OL in various ways, in part to systematize the language that MacBride uses in his arguments. We'll start with $2OL^=$, in which identity claims of the form "$F^n = G^n$" (for any n) are

4 This theory has been applied and developed in a number of more recent publications, including Linsky and Zalta (1995), Zalta (2006), Nodelman and Zalta (2014), Menzel and Zalta (2014), Zalta (2020), and elsewhere. These texts contain useful introductions to the theory.

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primitive.[5] We'll also treat states of affairs as 0-ary relations, and instead of using $F^0, G^0, \ldots$ as 0-ary relation variables, we'll use $p, q, \ldots$. So identity claims such as "$p = q$," asserting the identity of states of affairs, are well-formed. Moreover, we'll also make use of n-ary λ-expressions ($n \geq 0$), interpreted relationally; these are complex terms that denote relations and states of affairs.[6] And we'll let formulas be complex terms that denote states of affairs, so that when MacBride uses expressions like "$aRb = bRa$" and "$aRb \neq bRa$" (2014, 8), we can represent this talk precisely as identity and non-identity claims about the states of affairs denoted by the formulas flanking the identity symbol.[7] When we extend 2OL to OT in section 5, we'll add a new, primitive mode of predication and a primitive modal operator. Using OT, we'll define the primitive claims of the form "$F^n = G^n$" (for $n \geq 1$) and "$p = q$"; thus, we'll provide identity conditions for relations and states of affairs. I'll then be

5 Though logic texts Enderton (2001b) often formulate 2OL instead of $2OL^=$, Shapiro (1991, 64) and Väänänen (2019, sec. 2) mention that $2OL^=$, in which identity is taken as a primitive, is a simple extension of 2OL.

6 The definitions of the language of 2OL are easily adapted when we let $n = 0$, thereby including constants and variables ranging over states of affairs or propositions (where these are taken to be 0-ary relations). And there are extensions of 2OL in which n-ary λ-expressions have been included as complex names for n-ary relations ($n \geq 0$). This suggestion appears in Prior (1971, chaps. 3, 43–44), though Prior subsequently questions the ontological implications of λ-expressions (1971, 45). More recently, λ-expressions were adopted in Zalta (1983, chaps. III, IV; 1993, 407–409); in Menzel (1986, 7, 26; and Menzel 1993, 67–71) they are used in an untyped setting. And see Alama and Korbmacher (2023, sec. 9.3) for a discussion of the relational λ-calculus.

7 Thus, the language of $2OL^=$ that we'll need can be specified precisely in terms of a definition, by simultaneous recursion, of the notions of *formula* and *term*:

- Base clause for terms: every simple constant and variable is a term (i.e., individual constants and variables are individual terms, and n-ary relation constants and variables ($n \geq 0$) are n-ary relation terms).
- Base clauses for formulas: (a) for any $n \geq 0$, whenever $\kappa_1, \ldots, \kappa_n$ are any individual terms and Π^n is any n-ary relation term, $\Pi^n \kappa_1 \ldots \kappa_n$ is a formula, and (b) whenever κ and κ' are any individual terms, or Π and Π' are any n-ary relation terms (for some n), $\kappa = \kappa'$ and $\Pi = \Pi'$ are formulas.
- Recursive clause for formulas: if φ and ψ are any formulas and α is any variable, $\neg\varphi$, $\varphi \rightarrow \psi$, and $\forall\alpha\varphi$ are formulas.
- Recursive clauses for terms: where $\nu_1, \ldots, \nu_n$ ($n \geq 0$) are distinct individual variables and φ is any formula, then $[\lambda\nu_1 \ldots \nu_n\varphi]$ is an n-ary relation term and φ itself is a 0-ary relation term.

We define $\varphi \,\&\, \psi$, $\varphi \vee \psi$, $\varphi \equiv \psi$, and $\exists\alpha\varphi$ (α any variable) in the usual way. Note that by these definitions, formulas of the form $\exists p(p \equiv \varphi)$, where φ is any formula, are well-formed. Suitably restricted, this schema will serve as the 0-ary case of the comprehension principle for relations.

in a position to argue that OT thereby offers an analysis of "$aRb = bRa$" or "$aRb \neq bRa$" without facing any dilemmas.

It is also important to spend some time explaining how we plan to use the technical term *predicate*. First, we shall almost always be discussing the predicates of 2OL that serve to represent the predicates of natural language sentences. But the predicates of 2OL are not the same kind of expression as the predicates of natural language. When speaking of natural language sentences, it is traditional to distinguish the "subject" of a sentence from the "predicate." For example, in the sentence "John is happy," "John" is the subject and "is happy" is the predicate; and in the sentence "John loves Mary," "John" is the subject and "loves Mary" is the predicate. In the case of the latter sentence, one could also say that "loves" is the predicate, while "John" and "Mary" are the subjects (though "Mary" is often called the direct object). Thus, natural language predicates are not usually thought of as names or as nominalized expressions, for there is a sense in which these predicates are incomplete expressions.

But in what follows, we will be representing natural language predicates in terms of formal expressions that denote relations, and we'll be calling those formal expressions "predicates." Before I give the definition, however, let me mention that we shall *not* adopt the definition of *predicate* that MacBride introduces in the following passage (citing Dummett 1981, 38–39), in which he gives examples in terms of the expressions in a formal language:

> [W]hat is a second-order predicate? A first-order predicate (say of the form "$F\xi$") results from the extraction of one or more names ("a") from a closed sentence ("Fa") in which it occurs and inserting a variable in the resulting gap. A second-order predicate (say, of the form "$\exists x \Phi x$") results from the extraction of a first-order predicate ("$F\xi$") from a closed sentence ("$\exists x Fx$") and inserting a variable into the resulting gap. (MacBride 2022, 2–3)

In a footnote to this passage, MacBride makes it clear that open formulas, such as "Lax," "$\neg Rxa$," and "$Px \rightarrow Qy$" (in which x and y are the only variables), qualify as predicates. But in what follows, I shall distinguish between open formulas and predicates.

I shall use the term "predicate" to refer to a relation term Π (i.e., a relation constant, a relation variable, or a λ-expression) that can occur in an *atomic* predication. In classical logic, in which atomic predications take the form

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$\Pi\kappa_1 \ldots \kappa_n$, the expression Π is a predicate. So where "*L*" might be used to represent the *loves* relation, I'll distinguish between the predicate "*L*" and the open formula "*Lax*." The open formula is not a predicate and doesn't name a property (i.e., unary relation); we can't directly infer "$\exists F(Fx)$" or "$\exists F(Fa)$" from "*Lax*." The open formula "*Lax*" does have truth conditions and, given an assignment to the variable *x*, denotes a state of affairs. By contrast, when we add λ-expressions a bit later, we regard the complex unary relation term "$[\lambda x\, Lax]$" as a predicate. We can combine it with "*b*" to form the atomic predication "$[\lambda x\, Lax]b$" ("*b* exemplifies being an *x* such that *a* and *x* exemplify the *loves* relation," or more simply, "*b* exemplifies being loved by *a*").[8] And "$[\lambda xy\, \neg Lxy]$" is a predicate because we can form the atomic statement "$[\lambda xy\, \neg Lxy]ab$."

Thus, the predicates of 2OL and 2OL$^=$ denote properties and relations. Variables such as *F*, *G*, etc. are also predicates since the expressions "*Fa*," "*Gxy*," etc. are well-formed atomic formulas; the variables *F*, *G*, etc. denote properties and relations relative to an assignment to the variables. To consider a more complex example, let "*E*" denote *being even* and "*P*" denote *being prime*. Then, when we replace the constant "2" with "*x*" in the complex closed sentence "$E2 \,\&\, P2$" ("2 exemplifies being even and 2 exemplifies being prime"), we obtain "$Ex \,\&\, Px$." This latter expression isn't a predicate—it can't be predicated of anything since it is a conjunction of two statements. Relative to any variable assignment, "$Ex \,\&\, Px$" has truth conditions and denotes a (complex) state of affairs. Semantically, one can define a sense in which an individual in the domain can *satisfy* this open formula (namely, Tarski's sense), but this is not to say that the open formula can be *predicated* of that individual or predicated of the individual term "*a*." By contrast, the complex unary relation term "$[\lambda x\, Ex \,\&\, Px]$" can be combined with an individual constant to form a predication; that is, we can form the predication "$[\lambda x Ex\&Px]2$," which predicates the property denoted by the λ-expression of an individual. And in 2OL and 2OL$^=$, we can infer "$\exists F(F2)$" from "$[\lambda x\, Ex \,\&\, Px]2$." So whereas we call "$[\lambda x\, Ex \,\&\, Px]$" a predicate, we won't call "$Ex \,\&\, Px$" a predicate.

Similarly, we shall not say that the open formulas "*Fab*" and "$Fa \,\&\, Qb$" (where "*F*" is a free variable and the other letters are constants) are 2nd-order predicates. These are open formulas that denote states of affairs relative to

8 From Lax, we may directly infer, by the right-to-left direction of λ-CONVERSION (see section 2.2 below), that $[\lambda y Fyx]a$ and $[\lambda y Fay]x$, and from these latter, we can infer $\exists F(Fa)$ and $\exists F(Fx)$. But these existential claims are immediate consequences of the atomic exemplification predications $[\lambda y\, Fyx]a$ and $[\lambda y\, Fay]x$, in which $[\lambda y\, Fyx]$ and $[\lambda y\, Fay]$ are predicates.

an assignment to the free variable F. As such, these expressions are 0-ary relation terms, i.e., terms that denote states of affairs (relative to any variable assignment). By contrast, the higher-order λ-expressions "$[\lambda F\, Fab]$" and "$[\lambda F\, Fa \,\&\, Qb]$" are predicates of 3rd-order logic (3OL); these are expressions constructed from the open formulas "Fab" and "$Fa \,\&\, Qb$." The expressions "$[\lambda F\, Fab]$" and "$[\lambda F\, Fa \,\&\, Qb]$" are part of the language of 3OL because they denote properties of relations. These predicates can be used to form predications in 3OL such as "$[\lambda F\, Fab]R$," i.e., R exemplifies the property of being a relation F such that a and b exemplify F. We'll make use of these higher-order predicates later, at the point in the discussion when they become relevant.[9]

2. The First Horn

We can now outline and investigate MacBride's argument about the interpretation of the 2nd-order quantifiers. It proceeds under the reasonable assumption that 2nd-order quantification is a straightforward generalization of 1st-order quantification (MacBride 2022, 2). So let's suppose that the 1st- and 2nd-order quantifiers range over (mutually exclusive) domains and that the axioms and inference rules of the 2nd-order quantifiers mirror those of the 1st-order quantifiers. MacBride's argument, to the conclusion that we cannot interpret 2nd-order quantifiers as ranging over relations, goes by way of a dilemma.

9 It might be thought that such higher-order predicates are expressible in 2OL. One might point to the following passage in Shapiro (1991, 64–65):

> Second-order variables, as well as non-logical predicate, relation, and function names, may be called "higher-order terms," items that "denote" relations and functions. By way of analogy, this opens the possibility of relations of relations, functions on relations, etc. These may be called *higher-order non-logical terms*. An example would be a property TWO of properties such that TWO(P) "asserts" that P applies to exactly two things. A relevant "definition" would be:
>
> $$\text{TWO}(P) \equiv \exists x \exists y[x \neq y \,\&\, \forall z(Pz \equiv (z = x \lor z = y))]$$

But here Shapiro is talking loosely and signals that he is talking loosely by putting the word "denote" (and other terms) in quotation marks. The expression "TWO(P)" can be *defined* in 2OL, but it can't be interpreted as a *denoting term*, or as a term that denotes a property of properties, since there is no domain of properties of properties in the interpretation of 2OL. "TWO(P)" is simply an open formula that some properties satisfy and others don't. Moreover, in 2OL, the predicate $[\lambda F\, \text{TWO}(F)]$ isn't well-formed; the λ can only bind individual variables. There is no domain of properties of properties that could provide a denotation for such an expression.

Let's call this the DILEMMA FOR CONVERSES. He presents the dilemma as follows (MacBride 2022, 1–2):

> DILEMMA FOR CONVERSES
> Either pairs of mutually converse predicates, such as "ξ is on top of ζ" and "ξ is underneath ζ," refer to the same underlying relation or they refer to distinct converse relations. If they refer to the same relation, then we lack the supply of the higher-order predicates required to interpret second-order quantifiers as ranging over a domain of relations. […] If, by contrast, mutually converse predicates refer to distinct converse relations, then whilst we can at least make abstract sense of the higher-order predicates required to interpret quantifiers as ranging over a domain of relations, the implausible consequences for the content of lower-order constructions render this interpretation of higher-order quantifiers a deeply implausible semantic hypothesis

We need not state the full argument for each horn of the dilemma now because it can be shown that, given the reasonable assumption that non-symmetric relations exist, the condition leading to the first horn of the DILEMMA FOR CONVERSES doesn't hold in $2OL^=$. We spend the remainder of section 2 showing this, i.e., that mutually converse predicates do not refer to the same relation.

