

Vol 75 : 2 June 2021

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doi:10.48106/dial.v75.i2.01

Philipp Berghofer. 2021. "Intuitional Learning." *Dialectica* 75(2): 137–169. doi:10.48106/dial.v75.i2.01.



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Intuitional Learning

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Recently, perceptual learning has become a focus of philosophical investigations. This is because the phenomenon of perceptual learning sheds light on the nature of perception. It tells us that the physical objects that are perceived are not the sole causes of perceptual states, it reveals that previous experiences shape the way we perceive, and it brings into focus an experience's phenomenal character. Hence, there is no doubt that perceptual learning has crucial implications for philosophy of mind. Only very recently, however, have researchers begun to investigate whether we can draw genuinely epistemological lessons from perceptual learning. In the first section, we shall see that such epistemological lessons do indeed exist. The rest of the paper is devoted to showing that there are analogous cases of intuitional learning. By discussing simple examples, it is argued that such cases of intuitional learning suggest that intuitions are sui generis mental states, namely experiences that have a distinctive phenomenal character, and that it is this distinctive phenomenal character that makes intuitions a source of immediate justification.

1 Epistemological Lessons from Perceptual Learning

1.1 The Phenomenon of Perceptual Learning

Let us begin with an example based on true events.

TWINS. At a party, Tanya meets Christopher and Alexander. Christopher and Alexander are identical twins, and Tanya is not able to visually distinguish between them. To her, they both look exactly the same. Tanya then is in a relationship with Christopher for about five years. Now she is able to visually distinguish between them. They look different to her.

Such examples of perceptual learning have become increasingly popular in current philosophy of mind. According to psychologist Eleanor Gibson, per-

ceptual learning, broadly speaking, "refers to an increase in the ability to extract information from the environment, as a result of experience and practice with stimulation coming from it" (1969, 3).¹ Susanna Siegel discusses the hypothetical example in which one has never seen a pine tree before and gets hired to cut down pine trees (cf. 2010, 100). After several weeks, one is able to identify pine trees on sight and distinguish them visually from other trees. As Siegel rightly points out, one's experiences of pine trees before and one's experiences of pine trees after perceptual learning has taken place differ phenomenologically (cf. 2010, 101). By an experience's phenomenological or phenomenal character, I understand "what it is like subjectively to undergo the experience" (Tye 2021, sec.1). Kevin Connolly discusses the example of an expert birdwatcher who is looking at a wren. Connolly is in agreement with Siegel when he states that "the perception of an expert birdwatcher is phenomenally different from the perception of a layperson, even when viewed under the exact same background conditions" (2014, 1408).² Perceptual learning is a philosophically significant phenomenon that has obvious implications for philosophy of mind as it tells us something about the nature of perception. For instance, "the fact that perceptual learning occurs means that the causes of perceptual states are not just the objects in our immediate environment, as it seems at first glance. Rather, given the reality of perceptual learning, there is a long causal history to our perceptions that involves prior perception" (Connolly 2017, sec.3). When the expert birdwatcher and an ordinary person are looking at a wren from the same distance and angle, they are acquainted with the same object, but their respective experiences differ significantly. Perceptual experiences do not only present objects, they also shape future experiences. As Goldstone and Byrge put it: "Perception can be learned. Experience shapes the way people see and hear" (2015, 812).

I should make explicit that, in agreement with all these authors, I assume that perceptual learning involves *genuinely perceptual* changes. While this is the dominant position, there are scattered examples in the literature in which putative cases of perceptual learning are approached as changes in judgments/beliefs instead of changes in the experience's phenomenology.

¹ For more details on the characteristics of perceptual learning, cf. Connolly (2017).

² Of course, Siegel and Connolly differ in the lessons they draw from this phenomenological difference. While Siegel argues that cases of perceptual learning show that even high-level properties such as being a pine tree can be perceptual contents, Connolly attempts to explain the phenomenological difference in terms of a subject's "attentional pattern onto other low-level properties" (2014, 1408; cf. also 2019).

Jack Lyons, for instance, argues that when a herpetologist and a novice look at a copperhead, although "it looks like a copperhead" to the herpetologist and "only like a snake to" the novice, both, the herpetologist and the novice, "have identical visual experiences" (2009, 104).³ In a similar fashion, McDowell (2008, 3) and Smith (2002, 96–97) have denied or questioned perceptual changes in particular cases of perceptual learning. Furthermore, an anonymous referee of this journal has emphasized that psychologists often talk in terms of "changes in processing and/or behavior" instead of in terms of phenomenal changes, and that this is a terminology also adopted by some philosophers, as exemplified by Stokes (2021). This is because changes in behavior and processing can be measured. Phenomenal changes, on the other hand, cannot be quantified from the third-person perspective. Of course, researchers who avoid talking about phenomenal changes do not necessarily deny them, but these considerations put some pressure on my presupposition that perceptual learning is genuinely perceptual.

The most extensive and convincing defense of the claim that perceptual learning is genuinely perceptual is offered in Connolly (2019, chap. 2). Here Connolly elucidates "converging evidence that comes from different levels of analysis: from philosophical introspection, neuroscience, and psychology" (2019, 46). Concerning philosophical introspection, Connolly invokes the "multiplicity of philosophers from different times and places who independently argue, based on introspection, that" perceptual learning involves perceptual changes. Regarding neuroscience. Connolly discusses the "neuroscientific evidence that perceptual learning modifies the primary sensory cortices" (2019, 48), arguing that this is why most scientists do indeed consider perceptual learning a genuinely perceptual process. I take Connolly to have successfully shown that perceptual learning should be considered a process that involves perceptual changes. Importantly, the cases that I discuss in section 1.2 and on which my epistemological considerations are based in section 1.3 are cases in which it should be uncontroversial that the experiences occurring before and after perceptual learning differ phenomenologically.

While it is beyond doubt that perceptual learning is a philosophically interesting phenomenon with crucial implications for philosophy of mind, it is only very recently that researchers have begun to investigate whether we

³ Here I only want to stress that I take a statement such as "It looks like A to person P1 and like B [whereby $A \neq B$] to person P2, but P1 and P2 have identical visual experiences (in the sense that P1 and P2 have phenomenologically indistinguishable experiences)" to be highly implausible, even contradictory. For a criticism of Lyons's portrayal, cf. Vaassen (2016, 139).

can draw genuinely *epistemological* lessons from this (cf. Brogaard and Gatzia 2018; Chudnoff 2018; Siegel 2017; and Vaassen 2016). Does perceptual learning tell us something about experiential justification?⁴ The main thesis of this section is that perceptual learning has epistemological implications and does tell us something about experiential justification. The main thesis of this paper is that a phenomenon analogous to perceptual learning also takes place with respect to rational intuitions. But first things first.

In the present section, I am particularly concerned with the following phenomenon:

THESIS PERCEPTUAL LEARNING (TPL). Due to experience, practice, or gaining new information, the phenomenal character of my perceptual experiences of object O can change such that new characteristics C_i of O can be perceived in a way that my experiences with the "new" phenomenal character can justify me immediately in believing that O has C_i .

I may supplement TPL by specifying the following scenario:

SPL. Confronted with a physical object O, person S cannot see that O has the feature F. Due to experience, practice, or gaining new information, S can manage to see that O has F. Once S sees F, S is immediately justified in believing that O has F.

I say that an experience E provides immediate justification for believing proposition p if having E is sufficient for justifiably believing that p, which means that the belief that p is not in need of epistemic support from anything other than the underlying experience E.⁵

TPL and SPL refer to cases in which one is perceptually aware of an object but only after some time of experience and practice becomes perceptually aware of certain features of that object. Here I am interested in examples where it is uncontroversial that a *phenomenal* change—initially one's experiences did not have a presentive character concerning certain features of the object, but once the process of perceptual learning is finished, they do—is

⁴ By experiential justification I understand justification provided by experiences.

⁵ Of course, such immediate experiential justification is always prima facie justification, i.e., justification in the absence of defeaters. Also, there can be epistemic overdetermination. It is not ruled out that a basic (perceptual) belief gets *additional* justification by other experiences or other beliefs.

accompanied by an *epistemological* change—one becomes justified in believing that the object has these features. When discussing such examples in the next subsection, I will argue for a close connection between phenomenology and epistemology. TPL and SPL are meant to shed different perspectives on the same phenomenon. TPL emphasizes the phenomenal change, SPL the epistemological change.