Since MacBride's argument in the DILEMMA FOR CONVERSES involves claims about converse relations, let us define:

- G is a *converse of* F if and only if, for any objects x and y, x and y exemplify G iff y and x exemplify F, i.e.,

(2) $\mathit{ConverseOf}(G,F) \equiv_{df} \forall x \forall y(Gxy \equiv Fyx)$

In addition, the argument in the DILEMMA FOR CONVERSES concerns the identity and distinctness of converses and so involves statements of the form "$R = S$" and "$R \neq S$." Thus, to see that the condition leading to the first horn of the Dilemma is false, i.e., to see that it is not the case that mutually converse predicates refer to the same underlying relation, we only need to show that there are converses F and G that aren't identical:

(3) $\exists F \exists G(\mathit{ConverseOf}(G,F) \,\&\, G \neq F)$

Any predicates that witness this claim will show that not all predicates for converses denote the same underlying relation.

Though (3) is not a theorem of 2OL$^{=}$, it is implied by a theorem of 2OL$^{=}$ under the assumption that there are non-symmetric relations. To see how, let us first define:

- F is *non-symmetric* if and only if it is not the case that for any objects x and y, if x and y exemplify F, then y and x exemplify F, i.e.,[10]

(4) $\textit{Non-symmetric}(F) \equiv_{df} \neg\forall x\forall y(Fxy \rightarrow Fyx)$

Given this definition, the assumption and theorem needed to establish (3) may be represented as follows:

(5) $\exists F(\textit{Non-symmetric}(F))$
(6) $\forall F(\textit{Non-symmetric}(F) \rightarrow \exists G(\textit{ConverseOf}(G,F)\ \&\ G \neq F))$

As mentioned above, (5) is a reasonable assumption that MacBride adopts in his paper. So if we can show that (6), i.e., the formal representation of (1), is a theorem of 2OL$^{=}$, it then will be a simple matter to show that (3) follows from (5) and (6).

2.1. *The Reasoning*

Two facts about 2OL$^{=}$ have to be mentioned before we begin. First, 2OL$^{=}$ includes the standard two axioms that logic texts use to systematize identity claims, namely, the reflexivity of identity and the substitutivity of identicals.[11]

10 This is to be contrasted with:

- F is *asymmetric* if and only if for any objects x and y, if x and y exemplify F, then it is not the case that y and x exemplify F, i.e.,
  $\textit{Asymmetric}(F) \equiv_{df} \forall x\forall y(Fxy \rightarrow \neg Fyx)$

Russell discusses *asymmetric* relations in (1903, sec. 218). In what follows, however, we discuss the more general notion of non-symmetric relations now being defined in the main text.

11 The reflexivity of identity can be expressed by the schema $\alpha = \alpha$, where α is either an individual variable or an n-ary relation variable, for some n. So $F = F$ becomes an instance of the reflexivity of identity, where F is any relation variable of any arity. The substitutivity of identicals can be expressed by the schema $\alpha = \beta \rightarrow (\varphi \rightarrow \varphi')$, where α and β are both individual variables or both n-ary relation variables (for some n) and φ' is the result of substituting the variable β for one or more occurrences of α in φ, provided that β is substitutable for α in φ (i.e., doesn't get

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Second, where $n \geq 0$, 2OL$^=$ includes the following comprehension axiom schema of 2OL:

> *ConverseOf*$(G, F) \equiv_{df} \forall x \forall y(Gxy \equiv Fyx)$ **(CP) Comprehension Principle for Relations** $\exists F^n \forall x_1 \dots \forall x_n(F^n x_1 \dots x_n \equiv \varphi)$, provided F^n doesn't occur free in φ.

We may read this as: there exists an n-ary relation F such that any objects $x_1, \dots, x_n$ exemplify F if and only if φ. In the case where $n = 0$ and "p" is used as a 0-ary variable instead of "F^0," then (??) asserts $\exists p(p \equiv \varphi)$, i.e., there exists a state of affairs p such that p obtains if and only if φ. Note that we read "p" as it occurs in "$p \equiv \varphi$" as "p obtains," since (a) "p" occurs as a formula and (b) *obtains* for states of affairs is the 0-ary case of *exemplification*. The 0-ary case of (??) will be of service later, but for now we focus on the cases of (??) where $n \geq 1$.

Before we show how 2OL$^=$ yields (6) as a theorem, a few words about the role (??) plays in 2OL$^=$ are in order. First, it is often thought that 2OL and 2OL$^=$ require a large ontology of relations simply in virtue of including (??) as an axiom. After all, in the unary case, (??) has instances such as the following:

- $\exists F \forall x(Fx \equiv \neg Gx)$
  (Any given property) G has a negation.
- $\exists F \forall x(Fx \equiv Gx \,\&\, Hx)$
  (Any given properties) G and H have a conjunction.
- $\exists F \forall x(Fx \equiv \exists y Kyx)$
  There is a property that objects exemplify whenever a binary relation K is projected into its first argument place.

And in the binary case, (??) has instances like the following:

- $\exists F \forall x \forall y(Fxy \equiv Kyx)$
  (Any given relation) K has a converse.

"captured" by a variable-binding operator when substituted). So as instances of the substitutivity of identicals, we have $F = G \to (\varphi \to \varphi')$, where φ' is the result of substituting the variable G for one or more occurrences of F in φ, provided G is substitutable for F in φ.

From these two principles, one can derive that identity for relations is symmetric and transitive. For example, to derive symmetry, i.e., $F = G \to G = F$, assume $F = G$. Then consider the instance of the substitution of identicals $F = G \to (F = F \to G = F)$. From this instance and our assumption, it follows that $F = F \to G = F$. But from this and reflexivity, it follows that $G = F$. Hence, by conditional proof, $F = G \to G = F$.

Since these claims hold for any relations G, H, and K, it might seem that (??) commits one to a large ontology.

But in fact, the smallest models of 2OL and 2OL$^=$ require only that the domain of n-ary relations contains just two relations, for each n. In what follows, we'll focus on 2OL$^=$, though the same reasoning applies to 2OL. So how can it be that 2OL$^=$ requires only that the domain of n-ary relations contains just two relations, for each n? The answer is: the smallest models of 2OL$^=$ make (??) true by identifying properties and relations with the same extension. More specifically, in the smallest models of 2OL$^=$, (i) the domain of individuals contains just a single element, say b; (ii) the domain of unary relations contains just two properties—one exemplified by b and one exemplified by nothing; (iii) the domain of binary relations contains just two relations—one that relates b to itself and one that is empty; and so on. For example, if we let P_1 be the property that is exemplified by b and P_2 be the empty property, then P_2 is the negation of P_1 and vice versa. Moreover, the conjunction of P_1 with itself is just P_1; the conjunction of P_2 with itself is just P_2; and the conjunction of P_1 with P_2 (and the conjunction of P_2 with P_1) is just P_2, since nothing exemplifies both P_1 and P_2. And so on for the other unary instances of (??). Now for the case of binary relations, let R_1 be the relation that relates b to itself, and R_2 be the empty relation. Then R_1 is the negation of R_2, and vice versa. Moreover, R_1 and R_2 both have converses—each has itself as a converse. R_1 is a converse of itself because $R_1bb \equiv R_1bb$, and R_2 is a converse of itself for a similar reason, though in this second case, the biconditional $R_2bb \equiv R_2bb$ is true because both sides are false. And so on for the other binary instances of (??).

So if we don't add any distinguished, theoretical properties and relations, 2OL$^=$ doesn't commit us to much at all. But though 2OL$^=$ does commit us to the existence of converse relations, it does *not* commit us to the existence of non-symmetric relations. In the smallest models of 2OL$^=$, as we just saw, there are only two binary relations; we've called them R_1 and R_2. Note that both R_1 and R_2 are symmetric; they both satisfy the open formula $\forall x \forall y(Fxy \rightarrow Fyx)$. R_1 satisfies this formula because b is the only object that can instantiate the 1st-order quantifiers and $R_1bb \rightarrow R_1bb$ is a theorem of logic; it is an instance of the tautology $\varphi \rightarrow \varphi$ (note that the consequent is true and so the whole conditional is true). R_2 is symmetric because, again, b is the only object that can instantiate the 1st-order quantifiers and the tautology $R_2bb \rightarrow R_2bb$ is again a theorem of logic (note that the antecedent is false, and so the whole

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conditional is true). We can consider this same point *proof-theoretically*: the claim $\exists F(\textit{Non-symmetric}(F))$ is not a theorem of this logic.[12]

Of course, (6) can still be true even if there are no non-symmetric relations, by failure of the antecedent. But the key fact is not that (6) is true independently of the existence of non-symmetric relations, but that it is derivable as a theorem. The proof doesn't depend on the existence of non-symmetric relations, doesn't employ any analysis of predication, and doesn't require any particular semantic interpretation of the domain over which the relation variables range. I've put the proof in a footnote.[13] So the formal representation of (1), namely (6), is a theorem of $2OL^{=}$.

But the combination of (6) with the reasonable assumption (5) yields the conclusion that there are mutually converse predicates that don't refer to

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12 The claim that there are non-symmetric relations, i.e., $\exists F(\textit{Non-symmetric}(F))$, expands to the following, by definition (4):

$$\exists F \neg \forall x \forall y (Fxy \rightarrow Fyx)$$

Clearly, this claim is not an instance of (??) since it has the wrong form. Moreover, we can't derive the existence of non-symmetric relations from instances of (??), such as:

$$\exists F \forall x \forall y (Fxy \equiv \textit{Non-symmetric}(F))$$

This is not a well-formed instance of (??) either, but in this case, the problem is that the variable F is free in the formula $\textit{Non-symmetric}(F)$, violating the axiom's condition. :::

13 *Proof*. Pick an arbitrary relation R and assume R is non-symmetric. Then, by definition (4) and predicate logic, there are objects, say a and b, such that both $Rab \,\&\, \neg Rba$. Note independently that (??) implies that every relation has a converse, as follows: if we let φ be Gyx, where G is a free variable, then $\exists F \forall x \forall y (Fxy \equiv Gyx)$ is a binary instance of (??). It follows by universal generalization that:

$$\forall G \exists F \forall x \forall y (Fxy \equiv Gyx)$$

By instantiating to R, it follows that $\exists F \forall x \forall y (Fxy \equiv Ryx)$. Pick an arbitrary relation as a witness to this claim, say S, so that we know:

(A) $\forall x \forall y (Sxy \equiv Ryx)$

(A) implies, by definition (2), that $\textit{ConverseOf}(S, R)$. But we already know Rab, since it's the first conjunct of $Rab \,\&\, \neg Rba$. Hence, Sba, by instantiating b for x and a for y in (A). Now for reductio, assume $S = R$. Then it follows that Rba, by substitution of identicals. But this contradicts $\neg Rba$, which is the second conjunct of $Rab \,\&\, \neg Rba$. Hence $S \neq R$, by reductio. We've therefore established $\textit{ConverseOf}(S, R) \,\&\, S \neq R$. So by EXISTENTIAL INTRODUCTION, $\exists G(\textit{ConverseOf}(G, R) \& G \neq R)$. By conditional proof, then, it follows that $\textit{Non-symmetric}(R) \rightarrow \exists G(\textit{ConverseOf}(G, R) \,\&\, G \neq R)$. But since R was arbitrary, universally generalizing on R yields (6).

the same underlying relation. For let "R" be a witness to assumption (5), so that we know *Non-symmetric*(R). Then, by (6), we obtain the conclusion $\exists G(\mathit{ConverseOf}(G,R)\ \&\ G \neq R)$, which tells us that R has a distinct converse. But we're not quite done; the condition leading to the first horn of the Dilemma for Converses is about *predicates*, and to show that it is false, we need a bit more reasoning and semantic ascent. So let "S" be a witness to our last result, so that we know $\mathit{ConverseOf}(S,R)\ \&\ S \neq R$. Then, by semantic ascent, we have established that the predicates "R" and "S" denote converse relations that are distinct. Thus, the condition leading to the first horn of the Dilemma for Converses, namely that pairs of mutually converse predicates refer to the same underlying relation, fails in $2OL^=$ under any interpretation. We therefore need to consider only the second horn.

2.2. *Simplifying the Reasoning*

Before we turn to the second horn of MacBride's Dilemma for Converses in section 3, it is relevant, and of significant interest, that (1) can be represented, and its proof developed much more elegantly, if we add λ-expressions to $2OL^=$. λ-expressions are complex terms that denote relations, and they will play an important role in what follows. We begin the explanation of how λ-expressions simplify our definitions and theorems about converses by saying a few words about the logic that results when we add these expressions.[14] Assume, therefore, that we have added complex, n-ary relation terms of the form $[\lambda x_1 \dots x_n\, \varphi]$ to the definition of our language ($n \geq 0$) given in footnote 7. When $n \geq 1$, we read $[\lambda x_1 \dots x_n\, \varphi]$ as *being objects* $x_1, \dots, x_n$ *such that* φ; when $n = 0$, we read $[\lambda\, \varphi]$ as *that*-φ. Thus, λ-expressions do not denote functions, as in the functional λ-calculus, but rather relations, and in the 0-ary case, they denote states of affairs. A simple predication like "$[\lambda x\, \neg Px]y$" asserts that y exemplifies *being an object x that fails to exemplify P*, and "$[\lambda\, \neg Rab]$" denotes the state of affairs *that a and b don't exemplify R*.

By adding λ-expressions to 2nd-order logic, we can replace (??) by:

λ-Conversion (λC)
$[\lambda x_1 \dots x_n\, \varphi]x_1 \dots x_n \equiv \varphi$

14 In essence, we will be using the λ-calculus under the interpretation in which λ-expressions denote relations rather than functions. See again the nice discussion of this in Alama and Korbmacher (2023, sec. 9.3).

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This asserts: $x_1, \dots, x_n$ exemplify *being objects* $x_1, \dots, x_n$ *such that* φ if and only if φ. For example, $[\lambda xy\, \neg Fxy]xy \equiv \neg Fxy$ is an instance, and by universal generalization, it is a theorem of the relational λ-calculus that:

$$\forall F \forall x \forall y([\lambda xy\, \neg Fxy]xy \equiv \neg Fxy)$$

To see how this works, instantiate this theorem to an arbitrary binary relation R and then to arbitrary objects a and b. The result is the instance: $[\lambda xy\, \neg Rxy]ab \equiv \neg Rab$.[15]

As previously mentioned, (λC) eliminates the need for (??) since the latter becomes derivable. The proof is left to a footnote.[16] This applies even to the 0-ary case of (λC). When $n = 0$, (λC) asserts $[\lambda\, \varphi] \equiv \varphi$, i.e., that-$\varphi$ obtains if and only if φ.[17] For example, the formula $[\lambda\, \neg Lmj] \equiv \neg Lmj$ might be used to represent the claim: (the state of affairs) that-Mary-doesn't-love-John obtains if and only if Mary doesn't love John. Note that the 0-ary case of (??) immediately follows from the 0-ary case of (λC), by EXISTENTIAL INTRODUCTION.[18] Again, the 0-ary case of (λC) will play a role later, but for now, let's focus on the cases where $n \geq 1$.

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15 In what follows, I also assume two other principles of the λ-calculus (understood relationally), namely η-CONVERSION, which asserts $[\lambda x_1 \dots x_n\, \Pi^n x_1 \dots x_n] = \Pi^n$, for any n-ary relation term Π, and α-CONVERSION, namely, that alphabetically-variant λ-expressions denote the same relation. η-CONVERSION tells us that a λ-expression such as "$[\lambda xyRxy]$," in which all the free variables in the atomic exemplification formula "Rxy" are bound by the λ, denotes the same relation that "R" denotes, i.e., the identity "$[\lambda xy\, Rxy] = R$" holds. As an example of α-CONVERSION, we have "$[\lambda xy\, Rxy] = [\lambda yz\, Ryz]$."

16 Just universally generalize on $x_1, \dots, x_n$ in (λC) to conclude:

$$\forall x_1 \dots \forall x_n([\lambda x_1 \dots x_n\, \varphi]x_1 \dots x_n \equiv \varphi)$$

Then, we can existentially generalize on the λ-expression (provided F doesn't occur free in φ) so that we obtain (??):

$$\exists F \forall x_1 \dots \forall x_n(Fx_1 \dots x_n \equiv \varphi), \text{ provided } F \text{ doesn't occur free in } \varphi$$

If F were free in φ, it would get "captured" by the quantifier $\exists F$, and the resulting principle would be invalid, for it would have the contradictory instance $\exists F \forall x(Fx \equiv \neg Fx)$.

17 See Zalta (2014) for a full discussion of why this reading is justified and shows that the propositional version of the Tarski T-schema is a tautology.

18 We can existentially generalize on the 0-ary relation term $[\lambda \varphi]$ in $[\lambda \varphi] \equiv \varphi$ to obtain: $\exists p(p \equiv \varphi)$, i.e., there is a state of affairs p such that p obtains if and only if φ. Of course, the usual proviso applies, namely, that p not occur free in φ. If p were to occur free in φ, then we could generalize on $[\lambda\, \varphi]$ by introducing some other quantified variable that doesn't occur free in φ.

We can use λ-expressions to introduce a well-behaved *converse* operator $(\ )^*$ on predicates by taking advantage of λ-expressions. Where F is a binary relation, we may define the converse of F, i.e., F^*, as *being an x and y such that y and x exemplify F*, i.e.,

(7) $F^* =_{df} [\lambda xy\, Fyx]$

Note how this definition immediately implies that every relation has a converse, where this is expressible as $\forall F \exists G(G = F^*)$.[19] *A fortiori*, every non-symmetric relation has a converse. Thus, we can now represent and prove (1) more elegantly as the claim that for any binary relation F, if F is non-symmetric, then its converse F^* is distinct:[20]

(8) $\forall F(\mathit{Non\text{-}symmetric}(F) \rightarrow F^* \neq F)$

Again, I've put the proof in a footnote,[21] and I encourage the reader to compare the proof of (8) in footnote 21 with the proof of (6) in footnote 13 to confirm how λ-expressions simplify the reasoning. Thus, as soon as we instantiate the reasonable assumption (5) to an arbitrary predicate, say "R," to conclude *Non-symmetric*(R), we can immediately instantiate the new predicate "R^*"

19 Let R be an arbitrary relation. Then, in classical $2OL^{=}$, in which every term (including every λ-expression) has a denotation, we have, as an instance of the reflexivity of identity, that $[\lambda xy\, Ryx] = [\lambda xy\, Ryx]$. So by Existential Introduction, $\exists G(G = [\lambda xy\, Ryx])$. And by definition of R^*, it then follows that $\exists G(G = R^*)$. Since R was arbitrary, we have established $\forall F \exists G(G = F^*)$.

20 Of course, one could more strictly represent (1) as follows:

$$\forall F(\mathit{Non\text{-}symmetric}(F) \rightarrow \exists G(G = F^* \,\&\, G \neq F))$$

But the consequent of this quantified conditional, $\exists G(G = F^* \,\&\, G \neq F)$, is just equivalent to the consequent of claim (8) in the text, namely, $F^* \neq F$. The proof of both directions of the equivalence is straightforward. For the left-to-right direction, suppose $\exists G(G = F^* \,\&\, G \neq F)$. Let H be such a relation, so that we know both $H = F^*$ and $H \neq F$. Then, by substitution of identicals, $F^* \neq F$. For the right-to-left direction, assume $F^* \neq F$. Then, by reflexivity of identity, $F^* = F^* \,\&\, F^* \neq F$. Hence, by Existential Introduction, $\exists G(G = F^* \,\&\, G \neq F)$. Given the equivalence just established, we use the simpler $F^* \neq F$ as the consequent when representing (1) as (8).

21 *Proof*. Assume *Non-symmetric*(R), where R is arbitrary. Then, $\neg\forall x \forall y(Rxy \rightarrow Ryx)$, i.e., for some objects, say a and b, we know $Rab \,\&\, \neg Rba$. Now for reductio, assume $R^* = R$. Then, by symmetry of identity, $R = R^*$, and from Rab, it follows that R^*ab, by substitution of identicals. So by definition (7) of R^*, we know $[\lambda xy\, Ryx]ab$. But by (λC), this implies Rba. Contradiction. Hence, $R^* \neq R$. So by conditional proof, *Non-symmetric*(R) $\rightarrow R^* \neq R$. Since R is arbitrary, we may universally generalize to get (8).

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into (8) and then conclude $R \neq R^*$. So by semantic ascent, the condition leading to the first horn of the Dilemma for Converses is false.

Thus, when we add λ-expressions to 2OL$^=$, the concepts and claims simplify and clarify. I'll therefore use (8) as the clearer representation of (1) in what follows. But my analysis will apply to (6) as well. Both (6) and (8) have been established as formal theorems without any analysis of predication or any semantic arguments about converses.

3. The Second Horn

MacBride's Dilemma for Converses concludes that the quantifiers of 2OL don't range over relations, and we've now seen that the first horn of the dilemma fails in 2OL$^=$ (i.e., the logic needed to systematize talk about the identity or distinctness of relation converses). The argument in the second horn was sketched at the beginning of section 2 above. But a fuller sketch of the argument emerges later in the paper, beginning in the following passage:

> But even if pairs of mutually converse relations are admitted, thus avoiding the difficulties that arose from dispensing with them, higher-order predicates of the form '$a\,\Phi\,b$' are still required for the intelligibility of quantification into the positions of converse predicates, i.e., higher-order predicates capable of being true or false of a relation belonging to the domain independently of how that relation is specified. [...]
>
> [...] [D]o we have an understanding of higher-order predicates of the form "$a\,\Phi\,b$" which will enable us to interpret second-order quantification as quantification over a domain of relations? I will argue that we don't. (2022, 14)

Before we look at the specific way in which MacBride argues for this conclusion, let's first make the language that MacBride needs to present his argument a bit more precise.

3.1. *Third-Order Language and Logic (3OL)*

I shall suppose that MacBride's language is 3rd-order, since he wants to formulate higher-order predicates capable of being true or false of relations.

If we use λ-expressions, we can formally represent the higher-order property connected with the open formula "Fab" as $[\lambda F\, Fab]$. We read this λ-expression as: being a relation F such that a and b exemplify F. So let us take on board the resources of a 3rd-order language and logic (3OL), including monadic, higher-order λ-expressions of the form $[\lambda F\, \varphi]$ for denoting complex properties of relations. 3OL lets us quantify over, and denote, properties of relations such as $[\lambda F\, \forall x Fxx]$ ("being a relation F that is reflexive") and such as $[\lambda F\, \neg\forall x\forall y(Fxy \rightarrow Fyx)]$ ("being a relation that is non-symmetric"), etc.

In 3OL, λ-expressions of the form $[\lambda F\, \varphi]$ are governed by the following schema:

(Monadic) Third – Order λ-Conversion (3λC)
$[\lambda F\, \varphi]F \equiv \varphi$

I.e., F exemplifies *being a relation such that* φ if and only if F is such that φ. So by Universal Generalization, the following is a theorem schema of 3OL:

(9) $\forall F([\lambda F\, \varphi]F \equiv \varphi)$

With this formalization in mind, we can return to MacBride's argument.

MacBride argues that in order for "$\exists F(Fab)$" to be interpreted as quantifying over relations, we have to be able to grasp the higher-order predicate associated with the expression "Fab" as being true or false of relations independently of how such relations are named or picked out. He then proceeds to consider and reject a number of proposals for so understanding "Fab."

3.2. *The First Argument for the Second Horn*

The first proposal that MacBride considers, and rejects, appeals to the determinate-determinable distinction. Earlier in his paper, he defined "Fab" as having a *determinable* significance when it "is true of the referent R of a first-level predicate [...] just in case R relates [a] to [b] in *some manner or other* but without settling any determinate arrangement for them" (2022, 9). He now argues that the suggestion, that "Fab" has a determinable significance, gets the truth conditions wrong for non-symmetric relations. Let us use sentences numbered in square brackets to reference the numbered sentences in MacBride's paper and consider these two sentences:

[1] Alexander is on top of Bucephalus.

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[8] ¬ Bucephalus is on top of Alexander.

He says, in connection with these sentences:

> If 'Alexander Φ Bucephalus' has purely determinable significance, then 'Bucephalus Φ Alexander' does too, but they will mean the same. The latter will stand for a property that a relation has if it relates Bucephalus and Alexander in some manner or other. But a relation has the property of relating Bucephalus and Alexander in some manner or other iff it has the property of relating Alexander and Bucephalus in some manner or other—because the property of relating some things in some manner or other is order-indifferent. (2022, 15)

He then draws the conclusion that we can't explain the valid inference from [1] to [8] given this analysis, for whereas [1] says that *on top of* has the order-indifferent property of relating Alexander and Bucephalus in some manner or other, [8] says that this relation doesn't have that property.

MacBride quite rightly rejects the suggestion that "Fab" has a determinable significance, but for the wrong reasons. MacBride rejects the suggestion on the grounds that it can't explain the valid inference from [1] to [8], but I think we can reject the suggestion because, as we'll see below, (3λC) already shows that "Fab," "Fba," and "$\neg Fba$" have a *determinate* rather than a *determinable* significance. Before we examine this claim in more detail, let me first put one issue aside, to be revisited later (in the context of the next suggestion), namely, whether [1] and [8] say what MacBride claims that they say. I don't think they do, but we need not develop the issue at this point.

Instead, we can see that "Fab," "Fba," and "$\neg Fba$" have a *determinate* significance by considering the higher-order predicates of relations that can be constructed with the help of these formulas. We may represent the higher-order properties signified as $[\lambda F\, Fab]$, $[\lambda F\, Fba]$, and $[\lambda F\, \neg Fba]$. These higher-order properties are all well-defined. To see why, let φ in (9) be, successively, Fab, Fba, and $\neg Fba$, and instantiate the quantifier $\forall F$ to the relation R in each case. Then all of the following are theorems of 3OL derivable from (3λC):

(10) $[\lambda F\, Fab]R \equiv Rab$
(11) $[\lambda F\, Fba]R \equiv Rba$
(12) $[\lambda F\, \neg Fba]R \equiv \neg Rba$

These are *not* schemata. (10) says: relation R exemplifies *being a relation F such that a and b exemplify F* just in case a and b exemplify R. (11) says: R exemplifies *being a relation F such that b and a exemplify F* just in case b and a exemplify R. And (12) says: R exemplifies *being a relation F that b and a fail to exemplify* just in case b and a fail to exemplify R.

Thus, "Alexander Φ Bucephalus" ("Fab") and "Bucephalus Φ Alexander" ("Fba") have a determinate significance represented, respectively, by the higher-order properties $[\lambda F\, Fab]$ and $[\lambda F\, Fba]$. Moreover, they clearly don't mean the same; they aren't even materially equivalent. $[\lambda F Fab]$ is exemplified by R, given the fact that Rab and (10), and $[\lambda F\, Fba]$ fails to be exemplified by R, given the fact that $\neg Rba$ and (11). So we need not accept the proposal that "Alexander Φ Bucephalus" has a determinable significance, nor the premise about what that hypothesis implies for understanding [1] and [8]. The fact is, expressions of the form "Fab" can be interpreted in terms of *determinate* higher-order properties, as we have just done, and so (10) gives us the philosophical means for understanding the open formula "Fab" for an arbitrary relation R.