It is controversial whether the examples of perceptual learning we have discussed so far (twin, pine tree, and wren recognition) exemplify TPL. While Siegel argues that cases of perceptual learning show that even high-level properties, such as being a pine tree or being a wren, can be perceptual contents, i.e., represented by experience, she is hesitant to draw the conclusion that in such cases one is immediately justified in believing that the tree is a pine tree (cf. 2017). Chudnoff (2018) argues that, for experiential justification, it is not enough that an experience represents a content. What is needed is that the experience has a presentational phenomenology with respect to this content.⁶ In what follows, however, I will present three simple examples showing that TPL is highly plausible.⁷

1.2 Exemplifying TPL

Let us suppose you are looking at a piece of paper with two lines that slightly differ in length. At time t_1 , you are unable to spot the difference in length. It visually seems to you that both lines have the same length. Yet, after some practice, the phenomenal character of your experience has changed in a way that allows you to spot the difference at time t_2 . Now, it visually seems to you that the lines differ in length. Plausibly, at t_2 , you are immediately justified in

⁶ For Chudnoff, "what it is for an experience of yours to have presentational phenomenology with respect to p is for it to both make it seem to you that p and make it seem to you as if this experience makes you aware of a truth-maker for p" (2013, 37). It is to be noted that this is Chudnoff's characterization of the distinctive justification-conferring phenomenology of both perceptual experiences as well as rational intuitions. While in the case of perceptual experiences, the conception of a seeming awareness of a truth-maker seems to lead to an overintellectualization of perceptual experiences that does not adequately capture what it is like to undergo a perceptual experience, Chudnoff's characterization is much more plausible to me with respect to rational intuitions.

⁷ I would like to point out that it is not entirely clear whether the following examples qualify as cases of perceptual learning as introduced above since it is not clear whether the perceptual changes really are long-term changes. However, I am confident that the examples are close enough to justify my terminology.

believing that the lines differ in length. You are, simply because you can see it. You seem⁸ to be visually aware of two lines that differ in length.

Figure 1: Vernier Acuity9

Vernier acuity can be defined as "a measure of one's ability to detect failures of alignment between line segments" (Chudnoff 2018, 8). Vernier acuity is an experimentally well-studied phenomenon. McKee & Westheimer confronted test subjects with pictures like the one above, with the result that "every [test] subject showed some improvement in vernier acuity with practice" (1978, 259). More precisely, "the overall decline in threshold after 2,000–2,500 responses is about 40%" (1978, 259).

Say person S is looking at the picture and is unable to spot any failure of alignment. S seems to see one straight line. After some practice, S looks again at the picture and sees that there is a failure of alignment. S is now immediately justified in believing that there is a failure of alignment simply because she can see it. S seems to be visually aware of a failure of alignment.

⁸ I say "seem" to be visually aware to indicate that perceptual experiences are not factive mental states but can turn out to be illusions or hallucinations. For more details concerning the presentive, justification-conferring phenomenal character of perceptual experiences, cf. Berghofer (2020b).

⁹ This picture stems from Wikimedia Commons, licensed under: https://creativecommons.org/lice nses/by-sa/3.0/deed.en. Vernier acuity and its philosophical implications concerning perceptual learning and experiential justification are discussed in Chudnoff (2018).



Figure 2: Ambiguous Image¹⁰

Suppose at time t_1 , person S looks at the duck-rabbit image but is only able to see that this image shows a rabbit. S is unable to detect the duck. At t_1 , S is immediately justified in believing that the image shows a rabbit simply because she can see it. S may be *inferentially* justified in believing that this image also shows a duck (perhaps a trustworthy person has told her that this is an ambiguous image that shows a rabbit and a duck), but at t_1 , S is not immediately justified in believing that the picture shows a duck. At least she is not experientially justified in the sense that her experience provides her with immediate justification for this belief. At t_2 , you tell S that the rabbit's ears are the duck's beak. This information helps S to see the duck. Now S is immediately justified in believing that this picture also shows a duck simply because she can see it.

1.3 Epistemological Lessons

These three examples strongly speak in favor of TPL. If you believe that experiential justification is possible, these examples should convince you that TPL is true. More precisely, I take it that these examples support the following three theses:

P1. Perceptual learning is fundamentally linked to a change in the experience's phenomenal character.