3.3. *The Second Argument for the Second Horn*

The next proposal that MacBride considers and rejects is the suggestion that we understand "Fab" in terms of a higher-order property of relations in which ordinal notions ("first," "second") play some role. In particular, the proposal under consideration is that "Fab" is to be understood in terms of the higher-order property that a relation has if it applies to a first and b second. MacBride develops an extended argument (2022, 16–28) against this proposal by advancing a number of considerations. At the end, he concludes: "[…] we lack a grasp of the higher-order predicates required to characterize relations in a higher-order setting, a grasp that is appropriately rooted in our understanding of atomic statements" (2022, 25). This conclusion is then supposed to entail that we can't understand the quantified formula "$\exists F(Fab)$" as quantifying over relations.

Let's grant that the entailment holds. Then we can respond to the argument by showing that we do have a grasp of the higher-order predicates required to understand quantification over relations. Fortunately, we don't have to go through the extended argument in detail because we can demonstrate that our grasp of these higher-order predicates is embodied by (3λC). Over the next few paragraphs, I (a) show why (3λC) is the right principle, (b) defuse

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some reasons that might be offered as to why it isn't, (c) show how (3λC) helps us to undermine some of the claims MacBride makes during the course of his argument for the second horn, and (d) narrow our focus to a question that is, at least in part, driving MacBride's concern about quantification over relations.

Clearly, (3λC) is a logical principle, and it states exemplification (i.e., "application") conditions for the higher-order properties denoted by predicates of the form $[\lambda F\, \varphi]$. So, we do *not* lack a principled grasp of the higher-order predicate "$[\lambda F\, Fab]$" that is formulable from the open formula "Fab." We saw that (10) is an instance of (3λC) and so offers a principled statement of the application conditions of the higher-order property $[\lambda F\, Fab]$. Clearly, one must distinguish the open formula "Fab" from the closed predicate "$[\lambda F Fab]$" to even formulate (3λC).

MacBride does seem to recognize that (3λC) forms the basis of a genuine response to his argument, for he subsequently considers an *informal* version of (3λC). He writes:

> Might there be an alternative interpretation of higher-order predicates of the form '$a\Phi b$' over which we have more control and which will facilitate an interpretation of second-order quantifiers as ranging over a domain of relations? The ordinary language construction "—*bears*---*to*____," as it figures in
>
> [14] Alexander bears a great resemblance to Philip,
>
> might appear to be a promising candidate for a construction in which our understanding of a predicate of the form '$a\Phi b$' might be rooted. Roughly speaking, the idea is that a relation R satisfies the predicate '$a\Phi b$' just in case a bears R to b, whereas R satisfies '$b\Phi a$' just in case b bears R to a. (2022, 22–23)

MacBride then argues against this idea (2022, 23–24). But I will not examine the details of this particular argument, for it appears to challenge the intelligibility of a well-known logical principle, namely λ-CONVERSION (λC), in its higher-order guise as (3λC). I take both principles to be perfectly intelligible; they axiomatize complex predicates of the form $[\lambda\alpha\, \varphi]$ by precisely identifying their exemplification (or application) conditions. To my mind, the discussion in (2022, 23–24) doesn't clearly separate the logic from the way natural language is to be represented in that logic.

Note that one can't reject (3λC) on the grounds that it is trivial. One might argue that (3λC) trivially recasts the open formula as a higher-order predicate and so doesn't help us understand "Fab" or the higher-order property in question. But neither (λC) in 2OL nor (3λC) in 3OL are trivial. (λC) in 2OL is a significant principle that is an integral part of the λ-calculus of relations and thus one of the key axioms for axiomatizing relations (see Zalta 1983, 69; 1993, 406; Menzel 1986, 38; and Menzel 1993, 84). It is *stronger* than (??) (it implies (??), as we've seen, but (??) doesn't imply it), and it is not plausible to suggest that (??) is a trivial principle. (3λC) has a similar significance in 3OL.[22]

By systematizing the distinction between an open formula such as "Fab" and the higher-order predicate "$[\lambda F\, Fab]$," it becomes clear that (3λC) may even be an assumption of MacBride's paper that addresses the concern he raises, since the right-to-left direction of (3λC) tells us that if a relation R satisfies the open formula "Fab," then R exemplifies the higher-order property $[\lambda F\, Fab]$. And since (3λC) is a biconditional that implies the converse of this last claim, we forestall MacBride's conclusion that we lack a principled understanding of the application conditions of "Fab."[23]

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22 One referee for this journal suggested that MacBride would say:

> Schematic principles do not address these worries about relations [...] precisely because these principles are schematic, i.e., because they contain schematic letters which show what happens when a schematic letter is replaced with a predicate, in this case R. This means that schematic principles only speak to cases where relations are picked out by a predicate, but MacBride's point is that to grasp "$\exists\Phi(a\Phi b)$" as incorporating quantification, we need to grasp "$a\Phi b$" as being true or false of a relation in the domain even if no predicate can pick it out.

But this doesn't undermine (3λC) as a principle that yields an intelligible understanding of "Fab." The *instances* of (3λC) don't involve schematic letters. For example, "$[\lambda F\, Fab]F \equiv Fab$" directly governs the open formula "Fab," with the free variable F. No 1st-order predicate constant appears in this instance, and so no 1st-order relation has been specified by this instance. The two free occurrences of "F" in this instance refer to an arbitrary relation (i.e., whatever is assigned to the free variable "F"), independent of how that relation is specified ("F" is a variable, after all). Any relation in the domain could be assigned as a value for "F." Moreover, as we saw earlier, the universally quantified formula (9), i.e., $\forall F([\lambda F\, Fab]F \equiv Fab)$, is an immediate consequent of (3λC). It quantifies over *every entity in the domain* of the quantifier "$\forall F$," independently of how those entities are specified. So (3λC) is just the right principle to explain the higher-order property that MacBride says might be in play in our understanding of the open formula "Fab."

23 There is another way to forestall MacBride's conclusion without appealing to 3OL, namely by developing a precise semantics for the (open) formulas of 2OL that is grounded in a theory of relations and states of affairs. For example, the language in Zalta (1983) provides truth conditions,

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So if (3λC) gives a principled account of the significance of open formulas and the higher-order predicates we can build with such formulas, what then is really driving the concerns that MacBride has about quantifying over relations? To understand the root of the concerns, we have to consider one of the specific arguments that MacBride presents. He spends all of section 6 considering the consequences of supposing that relations hold between the objects they relate *in an order*. The underlying root of his concerns emerges when we consider the "untoward consequences" that allegedly result if we were to understand "Fab" in terms of a higher-order property that a relation has if it applies to a first and b second (2022, 17).

Now in the present paper, we're *not* committed to reading the formula "Fab" as "F applies to a first and b second." The notion of *applying to ... in an order* isn't a primitive of our logic; of course, one is tempted to say it is the *position* or *place* in the relation that a and b have to occupy rather than the order of application. But our logic isn't even committed to that much; it isn't committed to the existence of positions or places in a relation as entities (see Fine 2000, 16, for a defense of anti-positionalism). Our reading of "Fab" as "a and b exemplify F" doesn't explicitly *say* that a occupies the first position (or place) of F and b the second.[24] Similarly, when we read the predicate "$[\lambda F\, Fab]$" as "being an F such that a and b exemplify F," this doesn't require us to say further that F is such that a occupies its first position (or place) and b its second. But let's grant, for the sake of argument, that the higher-order predicate involves ordinal notions in the way MacBride suggests and

relative to an assignment to the variables, for the open formula "Fab." These are stated in terms of the relation that serves as the denotation of "F" relative to a variable assignment (the denotation of "F" relative to a variable assignment f is just the entity assigned to "F" by f). This semantics is grounded in the theory of relations that is expressible in the extended 2OL formalism developed in Zalta (1983). We'll discuss this theory later in the paper.

24 Are the *ordinal* concepts *first*, *second*, etc. assumed by the primitive notion of a relation? This is by no means clear. The *numerals* that serve as subscripts on "x_1,"...,"x_n" provide a way to have distinct variables; we could have used distinct letters instead. Moreover, the numeral "n," which serves as a superscript in "F^n" and as a subscript in "x_n" in atomic formulas of the general form $F^n x_1 \ldots x_n$, is *not* a variable that can be bound by a quantifier in 2OL. Instead of numerals, we could have placed a series of ticks on the predicate to indicate arity, so that a well-formed atomic formula includes as many arguments to the predicate as ticks. So, it looks like neither the ordinal concepts *first*, *second*, etc., nor the concept of *number* are primitives of the predicate calculus.

Thus, the expressions denoting relations have, at best, only an implicit notion of order that does little more than preserve the idea that "Fab" says something different from "Fba," and so on for relations of greater arity. That is, at a minimum, we require only that "a and b exemplify F" says something different than "b and a exemplify F." That may be the extent to which the theory of relations assumes ordinal notions.

read it as "being an *F* such that *F* applies to *a* first and *b* second." Under this reading, (3λC) remains true. MacBride then considers symmetric and non-symmetric relational statements and, in each case, finds reasons to question the understanding of "*Fab*" in terms of ordinal notions. For example, with respect to the symmetric relation *differs from*, he argues that "Darius differs from Alexander" and "Alexander differs from Darius" intuitively say the same thing, but given the understanding of the open formulas "*Fda*" and "*Fad*" that we're now considering, these formulas say different things. He argues:

> Since second-order logic permits existential quantification into the positions of symmetric predicates, it follows—assuming the proposed interpretation of higher-order predicates—that atomic statements in which symmetric predicates occur attribute to symmetric relations the property of applying to the things they relate in an order. But it is far from plausible that they do. Consider, for example,
>
> [9] Darius differs from Alexander
>
> and
>
> [10] Alexander differs from Darius.
>
> If predicates of the form "*a*Φ*b*" mean what they're proposed to mean, then [9] says that the relation picked out by "ξ differs from ζ" applies to Darius first and Alexander second, whereas [10] says that it applies to Alexander first and Darius second. But, as both linguists and philosophers have reflected, prima facie statements like [9] and [10] don't say different things but are distinguished solely by the linguistic arrangements of their terms. (2022, 17)

Although MacBride cites a number of authorities for his last claim, he also mentions that Russell (1903, sec. 94) argued against it and for the view that statements like [9] and [10] express distinct propositions.

Before I examine this argument, let me return to one issue. I don't accept that [9] says what MacBride claims it says. [9] does *not* say, nor can one *derive* in 2OL or 3OL that it says, "the relation picked out by 'ξ differs from ζ' applies to Darius first and Alexander second," as MacBride suggests. For one thing,

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[9] doesn't say anything about predicates picking out, or denoting, relations. Instead, [9] simply says Darius differs from Alexander (or, when regimented as $d \neq a$, [9] says "d and a exemplify being non-identical"). Of course, when we regiment [9] as "$d \neq a$" and use 3OL, we can also instantiate our sentence (9) in section 3.1 to the non-identity relation $\neq$ to obtain $[\lambda F\, Fda]{\neq} \equiv d \neq a$ and infer from this last fact and the representation of [9] that $[\lambda F\, Fda]{\neq}$, i.e., that the relation *differs from* exemplifies the higher-order property of being a relation Darius and Alexander exemplify. So, in what follows, I'll treat MacBride's reading of [9] not as what [9] says but as what [9] semantically implies in 3OL. And something similar applies to MacBride's sentence [10].

Clearly, the crux of MacBride's argument in the above passage is his view that [9] and [10] don't say different things. But surely there is at least a sense of "says" in which [9] and [10] do say different things. If we ignore the particular symmetric relation involved and consider a non-symmetric relation, then to say "John loves Mary" is not to say "Mary loves John." So MacBride's argument must turn on a notion of "says" in which [9] and [10] say the same thing. For the purposes of discussion, the notion in question has to be something like "denote the same state of affairs." He is convinced that they do, whereas I think this isn't at all clear. The point at issue concerns the identity of states of affairs; if one allows, for example, that necessarily equivalent states of affairs may be distinct, it is by no means a fact that [9] and [10] say the same thing.[25] Indeed, I hope to show in what follows that as long as we have a clear theory of relations and states of affairs (something that can be developed *without* the resources of 3OL), one can both (a) challenge the suggestion that [9] and

25 I don't think MacBride here is claiming that the state of affairs $d \neq a$ is identical to $a \neq d$ on the grounds that they are necessarily equivalent. That is, he does not give the following argument:

> Given the necessity of identity and a modal logic with the K and B axioms, it follows not only that $\forall x \forall y(x = y \rightarrow \Box x = y)$ but also that $\forall x \forall y(x \neq y \rightarrow \Box x \neq y)$. So from [9] ($d \neq a$) and [10] ($a \neq d$), it would follow that $\Box d \neq a$ and $\Box a \neq d$, respectively. But $(\Box\varphi \,\&\, \Box\psi) \rightarrow \Box(\varphi \equiv \psi)$, and so it would follow that $\Box(d \neq a \equiv a \neq d)$. Since necessarily equivalent states of affairs are identical, it would follow that $(d \neq a) = (a \neq d)$, thereby identifying the two states of affairs in question. This argument would hold for any symmetric relation like *differs from* that holds necessarily whenever it holds.

But MacBride doesn't argue this way, and even if he were to so argue, we do not suppose, in what follows, that necessarily equivalent states are identical. There are well-known counterexamples to the proposal that necessarily equivalent relations, properties, and states of affairs are identical. In what follows, we take such entities to be hyperintensional, i.e., entities that may be distinct even if necessarily equivalent.

[10] denote the same state of affairs *and* (b) argue that even if we leave the question open, we can still understand the application conditions of "Fab" and conclude that "$\exists F(Fab)$" quantifies over relations.[26]

But before we turn to the theory of relations and states of affairs that support this position, the second puzzling conclusion mentioned at the outset of the paper, namely the conclusion in MacBride (2014), becomes relevant. For the argument in that paper also turns, at least in part, on the question of the identity of states of affairs.

4. The Second Puzzling Conclusion

To state the second puzzling conclusion, which occurs in MacBride (2014), we have to recall the second of the three degrees of relatedness that MacBride distinguishes in that paper. He says, where R^* signifies the converse of R, that "to embrace the second degree is to make the existential assumption that every non-symmetric relation has a distinct converse ($R \neq R^*$)" (2014, 3). He then argues that relatedness in the second degree "spells trouble" and has "unwelcome consequences," namely, that it "commits us to a superfluity of converse relations and states" (2014, 4). Let's consider these claims in turn, i.e., by focusing first on the superfluity of relations and then on the superfluity of states.

Let me begin by suggesting that the superfluity of converse relations is not the main objection of the two. For recall that the conclusion in MacBride (2014) is that we should take relations and relation application as primitive. Since these notions are primitive in 2OL$^=$, the conclusion MacBride draws in (2014) doesn't eliminate the multiplicity of relations. For when (1) is represented as (6), it becomes a theorem of 2OL$^=$, as we saw in section 2.1. So the multiplicity of converse relations arises even when relations and relation application are primitive (given the assumption that non-symmetric relations

26 I note another reason for not accepting MacBride's reading of [9] as "what it says." If we were to accept his reading, then "$\exists F(Fab)$" would say that some relation has the higher-order property that a relation has when it applies to a first and b second. But "$\exists F(Fab)$" doesn't say this, not even semantically, for it says nothing about higher-order properties. The claim that MacBride attributes to "$\exists F(Fab)$" is representable in 3OL by the formula: $\exists G([\lambda F\, Fab]G)$. This does indeed say, given MacBride's hypothesis about the ordinal notions involved, that some relation G exemplifies the property of being a relation F that applies to a first and b second. But the semantics of 2OL doesn't explicitly require quantification over *properties of relations* when it assigns truth conditions to "$\exists F(Fab)$," and so one can interpret this claim in 2OL without invoking properties of relations. Of course, one needs Tarski's notion of satisfaction instead.

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exist). And this holds not only for binary non-symmetric relations but also non-symmetric relations of higher arity.[27] Though MacBride also suggests that we can't name the relations given such a multiplicity, in fact we can denote them using λ-expressions.[28] In any case, MacBride's argument that relations and relation application should be taken as primitive doesn't avoid the conclusion that there are a multiplicity of converse relations.

So the real problem about the fact that non-symmetric relations have distinct converses concerns the "profusion" of states of affairs. MacBride rehearses this problem by considering *on* and *under*, both of which are asymmetric (and hence non-symmetric if there are objects that stand in those relations):

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27 To see that the generalization of (6) remains a theorem for relations of higher arity, let F be any n-ary relation ($n \geq 3$) and let i and j be such that $1 \leq i < j \leq n$. Then we may define the i, j^{th}-converse of F, written $F^*_{i,j}$, as follows:

(ϑ) $F^*_{i,j} =_{df} [\lambda x_1 \ldots x_i \ldots x_j \ldots x_n\, Fx_1 \ldots x_j \ldots x_i \ldots x_n]$

And we can define F as *non-symmetric with respect to its* i^{th} and j^{th} places:

(ξ) $\mathit{Non\text{-}symmetric}_{i,j}(F) \equiv_{df} \neg\forall x_1 \ldots \forall x_i \ldots \forall x_j \ldots \forall x_n(Fx_1 \ldots x_i \ldots x_j \ldots x_n \rightarrow Fx_1 \ldots x_j \ldots x_i \ldots x_n)$

Then for any n-ary relation F ($n \geq 3$) and i, j ($1 \leq i < j \leq n$), it is provable that:

$$\forall F(\mathit{Non\text{-}symmetric}_{i,j}(F) \rightarrow F \neq F^*_{i,j})$$

The proof is just a generalization of the one given for (8) and goes as follows: Fix n, i, and j. Assume $\mathit{Non\text{-}symmetric}_{i,j}(F)$. Then by ($\xi$), there are objects $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, say $a_1, \ldots, a_i, \ldots, a_j, \ldots, a_n$, such that $Fa_1 \ldots a_i \ldots a_j \ldots a_n$ and $\neg Fa_1 \ldots a_j \ldots a_i \ldots a_n$. Assume, for reductio, that $F = F^*_{i,j}$. Then it follows by the substitution of identicals that $F^*_{i,j}a_1 \ldots a_i \ldots a_j \ldots a_n$. So by definition ($\vartheta$), it follows that:

$$[\lambda x_1 \ldots x_i \ldots x_j \ldots x_n\, Fx_1 \ldots x_j \ldots x_i \ldots x_n]a_1 \ldots a_i \ldots a_j \ldots a_n$$

Hence, by (λC): $Fa_1 \ldots a_j \ldots a_i \ldots a_n$. Contradiction.

28 MacBride says, "Each ternary non-symmetric relation has five mutual converses, and we don't have names for any of them" (2014, 4). But if S is a ternary non-symmetric relation, we can denote its converses as follows: $[\lambda xyz\, Sxzy]$, $[\lambda xyz\, Syxz]$, $[\lambda xyz\, Syzx]$, $[\lambda xyz\, Szxy]$, and $[\lambda xyz\, Szyx]$. The first of these can be read as: being objects x, y, and z such that x, z, and y exemplify S; the second as: being objects x, y, and z such that y, x, and z exemplify S; etc. van Inwagen (2006) would demur, but his argument doesn't engage (the coherency of) a precise theory of relations of the kind presented section 5 below.

> It's one kind of undertaking to put the cat on the mat, something else to put the mat under the cat, but however we go about it we end up with the same state. To bring the cat to the forefront of our audience's attention we describe this state by saying that the cat is on the mat; to bring the mat into the conversational foreground we say that the mat is under the cat. But whether it's the cat we mention first, or the mat, what we succeed in describing is the very same cat-mat orientation. That's intuitive but if—as the second degree describes—a non-symmetric relation and its converse are distinct, we must be demanding something different from the world, a different state, when we describe the application of the above relation to the cat and the mat from when we describe the application of the below relation to the mat and the cat. (2014, 4)

The worry is that converse relations commit us to the principle that if R is non-symmetric, then for any x and y, the state of affairs Rxy is distinct from the state of affairs R^*yx. We can formally represent the allegedly problematic principle as follows:

(13) $\forall F \Box(\mathit{Non\text{-}symmetric}(F) \rightarrow \forall x \forall y(Fxy \neq F^*yx))$

This, it is claimed, is counterintuitive, and MacBride cites Fine (2000) in support of his claim.[29] If this is the concern, why not adopt the following principle instead:

- For any binary relation F, necessarily, if F is non-symmetric, then for any x and y, the state of affairs *x and y exemplify F* is identical to the state of affairs *y and x exemplify F^**, i.e.,

29 In Fine (2000, 3), we find:

> What makes this consequence so objectionable, from a metaphysical standpoint, is a certain view of how relations are implicated in states or facts. Suppose that a given block a is on top of another block b. Then there is a certain state of affairs s_1, that we may describe as the state of a's being on top of b. There is also a certain state of affairs s_2 that may be described as the state of b's being beneath a. Yet surely the states s_1 and s_2 are the same. There is a single state of affairs s "out there" in reality, consisting of the blocks a and b having the relative positions that they do; and the different descriptions associated with s_1 and s_2 would merely appear to provide two different ways at getting at this single state of affairs.

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(14) $\forall F \Box (Non\text{-}symmetric(F) \rightarrow \forall x \forall y (Fxy = F^{*}yx))$

The answer MacBride gives is (2014, 4):

> We might attempt to defend the second degree by maintaining that the application of R and R^{*} does not give rise to different states with respect to the same relata but different decompositions of the same state. So whilst *above* and *below* are distinct, the relational configuration *cat-above-mat* is a decomposition of the same state as the configuration *mat-below-cat*. But these decompositions comprise what are ultimately different constituents—a non-symmetric relation and its converse are supposed to be distinct existences. But now we have the difficulty of explaining how such different decompositions can give rise to a *single* state.

So, again, the problem being raised is about the identity of states of affairs. In these cases, MacBride is confident that there is a single state involved.

Note that we've now connected up the issue on which MacBride's (2022) paper turns with the issue on which his (2014) paper turns, namely, the identity of states of affairs. What gives rise to this problem is that 2OL and $2OL^{=}$ don't have the resources to supply a good definition of the conditions under which states of affairs are identical, even if we add modality to the logic. For neither of the following definitions is a good one:

$$p = q \equiv_{df} p \equiv q$$

$$p = q \equiv_{df} \Box(p \equiv q)$$

It is reasonable to suppose that the state of affairs *there is a barber who shaves all and only those who don't shave themselves* ($\exists x(Bx \,\&\, \forall y(Sxy \equiv \neg Syy))$) is distinct from the state of affairs *there is a brown and colorless dog* ($\exists x(Dx \,\&\, Bx \,\&\, \neg Cx)$), yet these are not just equivalent but necessarily equivalent (since both are necessarily false).

So whereas both of the above definitions might be used to explain why $Fxy = F^{*}yx$ (e.g., "they are identical because they are necessarily equivalent"), the definitions fail when states of affairs (or propositions) are regarded as hyperintensional entities. The identity conditions for states of affairs are more fine-grained than material or necessary equivalence. Furthermore, when F is non-symmetric, there is no obvious way to account for the identity of Fab

and F^*ba by appealing to some notion of "constituents." On what grounds, expressible in 2OL, would one claim that the distinct constituents F, F^*, a, and b can be combined so that the identity $Fab = F^*ba$ holds?[30] And how can one state hyperintensional identity conditions for states of affairs that also allow us to assert, in the case of a non-symmetric relation F, that $Fab = F^*ba$?

MacBride, as noted at the outset, finalizes this problem for any analysis of the identity (or non-identity) of states of affairs as a *dilemma*. We earlier provided an edited version of the argument to give the reader the general idea. But the passage posing the dilemma goes as follows, in full:

> What vexes the understanding is the difficulty of disentangling one degree of relatedness from another when we try to provide an *analysis* of the fundamental fact that $aRb \neq bRa$ for non-symmetric R. We can usefully distinguish, albeit in a rough and ready sense, between two analytic strategies for explaining this fundamental fact—that the world exhibits relatedness in the first degree. *Intrinsic* analyses aim to account for the fact that $aRb \neq bRa$ by appealing to features of those states themselves; *extrinsic* analyses attempt to account for their difference by appealing to features that aren't wholly local to them. Anyone who wishes to give an analysis of the fact that $aRb \neq bRa$ faces a dilemma. If they adopt the intrinsic strategy then they will find it difficult to avoid a commitment to either R's converse or an inherent order in which R applies to the things it relates. Alternatively our would-be analyst can avoid entangling the first degree with the second and third by adopting the extrinsic strategy. But this approach embroils us in other unwelcome consequences. Since neither intrinsic nor extrinsic analyses are satisfactory, this recommends our taking the fact that $aRb \neq bRa$ to be primitive. (2014, 8, italics in original)

I think MacBride reaches this conclusion because he doesn't have a precise theory of relations and states of affairs to provide an answer. In the remainder

30 You can't assert the principle $Fxy = Gzw \equiv (F = G \,\&\, x = z \,\&\, y = w)$, for the scenario in which cat-on-mat (Ocm) and mat-under-cat (O^*mc) are identical constitutes a counterexample. For the principle would imply the instance $Ocm = O^*mc \equiv (O = O^* \,\&\, c = m \,\&\, m = c)$. And from the fact that $O \neq O^*$, or the fact that $c \neq m$, it would follow that $Ocm \neq O^*mc$. So this is no help, since we're trying to explain how we can have, simultaneously, $O \neq O^*$ and $c \neq m$, and yet $Ocm = O^*mc$.

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of the paper, I show how object theory (OT) takes *n*-ary relations as primitive (including states of affairs, understood as 0-ary relations), takes relation application (predication) as primitive, but defines identity for relations and states of affairs. These identity conditions don't appeal to "decompositions" or "constituents." Nevertheless, they allow one to *consistently* assert that (some) necessarily equivalent relations and states may be distinct. Using this theory of relations and states, we can address the "profusion of states" problem (in MacBride 2014) in either of two ways and address the problem underlying the first puzzling conclusion (in MacBride 2022) as well. As we shall see, a precise theory of relations and states may leave certain identity questions open, just as the precise theory of sets ZFC leaves open certain identity questions. The solution in ZFC is not to conclude that its quantifiers can't range over sets but to find and justify axioms that help decide the open questions within the precise, but extendable, framework ZFC provides (i.e., one that clearly quantifies over sets). Something similar happens in OT.