¹⁰ This picture stems from Wikimedia Commons, licensed under: https://creativecommons.org/licenses/by-sa/3.0/deed.en. No changes were made.

P2. Perceptual learning can have an influence on the experience's justificatory force, such that before the learning process, your experience did not immediately justify you in believing that p, but after the learning process, it does.

P3. Perceptual experiences gain their justificatory force precisely by virtue of their distinctive justification-conferring phenomenal character.

P1 and P2 should be uncontroversial. All this suggests P3.¹¹ P3 is a strong claim that is genuinely internalist and opposed to currently popular positions such as reliabilism. Here, I cannot provide a detailed defense of P3. However, it is important to note that P₃ offers the most natural explanation of the link between perceptual learning and experiential justification. All three examples had the following structure: At t_1 , one's perceptual experience does not have a presentive¹² phenomenology with respect to p, and at t_1 , one is not experientially justified in believing p. At t_2 , due to perceptual learning, the perceptual character has changed such that one's perceptual experience now has a presentive phenomenology with respect to p, and at t_2 , one is experientially justified in believing *p*. Thus, in each of these cases, a shift in the experience's phenomenal character from not having a presentive character with respect to *p* to having a presentive character with respect to *p* is accompanied by a shift in the experience's justificatory force from not justifying p to justifying p. P₃ is the most natural explanation because it states the obvious: Perceptual justification is linked to the experience's phenomenal character in the sense that certain experiences gain their justificatory force simply by virtue of their distinctive phenomenal character. As Chudnoff has recently put it, "the phenomenology grounds the epistemology" (2016, 117). Perceptual experiences justify by virtue of their presentive phenomenology, and they justify precisely those propositions with respect to which they have a presentive phenomenology.

¹¹ For recent works that elaborate on and defend P₃, cf. Bengson (2015b), Chudnoff (2013), Church (2013), Koksvik (2011), and Berghofer (2023). The same basic idea can be found in Husserl (2008, 343; 1982, 36–37).

¹² For a detailed account of the distinctive, justification-conferring phenomenology of perceptual experiences, cf. Berghofer (2020b).

The main thesis of this paper is that parallel claims are true with respect to rational intuitions. In section 5, I will argue for this thesis by discussing concrete examples, analogously to how I proceeded in this section.

2 What Are Intuitions?

Rational or a priori intuitions have always been of central philosophical interest. From Plato to Augustine, to Descartes, to Kant, to Husserl, for all these thinkers, the nature and epistemic role of intuitions were a central theme of their philosophical investigations. The significance of intuitions may be particularly obvious in current analytic philosophy. This is because, regarding orthodox philosophical methodology, there is considerable agreement that "intuitions are presented as our evidence in philosophy" (Williamson 2007, 214) and that "analytic philosophy without intuitions just wouldn't be analytic philosophy" (Weinberg 2007, 318; cf. also Pust 2000, xiii). The reliance on intuition is considered as one of the defining features of philosophy: "One thing that distinguishes philosophical methodology from the methodology of the sciences is its extensive and avowed reliance on intuition" (Goldman 2007, 1).¹³ Despite this consensus on the significance of intuitions, there is no agreement concerning even the most fundamental questions, such as: What are intuitions? Can intuitions be a source of immediate justification? Can intuitions be a source of justification at all? Can intuitions tell us something about reality?

Concerning the nature of intuition, the question of what intuitions are, the two opposing views are sui generism and reductivism. According to sui generism, intuitions are sui generis mental states that cannot be reduced to other, more fundamental types of mental states. Intuitions are irreducible. Reductivism is usually introduced as a form of doxasticism, according to which intuitions can be reduced to doxastic states such as judgments (cf. Williamson 2007), beliefs or opinions (cf. Lewis 1983), or inclinations to believe (cf. Inwagen 1997; and Sosa 2007, 54). Of course, the answer to the question of what intuitions are has significant implications for the questions of whether and how intuitions can be a source of justification. If intuitions are merely beliefs, they cannot be a source of immediate justification. However, if intuitions are sui generis mental states and perhaps even a type of experience

¹³ To be sure, there are scholars such as Cappelen (2012), Deutsch (2010), and Ichikawa (2014) who actively and explicitly dispute the claim that intuitions are essential for philosophical inquiry. However, these authors are clearly in the minority.

in the sense of having a distinctive phenomenal character, intuitions, just as perceptual experiences, may be a source of immediate justification. In what follows, we shall discuss cases of intuitional learning that are parallel to the cases of perceptual learning we have discussed above. By discussing such cases of intuitional learning, I will show that intuitions are sui generis mental states that have a distinctive phenomenal character. My findings will indicate that *intuitions are a source of immediate justification by virtue of their distinctive phenomenal character*.