5. The Theory of Relations and States of Affairs

This section can be skipped by those familiar with OT since the material contained herein has been outlined and explained in a number of publications [e.g., Zalta (1983); -Zalta (1988); -Zalta (1993); Bueno, Menzel and Zalta (2014); Menzel and Zalta (2014); and others]. For those completely unfamiliar with it, OT may be sketched briefly by saying that it extends 2OL, not 2OL$^=$, since identity isn't taken as a primitive. OT adds to 2OL new atomic formulas of the form "xF," which represent a new mode of predication that can be read as "*x encodes F*," where "F" can be replaced by any unary predicate. Intuitively, "xF" expresses the idea that F is one of the properties by which we conceive and characterize an abstract, intentional object x.[31] OT also includes a distinguished unary relation constant "$E!$" for *being concrete*, a primitive necessity operator ($\Box$), and a defined possibility operator ($\Diamond$). OT then defines *ordinary* objects ("$O!x$") as objects x that might exemplify concreteness and defines *abstract* objects ("$A!x$") as objects x that couldn't exemplify concreteness. It is axiomatic that ordinary objects necessarily fail to encode properties

31 For example, consider the *content* of the mental image we have of Mark Twain and ask, How does the property of having a walrus mustache characterize that content? The content of the image is characterized by the property, but the content doesn't exemplify the property—Mark Twain exemplifies the property. But I would say that the content encodes the property, and since encoding is a mode of predication, the property characterizes the content.

$(O!x \rightarrow \Box\neg\exists FxF)$, though the theory allows that abstract objects can both exemplify and encode properties. It is also axiomatic that if x encodes a property, it necessarily does so $(xF \rightarrow \Box xF)$.

But the key principle for abstract objects is the comprehension schema that asserts, for any condition (formula) φ in which x doesn't occur free, that there exists an abstract object that encodes all and only the properties such that φ:

(15) $\exists x(A!x \,\&\, \forall F(xF \equiv \varphi))$

Here are some instances, expressed in technical English:

- There exists an abstract object that encodes all and only the properties that y exemplifies. $\exists x(A!x \,\&\, \forall F(xF \equiv Fy))$
- There exists an abstract object that encodes just the property G. $\exists x(A!x \,\&\, \forall F(xF \equiv F = G))$
- There is an abstract object that encodes all the properties necessarily implied by G. $\exists x(A!x \,\&\, \forall F(xF \equiv \Box\forall x(Gx \rightarrow Fx)))$
- There is an abstract object that encodes all and only the propositional properties constructed out of true propositions. $\exists x(A!x \,\&\, \forall F(xF \equiv \exists p(p \,\&\, F = [\lambda x\, p])))$

And so on. Intuitively, for any group of properties you can specify to describe an abstract object, there is an abstract object that encodes just those properties and no others.

The other principles of this theory that will play an important role in what follows are the definitions of identity for individuals and the principles (existence and identity conditions) for relations. First, the theory of identity for individuals includes a definition stipulating that x and y are identical if and only if they are both ordinary objects that necessarily exemplify the same properties or they are both abstract objects that necessarily encode the same properties:

(16) $x = y \equiv_{df} (O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)) \vee (A!x \,\&\, A!y \,\&\, \Box\forall F(xF \equiv yF))$

Second, the theory of relations consists of *existence* and *identity* conditions for relations. The existence conditions are *derived* since OT includes the resources of the relational λ-calculus; λ-expressions of the form $[\lambda x_1 \ldots x_n \varphi]$ are well-formed, but only if φ doesn't have any encoding subformulas.[32] So (λC), as

32 In the latest version of OT, currently under development (Zalta 2024), every formula φ becomes a permissible matrix of a λ-expression, but not every λ-expression has a denotation. If the variables

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stated above, is the main axiom governing λ-expressions. One can derive from (λC) a *modal* version of (??). This theorem schema, ($\Box$CP), asserts existence conditions for relations as follows:[33]

> MODAL COMPREHENSION FOR RELATIONS ($\Box$CP)
> $\exists F^n \Box \forall x_1 \ldots \forall x_n (F^n x_1 \ldots x_n \equiv \varphi)$, provided F doesn't occur free in φ and φ doesn't contain any encoding subformulas.

When $n = 1$ and $n = 0$, respectively, this principle asserts existence conditions for properties and states of affairs:

> $\exists F \Box \forall x (Fx \equiv \varphi)$, provided F doesn't occur free in φ and φ doesn't contain any encoding subformulas.

> $\exists p \Box (p \equiv \varphi)$, provided p doesn't occur free in φ and φ doesn't contain any encoding subformulas.

In other words, any formula free of encoding conditions can be used to produce a well-formed instance of ($\Box$CP). It is of some interest that there are still very small models of OT; for example, the smallest model involves one possible world, one ordinary object, two 0-ary relations, two unary relations, two binary relations, etc., and four abstract objects. Though the models grow when OT is applied, minimal models show that without further axioms, the theory doesn't commit one to much. Thus, relations, properties, and states of affairs exist under conditions analogous to those in classical, modal 2OL.[34]

The identity conditions for relations are stated by cases: (a) for properties F and G, (b) for n-ary relations F and G ($n \geq 2$), and (c) for states of affairs p and q. Identity for relations and states of affairs is defined in terms of identity for properties. The definitions are as follows:

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bound by the λ don't occur as primary terms in an encoding formula in φ, the resulting λ-expression is stipulated to denote a relation. So in the latest versions of the theory, λ-expressions are governed by a free logic. But for this paper, the published versions of the theory suffice; the logic of well-formed λ-expressions is classical.

33 The proof of this principle from (λC) is analogous to the proof in footnote 16, except that you use the RULE OF NECESSITATION after universally generalizing on $x_1, \ldots, x_n$ and just before existentially generalizing on the λ-expression.

34 Again, in the latest version of OT, under development in Zalta (2024), one can derive that every formula denotes a state of affairs—even formulas containing encoding subformulas. But this doesn't hold for property and relation comprehension though; not every formula with free variables $x_1, \ldots, x_n$ can be turned into a λ-expression that is guaranteed to denote.

- Properties F and G are identical if and only if F and G are necessarily encoded by the same objects, i.e.,

(17) $F = G \equiv_{df} \Box\forall x(xF \equiv xG)$

- n-ary relations F and G ($n \geq 2$) are identical just in case, for any $n-1$ objects, every way of applying F and G to those $n-1$ objects results in identical properties, i.e.,

(18) $F = G \equiv_{df} \forall y_1 \dots \forall y_{n-1}([\lambda x\, Fxy_1 \dots y_{n-1}] = [\lambda x\, Gxy_1 \dots y_{n-1}]\ \&$
$[\lambda x\, Fy_1xy_2 \dots y_{n-1}] = [\lambda x\, Gy_1xy_2 \dots y_{n-1}]\ \&\ \dots\ \&$
$[\lambda x\, Fy_1 \dots y_{n-1}x] = [\lambda x\, Gy_1 \dots y_{n-1}x])$

- States of affairs p and q are identical whenever (the property) *being an individual z such that* p is identical to (the property) *being an individual z such that* q, i.e.,

(19) $p = q \equiv_{df} [\lambda z\, p] = [\lambda z\, q]$

From these definitions, it can be shown that the reflexivity of identity holds universally, i.e., that $x = x$ is derivable from (16), that $F = F$ is derivable from each of (17) and (18), and that $p = p$ is derivable from (19). So OT asserts only the substitution of identicals as an axiom governing identity. It therefore has all the theorems about identity that are derivable in 2OL$^=$. Identity is provably symmetric, transitive, etc., and since every term of the theory is interpreted rigidly, substitution of identicals holds in any (modal) context whatsoever.

Since (λC) is an axiom of OT, the foregoing facts make it clear that (8) is also a theorem of OT, by the same reasoning used in the proofs given earlier in the paper. So as soon as one adds the hypothesis that a particular binary relation, say R, is non-symmetric, OT also implies that $R^* \neq R$. And so on for ternary relations. The multiplicity of relations is just a fact about both 2OL$^=$ and OT when these systems are extended with the claim that non-symmetric relations exist. So taking relations and relation application as primitive still yields multiple converse relations for n-ary relations ($n \geq 2$). This is a consequence one should accept if we take relations and relation application as primitive and treat them as hyperintensional entities.[35] This multiplicity isn't egregious,

35 Recall the passages in MacBride (2014), where he says, "We simply have to accept as primitive, in the sense that it cannot be further explained, the fact that one thing bears a relation to another" (2014, 2); "[…] we should just take the difference between *aRb* and *bRa* as primitive" (2014, 14);

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in any case, for as we've seen, λ-expressions give us the expressive power to distinguish among the converses of (non-symmetric) relations. So let's return to the questions about the identity of states of affairs to see how they fare with a precise theory of relations and states of affairs in hand.

6. Asserting the Identity of States

Recall that the puzzling conclusion reached in MacBride's (2022) paper turned on the question of whether the states of affairs denoted by [9] and [10] are the same or distinct. This question can now be posed without discussing the converses of relations and without invoking 3OL. Let R be *any* symmetric relation, and let a and b be two particular and distinct objects. Then consider the states of affairs Rab and Rba (or, if you prefer, $[\lambda\ Rab]$ and $[\lambda\ Rba]$). MacBride apparently has no doubt they are the same state. So let's suppose they are, i.e., that $Rab = Rba$. And let's again grant him the ordinalized readings of relational claims. What happens to the argument in which he concludes that if we understand "Fab" in terms of ordinalized, higher-order properties, then "Rab" and "Rba" don't express the same state of affairs? Answer: it has no force against the theory of states of affairs in OT. For in OT, all that is relevant to the truth of "$Rab = Rba$" is principle (19), i.e., the question of whether the properties $[\lambda z\, Rab]$ and $[\lambda z\, Rba]$ are identical, i.e., by (17), whether there might be objects that encode $[\lambda z\ Rab]$ without encoding $[\lambda z Rba]$ (or vice versa). Given these definitions, one could, should one wish to do so, simply use OT to assert, as an axiom, that when R is symmetric, $[\lambda z Rab]$ and $[\lambda z Rba]$ are identical, i.e., that no abstract object encodes $[\lambda z Rab]$ without also encoding $[\lambda z\ Rba]$, and vice versa.

Does this mean we don't understand the open formula "Fab" or the quantified claim "$\exists F(Fab)$"? Not at all. First, the semantics of OT is perfectly precise on this score. Let "$\boldsymbol{a}$" and "$\boldsymbol{b}$" be the semantic names of the objects assigned to "a" and "b." Now consider some assignment f to the variables of the language, and suppose that "$\boldsymbol{R}$" is the semantic name of the relation assigned to the variable "F" by f. Then the open formula "Fab" is true relative to f if and only if the state of affairs $\boldsymbol{Rab}$ obtains.[36] And "$\exists F(Fab)$" is true just in case

and "The difficulties that result from attempting to analyse the first degree suggest that that the operation of relational application should itself be taken as primitive" (2014, 15).

36 OT does have a formal semantics, but its primary purpose is to establish that the theory has a set-theoretic model. Given the assignments to "a," "b," and "F" mentioned in the text, the formal semantics implies that "Fab" is true relative to f if and only if the ordered pair $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$ is in the

some relation in the domain satisfies the open formula "Fab," no matter how that relation is specified.

Second, OT doesn't require a formal semantics to be intelligible, just as ZF is intelligible when we express its primitive notions and axioms within first-order logic. The axioms and theorems of OT give us an understanding of the open formula "xF" and, in turn, give us an understanding of the identity conditions for states of affairs expressed in (19). To suggest otherwise would be like suggesting that we don't understand "$x \in y$." This is a primitive of set theory; set identity is stated in terms of this primitive, in the form of the principle of extensionality. The more we work through the consequences of the axioms (i.e., the more theorems we prove in set theory), the better we understand "$x \in y$." Analogous observations hold with respect to OT. The formula "xF" is a primitive mode of predication, and the identity conditions for properties and relations are stated in terms of this primitive. The more we work through the consequences of the axioms, the better we understand this form of predication.

So if one is inclined to accept MacBride's view that the states of affairs expressed by [9] and [10] are identical, one should then be inclined to accept the following general principle:

(20) $\forall F \Box(\mathit{Symmetric}(F) \rightarrow \forall x \forall y(Fxy = Fyx))$

(20) is consistent with OT. We need not conclude that the open formula "Fab" is unintelligible or that the second-order quantifiers don't range over relations. Instead, we make use of a theory of relations and states of affairs in which relation application is primitive but identity is defined. And we address the problem by asserting a principle, not by concluding that the language is unintelligible; indeed, it seems to be the principle that MacBride is relying upon to make his case.

This generalizes to non-symmetric relations. For recall the objection to (14), which is the claim:

(14) $\forall F \Box(\mathit{Non\text{-}symmetric}(F) \rightarrow \forall x \forall y(Fxy = F^{*}yx))$

exemplification extension of the relation $\boldsymbol{R}$. And this latter holds if and only if the extension of the 0-ary relation $\boldsymbol{Rab}$ is The True. But these semantic conditions only give us a set-theoretic representation of the truth conditions; they are not a substitute for the metaphysics of relations, predication, and states of affairs.

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The problem with (14), according to MacBride, is to explain how different decompositions can give rise to the same state [-MacBride (2014), 4; quoted above]. But no such explanation is needed, since the identity of states of affairs is not a matter of decompositions and constituents. If F is non-symmetric, then the above principle implies, by definition (19), that $[\lambda z Fxy] = [\lambda z F^* yx]$, for any objects x and y. That is consistent with OT.

Why does this address the difficulty in MacBride (2014, 4)? The answer: because we're not attempting to *explain* how "distinct existences" (i.e., a non-symmetric relation F, its converse F^*, and objects x and y) can "give rise" to the same state; we're instead proposing that one adopt a principle (indeed, a principle on which MacBride relies) that asserts that they do, without appealing to "decompositions," "constituents," etc. The definitions of identity for abstract objects (16) and for properties (17) place reciprocal bounds on the existence of these entities. The theory's comprehension principle and identity conditions for abstract objects tell us that *any* (expressible) condition on properties can be used to define an abstract object. If we think of abstract objects as objects of thought or as logical objects, then the theory implies that if properties F and G are distinct, then there is a logical, abstract object of thought that encodes F and not G (and vice versa). And if F and G are identical, then no logical, abstract object of thought encodes F without encoding G. So if the properties $[\lambda z\, Fxy]$ and $[\lambda z\, F^* yx]$ are identical, then no logical, abstract object of thought encodes the one without encoding the other.[37]

By adopting (14), one can *use* OT's theory of identity for states of affairs to give a precise, theoretical answer to a philosophical question ("Under what conditions are states of affairs identical?") which, if left unanswered, would leave one open to MacBride's concerns about the intelligibility of 2OL and $2OL^=$.[38]

37 One practical consequence of this identification is this: it prevents one from telling a consistent story about a fictional object, say c, in which Fxy is true in the story but $F^* yx$ is not, for some relation F and objects x and y. For example, if you believe *cat-on-mat* is identical to *mat-under-cat*, then you can't tell a consistent story in which one is true and the other is not, or consistently describe a fictional object such that one is true while the other is not. I'm not ruling out stories where some fictional character *believes* that Rab and doesn't believe that $R^* ba$, for in that case, we're not talking about the states denoted by "Rab" and "$R^* ba$," but about the senses of these expressions. And OT represents these as *abstract* states of affairs, which requires the typed version of OT. See Zalta (1988), Zalta (2020).

38 This answer, if adopted, would put to rest another of MacBride's concerns, namely, that endorsing distinct converses for non-symmetric relations requires a commitment to a "substantive linguistic doctrine," namely, that when we switch from the active "Antony loves Cleopatra" to the passive "Cleopatra is loved by Antony," we "introduce a novel subject matter" (MacBride 2014, 5). But

Before we turn, finally, to the intuition that states of affairs like those expressed by [9] and [10] are distinct, there is one final way to formulate the concern that MacBride has raised, given his understanding of the identity of states of affairs. Consider the property $[\lambda z\, Fzy]$, i.e., being an object z such that z and y exemplify F. Now predicate that property of x to obtain the state of affairs $[\lambda z Fzy]x$, i.e., x exemplifies the property of being a z such that z and y exemplify F. Put this aside for the moment and now consider the property $[\lambda z\, Fxz]$, i.e., being an object z such that x and z exemplify F. Now predicate that property of y to obtain the state of affairs $[\lambda z\, Fxz]y$, i.e., y exemplifies the property of being a z such that x and z exemplify F. Now, we might ask:

(A) What is the relationship between the states of affairs Fxy, $[\lambda z\, Fzy]x$, and $[\lambda z\, Fxz]y$—are they all the same or are they all pairwise distinct?

If you accept MacBride's view about the identity of states of affairs, then you would answer (A) by adopting the following principles:

(21) $\forall F\Box(Fxy = [\lambda z\, Fzy]x)$
(22) $\forall F\Box([\lambda z\, Fzy]x = [\lambda z\, Fxz]y)$

From these principles, it also follows, by the transitivity of identity, that $\forall F\Box(Fxy = [\lambda z\, Fxz]y)$.

I'm not suggesting that this is the only or best answer to (A) because there may be contexts where one might wish to distinguish these states of affairs (see the next section). But the general point is clear. Some precise, axiomatized theories leave open certain questions of identity, and those questions can be answered by looking for principles rather than questioning whether the quantifiers of the theory range over the entities being axiomatized. ZFC has precise identity conditions for sets but leaves open the Continuum Hypothesis ("CH"), and yet we can still interpret the quantifiers in set theory as ranging over sets. CH can be formulated as the claim $2^{\aleph_0} = \aleph_1$, and though CH and its negation are consistent with ZFC, we don't give up the interpretation of the quantifiers of ZFC as ranging over sets just because CH is an open question;

our solution allows one to agree with MacBride that if the subject matter is defined by the state of affairs being referenced, there is no change—one can move from "Antony loves Cleopatra" to "Cleopatra is loved by Antony" without changing the subject matter, since those sentences designate, on this view, the same state of affairs.

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instead, we look for axioms that will help decide the issue. The same applies to the theory of relations.[39]

As it turns out, there is an alternative way to respond to the problems MacBride has raised. It may be of interest to some readers to consider what happens to his arguments if one instead asserts that $Fxy \neq Fyx$ when F is symmetric, or accepts that $Fxy \neq F^*yx$ when F is non-symmetric, or generally accepts that $Fxy \neq [\lambda z\, Fxz]y \neq [\lambda z\, Fzy]x$. In the final section, then, I show that, with OT's theory of states of affairs,

- one may alternatively assert these non-identities;
- one can account for the intuition that there is *one* part of the world that makes these distinct states true when they are true; and, consequently,
- one can disarm the worry about a "profusion" of states of affairs and clear the path for understanding the quantifiers of 2OL and 2OL$^=$ as quantifying over relations.

7. Distinct States, One Situation

What is driving MacBride's certainty that (a) $Fxy = Fyx$ when F is symmetric, (b) $Fxy = F^*yx$ when F is non-symmetric, and (c) $Fxy = [\lambda z\, Fxz]y = [\lambda z\, Fzy]x$ generally? The argument is most clearly stated for the case of non-symmetric relations, where he argues that if non-symmetric relations have distinct converses, then we end up with "a profusion of states of affairs." We laid out the argument in section 4, in the quote from (2014, 4), about there being one state of affairs (i.e., one cat-mat orientation) despite there being two kinds of undertakings (putting the cat on the mat and putting the mat under the cat). Since to undertake to do something is to attempt to bring about a state of affairs, one might then conclude that there are two distinct undertakings precisely because there are two distinct states of affairs to be brought about. But, as we saw earlier, MacBride and Fine both conclude that there is only one state and that to claim otherwise is counterintuitive. And we saw that the concern is that converse relations commit us to the principle that if F is non-symmetric, then the state of affairs Fxy is distinct from the state of affairs F^*yx. We have formally represented the principle that concerns them as follows:

39 I'm indebted to Daniel Kirchner, who was able to use his implementation of OT in Isabelle/HOL (2017) to confirm the consistency of separately adding (14), (20), (21), and (22) to OT.

(13) $\forall F\Box(\textit{Non-symmetric}(F) \rightarrow \forall x\forall y(Fxy \neq F^*yx))$

But notice that the cases MacBride (and Fine) discuss involve necessarily non-symmetric relations, such as *on*, *on top of*, *above*, etc. So when we instantiate (13) to a necessarily non-symmetric relation, say R, it would follow by the K axiom of modal logic that $\Box\forall x\forall y(Rxy \neq R^*yx)$. But of course, we can also infer, from the fact that (λC) is a universal, necessary truth, that $\Box\forall x\forall y(Rxy \equiv R^*yx)$.[40] So we can generalize to conclude that whenever we assert that R is a necessarily non-symmetric relation, (λC) and (13) combine to ensure that Rxy and R^*yx are necessarily equivalent but distinct states of affairs, for any values of the variables x and y.

The real problem is now laid bare: the hyperintensionality of states of affairs appears to undermine the intuition that in these cases, there is only one piece of the world (e.g., one *cat-mat* orientation) that accounts for the truth of the relational claims "Rab" and "R^*ba" when they are true. Note that this same problem arises for the other cases we're considering. I take it MacBride would similarly be concerned about the following principle regarding *symmetric* relations:

(23) $\forall F\Box(\textit{Symmetric}(F) \rightarrow \forall x\forall y(Fxy \neq Fyx))$

And the concern extends generally to principles such as the following, which would govern every binary relation:

(24) $\forall F\Box\forall x\forall y(Fxy \neq [\lambda z\, Fzy]x)$
(25) $\forall F\Box\forall x\forall y([\lambda z\, Fzy]x \neq [\lambda z\, Fxz]y)$

In each case, a "profusion" of states of affairs will arise, for it can be shown (a) that (λC) and (23) imply that for any necessarily symmetric relation R, Rxy and Ryx are necessarily equivalent but distinct;[41] and (b) that (λC), (24), and

40 This holds for any binary relation F. As an instance of (λC), we know $[\lambda xy\, Fyx]xy \equiv Fyx$. So by definition (7), $F^*xy \equiv Fyx$, which, by the commutativity of the biconditional, implies $Fyx \equiv F^*xy$. So by applying, in order, the Rule of Generalization (2×) and the Rule of Necessitation, we obtain $\Box\forall y\forall x(Fyx \equiv F^*xy)$, which is an alphabetic variant of $\Box\forall x\forall y(Fxy \equiv F^*yx)$.

41 Suppose $\Box\textit{Symmetric}(R)$. Then, by the definition of a symmetric relation, both $\Box\forall x\forall y(Rxy \rightarrow Ryx)$ and $\Box\forall x\forall y(Ryx \rightarrow Rxy)$, where the latter follows by universal quantifier commutativity and substitution from $\Box\forall y\forall x(Ryx \rightarrow Rxy)$, which is an alphabetic variant of the former. So $\Box\forall x\forall y(Rxy \equiv Ryx)$. But by (23) and the K axiom, $\Box\forall x\forall y(Rxy \neq Ryx)$. So again, we have that Rxy and Ryx are necessarily equivalent, but distinct.

(25) imply that for any relation R, the states Rxy, $[\lambda z\, Rxz]y$, and $[\lambda z\, Rzy]x$ are all pairwise necessarily equivalent but all pairwise distinct.[42]

So if one accepts (13) and (23)–(25), can we account for the intuition that there is only one piece of the world in virtue of which the necessarily-equivalent-but-distinct states of affairs are true when they are true? To answer this question, we shall not invoke "decompositions" and "constituents," for the identity for states of affairs is given by (19). But we *can* address the intuition driving MacBride, Fine, and no doubt others, by appealing to the notion of a *situation* and defining the conditions under which a state of affairs p *obtains in* a situation s (i.e., the conditions under which s *makes* p *true*). Once these notions are defined, we can identify, for any state of affairs p, a canonical situation s in which obtain all and only the states of affairs necessarily implied by p. Then, the canonical situation in which obtain the states necessarily implied by Rab will be identical to the canonical situation in which obtain the states necessarily implied by R^*ba; this will follow from the fact that Rab and R^*ba are necessarily equivalent. And similar results follow for states arising from necessarily symmetric relations and for the states Rab, $[\lambda x\, Rxb]a$, and $[\lambda x\, Rax]b$. As I develop this response, I'll use R as an arbitrary binary relation, which is necessarily non-symmetric, or symmetric, or unspecified, as the case may be.

In OT (Zalta 1993, 410), situations are defined as abstract objects that encode only properties constructed out of states of affairs, i.e., encode only properties F of the form $[\lambda z\, p]$, where p ranges over states of affairs:

(26) $Situation(x) \equiv_{df} A!x \;\&\; \forall F(xF \rightarrow \exists p(F = [\lambda z\, p]))$

42 The states Fxy, $[\lambda z\, Fzy]x$, and $[\lambda z\, Fxz]y$ are all necessarily equivalent by (λC) and the RULE OF NECESSITATION, but they are pairwise distinct by (24) and (25). Note that philosophers have argued for (24) and (25); Menzel (1993, 81–83) considers the case of:

[17] 100 is less than 1000.
[3] 100 is submillenial.

He then suggests that the proposition expressed by [17] (Lht) differs (structurally) from the proposition expressed by [3] ($[\lambda x\, Lxt]h$)—the former is a binary predication, whereas the latter is a unary or monadic predication. It is of interest to note that Menzel's system rejects η-CONVERSION—it doesn't endorse, for example, $[\lambda xy\, Fxy] = F$ (Menzel 1993, 82). Daniel Kirchner notes (personal communication) that it would be easier to model (24) and (25) in the Isabelle/HOL implementation of OT if one were to generally drop η-CONVERSION. This is an interesting avenue of research.

A situation, thus defined, is not a mere mereological sum because encoding is a mode of predication; a situation is therefore *characterized* by the state-of-affairs properties of the form $[\lambda z\ p]$ that it encodes. In addition, a state of affairs p *obtains in* a situation s ("$s \models p$") just in case s encodes *being a z such that p* (Zalta 1993, 411):

(27) $s \models p \equiv_{df} s[\lambda z\ p]$

In what follows, therefore, we sometimes extend the notion of encoding by saying that s encodes a state of affairs p, or that s *makes p true*, whenever p obtains in s. That is, when $s \models p$, we can say either s encodes $[\lambda z\ p]$, or s encodes p, or s makes p true.