It is important to note that the term "intuition" is usually used in a very broad sense. People are said to rely on their intuitions when they deem it irrational to propose p and not-p, when they are convinced that 2 + 2 = 4, when they claim that Gettier cases are not cases of knowledge, when they understand that it is not a good idea to build a house on sand, when they condemn their neighbor for torturing his cat just for fun, when they feel that it will rain tomorrow, or when they decide that their pants match their shirt. Sometimes the term "intuition" is used in the sense of a quasi-perceptual experience that reveals an a priori truth; often it is used in the sense of a gut feeling or strong conviction. By the term "intuition," people may refer to

- (i) gut feelings
- (ii) strong convictions
- (iii) intuitional experiences that have a distinctive phenomenal character.

Note that these different usages of "intuition" can occur in the very same field, for instance, mathematics. A mathematician's "intuition" that the continuum hypothesis is true may simply be a gut feeling. A novice's "intuition" that $(-1) \cdot (-1) = 1$ may be nothing but a strong conviction. As I use the term, an intuition is an intuitional experience that has a distinctive phenomenal character. Of course, many current epistemologists deny that there are intuitions in this sense. While the view that intuitions are some kind of intellectual perception has been popular among many of the most influential historical philosophers, such as Plato, Descartes, and Husserl, this view, under the pressure of moderate and radical empiricism, has nearly vanished in the second half of the twentieth century. However, the end of the twentieth century has seen a revival of rationalism that has been famously dubbed a "rationalist renaissance" (Bealer 2002, PAGE-NUMBER). It is one of the cornerstones of this revival that intuitions are viewed as a source of justification. Of course, not every rationalist and not every philosopher who holds that intuitions are

justifiers subscribes to the view that intuitions are some kind of intellectual perception. According to Bengson, the view that "intuition is a form of intellectual perception, affording epistemic access to abstract truths without mediation by conceptual understanding, remains relatively unpopular—and unexplored—by comparison" (2015a). Having clarified that I mean by "intuitions" intuitional *experiences* that have a distinctive phenomenal character, I will now shed light on this distinctive phenomenal character. The focus is on *mathematical* intuitions.

3 Mathematical Intuitions—Towards a Phenomenological Clarification

By mathematical intuitions, I understand intuitions with contents such as "1 + 1 = 2," "3 < 4," "2 is an even number," or "2 is the only even prime number." In the literature, there is no consensus on whether such mathematical intuitions are merely beliefs, inclinations to believe, or sui generis mental states, namely experiences with a distinctive phenomenal character (cf. Pust 2017). I am committed to the latter view. Let us call it the experience-view. In this section, I will not defend the experience-view but presuppose it, aiming at clarifying the phenomenal character of these experiences.¹⁴ In the following sections, I will motivate the experience-view via the phenomenon of intuitional learning.

In current debates, perhaps the most popular version of the experience-view is what we may call the seeming-view. In the twenty-first century, the seeming-view has been made popular by the works of George Bealer and Michael Huemer. According to the seeming-view, a priori intuitions are intellectual seemings. For Huemer, it is of crucial significance that seemings are neither beliefs nor inclinations to beliefs but irreducible mental states, namely, some kind of experience (cf. Tucker 2013a). Huemer argues that every seeming is a source of prima facie justification: "If it seems to *S* as if *P*, then *S* thereby has at least prima facie justification for believing that *P*" (2001, 99). In this picture, a priori intuitions are a subclass of seemings. Mathematical intuitions, in turn, are a subclass of a priori seemings.

Although Huemer's approach enjoys much popularity, objections have been put forward that the notion of "seeming" is too broad and that declaring every seeming a source of prima facie justification is too liberal since this

¹⁴ For a defense of the experience-view, cf. Chudnoff (2013, 2014).

opens the door to various counterexamples (cf., e.g., Markie 2005). My main concern is that the seeming terminology is not an adequate *phenomenological* description of what it means to undergo a mathematical intuition. When I have the intuition that 1 + 1 = 2, it does not simply seem to me that this is the case. This intuition has a more distinctive phenomenal character that seems to make me aware of *why it must be so*.