Now consider some state of affairs, say Rab. Given the foregoing definitions, OT implies that there exists a situation s such that a state of affairs p obtains in s if and only if p is necessarily implied by Rab. To see this, note that the comprehension principle for abstract objects asserts that there is an abstract object that encodes exactly those properties F such that F is a property of the form $[\lambda z\ p]$ when p is some state of affairs necessarily implied by Rab:

(28) $\exists x(A!x \,\&\, \forall F(xF \equiv \exists p(\Box(Rab \rightarrow p) \,\&\, F = [\lambda z\ p])))$

Let s_1 be such an object, so that we know:

(29) $A!s_1 \,\&\, \forall F(s_1F \equiv \exists p(\Box(Rab \rightarrow p) \,\&\, F = [\lambda z\ p]))$

Since s_1 is abstract and every property it encodes is a property of the form $[\lambda z\ p]$, it follows that s_1 is a situation by definition (26). Moreover, the theory implies that s_1 is unique, i.e., that any abstract object that encodes all and only those states of affairs necessarily implied by Rab is identical to s_1. Since situations are abstract objects, they are identical whenever they encode the same properties.[43] And since situations, by (26), encode only properties F such that $\exists p(F = [\lambda z\ p])$, they obey the principle: s and s' are identical just in case the same states of affairs obtain in s and s' (Zalta 1993, 412, Theorem 2). So there can't be two distinct abstract objects that encode all and only the states of affairs necessarily implied by Rab. Since (28) has a unique witness,

43 Strictly speaking, the definition of identity (16) implies that abstract objects x and y are identical if and only if *necessarily* they encode the same properties. But since $xF \rightarrow \Box xF$ is an axiom of OT, it follows that if x and y encode the same properties, they necessarily encode the same properties, and so it is sufficient to show $\forall F(xF \equiv yF)$ to establish that $x = y$, for abstract x and y.

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we may treat s_1 as a name of this witness (introduced by definition) and treat (29) as a fact about s_1 implied by the definition.

Two modal facts about s_1 become immediately relevant:

- A state of affairs obtains in s_1 if and only if it is necessarily implied by Rab, i.e.,

(30) $\forall p(s_1 \models p \equiv \Box(Rab \rightarrow p))$.

- s_1 is *modally closed* in the following sense: for any states of affairs p and q, if p obtains in s_1 and p necessarily implies q, then q obtains in s_1, i.e.,

(31) $\forall p \forall q((s_1 \models p) \,\&\, \Box(p \rightarrow q) \rightarrow (s_1 \models q))$.

The proof of (30) is straightforward and, interestingly, relies on the object-theoretic definition for the identity for states of affairs (19).[44] Note that it immediately follows from (30) that Rab obtains in s_1, since $\Box(Rab \rightarrow Rab)$ is an instance of the modal principle $\forall p\Box(p \rightarrow p)$. The proof of (31) relies on both the definition of identity for states of affairs (19) and the fact that necessary implication is transitive, i.e., the fact that:

- $\forall p \forall q \forall r(\Box(p \rightarrow q) \,\&\, \Box(q \rightarrow r) \rightarrow \Box(p \rightarrow r))$

The proof of (31) is left to a footnote.[45]

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44 We prove the universal claim by showing that the biconditional holds for an arbitrary state of affairs, say q_1. To show the left-to-right direction, assume $s_1 \models q_1$, to show $\Box(Rab \rightarrow q_1)$. Then, by definition of *obtains in* (27), $s_1[\lambda z\, q_1]$. So by a fact about s_1, namely the second conjunct of (29), it follows that $\exists p(\Box(Rab \rightarrow p) \,\&\, [\lambda z\, q_1] = [\lambda z\, p])$. Let q_2 be such a state of affairs, so that we know $\Box(Rab \rightarrow q_2) \,\&\, [\lambda z\, q_1] = [\lambda z\, q_2]$. By the definition of identity for states of affairs (19), the second conjunct implies $q_1 = q_2$. But then, substituting identicals into the first conjunct, we obtain $\Box(Rab \rightarrow q_1)$.

For the right-to-left direction, assume $\Box(Rab \rightarrow q_1)$. By the reflexivity of identity, $[\lambda z\, q_1] = [\lambda z\, q_1]$. Hence $\Box(Rab \rightarrow q_1) \,\&\, [\lambda z\, q_1] = [\lambda z\, q_1]$. So $\exists p(\Box(Rab \rightarrow p) \,\&\, [\lambda z\, q_1] = [\lambda z\, p])$. Then by a fact about s_1, namely the second conjunct of (29), $s_1[\lambda z\, q_1]$, and by definition of *obtains in* (27), $s_1 \models q_1$.

45 We prove the doubly-universal claim by showing that it holds for arbitrary states of affairs p_1 and q_1. So assume both

(a) $s_1 \models p_1$
(b) $\Box(p_1 \rightarrow q_1)$

to show $s_1 \models q$. By definition (27), (a) implies $s_1[\lambda z\, p_1]$. From this fact and the second conjunct of (29), it follows that $\exists p(\Box(Rab \rightarrow p) \,\&\, [\lambda z\, p_1] = [\lambda z\, p])$. Suppose r_1 is an arbitrary such state

It is an immediate consequence of (30) that:

- if R is necessarily non-symmetric, then R^*ba obtains in s_1, for it is necessarily equivalent to, and so necessarily implied by, Rab;
- if R is necessarily symmetric, then Rba obtains in s_1, for it is necessarily equivalent to, and so necessarily implied by, Rab; and
- if R is any binary relation whatsoever, then $[\lambda x\, Rxb]a$ and $[\lambda x\, Rax]b$ both obtain in s_1, since these are both necessarily equivalent to, and so necessarily implied by, Rab.

Moreover, when R is necessarily non-symmetric, it follows that neither Rba nor R^*ab obtain in s_1, since neither is necessarily implied by Rab in that case.

It is interesting to observe that in each of the above scenarios, any one of the necessarily equivalent states of affairs in question can be used to define the unique situation in which they all obtain. The resulting situations become identified, since it is a theorem of modal logic that necessarily equivalent states of affairs necessarily imply the same states of affairs:

(32) $\forall p \forall q(\Box(p \equiv q) \rightarrow \forall r(\Box(p \rightarrow r) \equiv \Box(q \rightarrow r)))$

To see why this fact helps us to show that the resulting situations are all identified, consider the case of necessarily non-symmetric R and consider the situation that can be introduced in a manner similar to s_1 but with R^*ba instead of Rab:

$$\exists x(A!x \;\&\; \forall F(xF \equiv \exists p(\Box(R^*ba \rightarrow p) \;\&\; F = [\lambda z\, p])))$$

This is the (provably unique) situation that makes all and only the states of affairs necessarily implied by R^*ba true. Call this s_2. Clearly, facts analogous to (30) and (31) hold for s_2: a state of affairs p obtains in s_2 if and only if R^*ba necessarily implies p, and s_2 is modally closed.

But OT implies that $s_1 = s_2$.[46] Moreover, the reasoning in the proof applies to all the other canonical situations definable in terms of the necessarily

of affairs, so that we know $\Box(Rab \rightarrow r_1) \;\&\; [\lambda z\, p_1] = [\lambda z\, r_1]$. The second conjunct of this last result implies, by the identity of states of affairs (19), that $p_1 = r_1$. Hence $\Box(Rab \rightarrow p_1)$. But this last fact and (b) jointly imply $\Box(Rab \rightarrow q_1)$, by the transitivity of necessary implication. Hence $\Box(Rab \rightarrow q_1) \;\&\; [\lambda z\, q_1] = [\lambda z\, q_1]$, by reflexivity of identity and conjunction introduction. So $\exists p(\Box(Rab \rightarrow p) \;\&\; [\lambda z\, q_1] = [\lambda z\, p])$. But this implies, by the second conjunct of (29), that $s_1[\lambda z\, q_1]$. Hence $s_1 \models q_1$, by definition of *obtains in* (27).

46 *Proof*. To show $s_1 = s_2$, it suffices to show that they encode the same properties, for as we noted earlier in footnote 43, the object-theoretic principle $xF \rightarrow \Box xF$ implies that if s_1 and s_2 encode

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equivalent states of affairs mentioned above: these canonical situations are pairwise identical. Thus, in each example, there is a single canonical situation in which all of the states of affairs mentioned in the example obtain.

Finally, to account for the intuition that the situation in which the necessarily equivalent states obtain is *part of* the actual world, we turn to the principles (theorems and definitions) governing *part of*, *actual situations*, and *possible worlds*. Since "x is a part of y" is defined as $\forall F(xF \rightarrow yF)$, it follows that a situation s is *part of* a situation s' ($s \trianglelefteq s'$) just in case every state of affairs that obtains in s also obtains in s' (Zalta 1993, 412, Theorem 4). Moreover, an *actual situation* is a situation s such that every state of affairs that obtains in s obtains *simpliciter* (Zalta 1993, 413). And a *possible world* is a situation s that might be such that it makes true all and only the truths (Zalta 1993, 414). Formally:

$$s \trianglelefteq s' \equiv \forall p(s \models p \rightarrow s' \models p)$$

$$Actual(s) \equiv_{df} \forall p(s \models p \rightarrow p)$$

$$PossibleWorld(s) \equiv_{df} \Diamond \forall p(s \models p \equiv p)$$

OT then yields, as theorems (1993, Theorem 18 and 19):

There is a unique actual world, i.e.,

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the same properties, then necessarily they encode the same properties. To show s_1 and s_2 encode the same properties, we show, for an arbitrarily chosen property, say P, that $s_1P \equiv s_2P$. Without loss of generality, we show only $s_1P \rightarrow s_2P$, since the proof of the converse is analogous. So, assume s_1P. Then, by definition of s_1,

$$\exists p(\Box(Rab \rightarrow p) \,\&\, P = [\lambda y\, p])$$

Let q_1 be such a state of affairs, so that we know $\Box(Rab \rightarrow q_1)$ and $P = [\lambda y\, q_1]$. Now, earlier we saw that when R is necessarily non-symmetric, $\Box(Rxy \equiv R^*yx)$. Hence, $\Box(R^*ba \equiv Rab)$. So by an appropriate instance of (32), it follows that $\forall r(\Box(R^*ba \rightarrow r) \equiv \Box(Rab \rightarrow r))$. Instantiating this last result to q_1, it follows that $\Box(R^*ba \rightarrow q_1) \equiv \Box(Rab \rightarrow q_1)$. But we already know $\Box(Rab \rightarrow q_1)$. Hence, $\Box(R^*ba \rightarrow q_1)$. So we have established:

$$\Box(R^*ba \rightarrow q_1) \,\&\, P = [\lambda y\, q_1]$$

By existential generalization:

$$\exists p(\Box(R^*ba \rightarrow p) \,\&\, P = [\lambda y\, p])$$

But then, by definition of s_2, it follows that s_2P.

$$\exists!s(PossibleWorld(s)\ \&\ Actual(s)) \qquad (\text{“}w_\alpha\text{”})$$

Every actual situation is a *part of* the actual world, i.e.,
$\forall s(Actual(s) \rightarrow s \trianglelefteq w_\alpha)$

The proof of the first theorem rests on the fact that there is a unique situation that encodes all and only the states of affairs that obtain, i.e., there is a unique situation s such that all and only the states that obtain in s are states that obtain *simpliciter*.[47]

So the canonical situations that exist in each of the examples validate the following claims:

- When R is necessarily non-symmetric and Rab obtains, there is a unique situation that (a) encodes all and only the states of affairs necessarily implied by Rab, (b) is actual, (c) is a part of the actual world, and (d) makes both Rab and R^*ba true.
- When R is necessarily symmetric and Rab obtains, there is a unique situation that (a) encodes all and only the states of affairs necessarily implied by Rab, (b) is actual, (c) is a part of the actual world, and (d) makes both Rab and Rba true.
- When R is any binary relation and Rab obtains, there is a unique situation that (a) encodes all and only the states of affairs necessarily implied by Rab, (b) is actual, (c) is a part of the actual world, and (d) makes Rab, $[\lambda x\, Rxb]a$, and $[\lambda x\, Rax]b$ true.

This addresses the intuition that served as the obstacle to treating states of affairs as hyperintensional entities. It lays to rest the claim that we don't understand the open formula “Fab” and the claim that we can't interpret the quantifier in “$\exists F(Fab)$” as ranging over relations.

The foregoing analysis therefore preserves the conclusion that Russell developed concerning non-symmetric relations when he said (1903, sec. 219) regarding the terms *greater* and *less*:

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47 The proof goes by way of an instance of comprehension that asserts:

$$\exists x(A!x\ \&\ \forall F(xF \equiv \exists p(p\ \&\ F = [\lambda y\, p])))$$

One can then prove that any such object, call it a, is a possible world, is actual (i.e., every state of affairs that *obtains in* a obtains *simpliciter*), and that any other situation that is a possible world and actual is identical to a. So one can then legitimately introduce the name w_α in terms of the description: *the* actual world.

> These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and are certainly relations. Hence if we are to hold that "*a* is greater than *b*" and "*b* is less than *a*" are the same proposition, we shall have to maintain that both greater and less enter into each of these propositions, which seems obviously false.

One might reframe Russell's point by noting that if non-synonymous relational expressions signify or denote different relations, then the simple statements we can make using those expressions signify different states of affairs. That principle has been preserved, without sacrificing any contrary intuitions.

8. Conclusion

I think relations and predication are so fundamental that they cannot be analyzed in more basic terms. They can only be axiomatized, and the most elegant formalism we have for doing so is the language of 2OL. The suggestion that the quantifiers of 2OL *can't* range over relations doesn't get any purchase against OT. The latter is a friendly extension of 2OL and provides 2OL with the additional expressive power needed to assert a precise theory of relations and states of affairs that includes plausible existence and identity conditions for these entities. OT therefore offers a natural formalism for intelligibly quantifying over relations and states of affairs and thus provides a deeper understanding of the open and quantified formulas of 2OL. So the suggestion that the quantifiers of 2OL *can't* be interpreted as ranging over relations fails to engage with at least one theory that shows that they can and, without any heroic measures, do.*

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