In the works of Elijah Chudnoff and in BonJour (2005), we find similar claims, demanding a phenomenological characterization of a priori intuitions that goes beyond characterizing them as seemings. For Chudnoff, justification-conferring experiences gain their justificatory force by what he calls their "presentational phenomenology." With respect to intuitions, this means: "If your intuition experience representing that p justifies you in believing that p, then it does so because it has presentational phenomenology with respect to p" (Chudnoff 2013, 94). For Chudnoff, it is essential that presentational phenomenology goes beyond simply making it seem to you that p. An experience has presentational phenomenology only if it also "make[s] it seem to you as if this experience makes you aware of a truth-maker for p" (2013, 37).

Recently, Laurence BonJour has provided a similar characterization:

[A priori insights] are not supposed to be merely brute convictions of truth, on a par with the hunches and fears that may simply strike someone in a psychologically compelling way. On the contrary, a priori insights at least purport to reveal not just that the claim is or must be true but also, at some level, *why this is and indeed must be so*. (BonJour 2005, 179, my emphasis)

The basic idea is that a priori intuitions do not merely make it seem that *p*, or push you towards believing that *p*, but reveal why *p* must be true.¹⁵ What all versions of the experience-view have in common is the analogy to perceptual experience: A priori intuitions are a source of immediate prima facie justification, exhibiting a distinctive phenomenal character. In current debates, proponents of the experience-view tend to characterize the phenomenal characters of perceptual experiences and a priori intuitions not only analogously but identically (cf., e.g., Chudnoff 2013; Church 2013; Huemer 2001; and Koksvik 2011). I believe that this is a mistake. Although sharing many important epistemic features (such as being a source of immediate

¹⁵ In this context, cf. also Husserl (2002, 410; 2008, 120).

prima facie justification), perceptual experiences and a priori intuitions differ significantly in how they present their contents.

When I look at my desk, I am visually aware of a black book lying on the desk. This perceptual experience has a presentive phenomenal character concerning the proposition "There is a black book on the desk." However, I do not seem to see *why* there is a book or any reason why it could not be different. When I intuit that 2 + 3 = 5, I am not simply aware of this fact: I can see why it must be so and could not be different. The metaphysical difference that one is a contingent fact while the other is a necessary truth is reflected phenomenologically in how these contents are presented by the respective experiences.

For mathematical intuitions, I postulate the following phenomenal character:

PHENOMENAL CHARACTER OF MATHEMATICAL INTUITIONS. If S has the intuition I with respect to p, I not only presents p as true but seems to reveal why p must be true.¹⁶

Accordingly, we say that a subject's mental state is a mathematical intuition in the sense that I use this term if and only if this mental state is (i) intentionally directed at a mathematical proposition and (ii) seems to reveal why this mathematical proposition must obtain.

So far, I have only stated how I conceive of intuitions and how I understand their phenomenal character. In what follows, by discussing what I call the phenomenon of intuitional learning, I will motivate the following claims: Intuitions are (i) sui generis mental states that (ii) have a distinctive phenomenal character, and they (iii) provide immediate prima facie justification for those propositions with respect to which they have such a distinctive phenomenal character.

4 Intuitional Learning

I use the term "intuitional learning" such that it precisely parallels what I have claimed concerning perceptual learning. In analogy to the thesis TPL from section 1, I introduce the following thesis concerning intuitional learning:

¹⁶ I say that an intuition "seems" to reveal why its content must be true in order to indicate that here we are talking about the experience's phenomenal character. There is no systematic reason to think that intuitions are infallible.

TIL. Due to experience, practice, or gaining new information, my contemplating an a priori truth T can change phenomenologically such that T can be intuited in a way that my intuition can justify me immediately in believing that T obtains.

I may supplement TIL by specifying the following scenario:

SIL. Confronted with an a priori truth T, person S cannot intuit that T obtains. Due to experience, practice, or gaining new information, S can manage to intuit that T obtains. Once S intuits T, S is immediately justified in believing that T obtains.

I would like to point out one crucial difference between perceptual learning and intuitional learning. Let us simplify matters and say that at time t, perceptual/intuitional learning takes place. In the case of perceptual learning, I have perceptual experiences of an object O before t and after t, and all that changes is the phenomenal character of my experiences. The type of mental state that is directed at O remains the same-perceptual experiences. In the case of intuitional learning, the mental states directed at the a priori truth T are not the same before and after t. Only after t, i.e., after intuitional learning has taken place, I am able to intuit T. Remember that I use the term "intuition" in the sense of an experience having a distinctive phenomenal character. Before t, I am only contemplating T but fail to intuit it. When intuitional learning takes place, the intuition is an *emergent* mental state resulting from contemplation, practice, etc.¹⁷ In the following section, I will discuss cases exemplifying TIL. First, let me briefly clarify some crucial terms that play a role in this and the following sections. We need to be aware of the distinctions between contemplating a theorem, believing a theorem, understanding a theorem, proving a theorem, and *intuiting* a theorem.

By contemplating a theorem, I basically mean thinking about it, being intentionally directed at it. When you read in your textbook that 2 is the only even prime number, and you wonder whether this statement is true and how it could be proved, you are contemplating it. By believing a theorem, I simply mean taking it to be true. There are several reasons why you might believe a theorem. You might believe it because your textbook or teacher says it is true, you might believe it because you have proved it, or you might believe

¹⁷ Note that this does not imply that I can never intuit a truth T at once without contemplation. But in cases of intuitional *learning*, intuitions are emergent states.

it because you can intuit it. By understanding a theorem, I mean grasping the content of a theorem, which implies being familiar with all the terms that are involved. When contemplating the theorem that 2 is the only even prime number, you need to understand the terms "2" and "prime number" to understand the theorem. Understanding a theorem does not imply believing a theorem. You may understand Goldbach's conjecture without believing it to be true.

What it means to prove a theorem is much more controversial. In its most rigorous sense, proving a theorem means to show that it follows from the most basic axioms. The most basic axioms of current mathematics are the set-theoretic axioms of ZFC. Only (a few) trained mathematicians would be capable of showing how a statement as simple as 1 + 1 = 2 follows from ZFC. In its perhaps most liberal sense, proving a theorem could be understood as providing the means, e.g., an argument or a diagram, such that a person contemplating this argument/diagram can be helped to see why the respective statement must be true. What this means will become clearer shortly when we discuss picture proofs.

Determining the relationship between intuiting and proving is difficult since there are many different forms of proof. Concerning rigorous symbolic proofs, it can be argued that the relationship is such that a proof P can justify a subject S in believing a mathematical theorem only if S intuitively grasps every step of P (cf. Berghofer 2020a). The relationship between picture proofs and intuitions will be determined below.

Importantly, there is a distinction between intuiting a theorem and understanding why a (type of) proof works. Consider a proof by exhaustion with a large but finite number n of cases. When a theorem can be split into n cases and for each case it can be shown that the theorem holds, this means that the theorem is true.¹⁸ For instance, the infamous four color theorem can be proved by splitting it into 1936 cases, showing that it holds in each of them. However, even if you had personally proved the statement for each of the 1936 cases, after having finished this procedure, you still could not intuit the theorem. You may have proved the theorem, and due to your proof, you may be (inferentially) justified in believing the theorem is not presented to you as true in a way that you can see why it must be true. In some sense,

¹⁸ In fact, I claim that you can intuit that proof by exhaustion is a valid form of proof. You can see that it could not be that a theorem that holds in all possible cases is not true.

you know why it must be true—because you know now that it holds in all possible cases—but this does not mean that the theorem is presented to you in an intuitive manner. There is a clear phenomenological difference between understanding that a proof works for a theorem and intuiting the theorem. However, in the case of very simple proofs, proofs in which every step can be grasped at once, proving and intuiting may coincide. Now we shall motivate all this by discussing concrete cases of what I call intuitional learning.

5 Cases of Intuitional Learning

As pointed out in section 2, the term "intuition" is used rather ambiguously by ordinary people as well as by philosophers. In the literature, examples such as 2 + 2 = 4 are used as prime examples for intuitions. I believe that the use of such examples is problematic since, especially in cases that are so obviously true and well-known as the proposition that 2 + 2 = 4, it is very difficult not to confuse different mental states such as strong convictions and genuine intuitions.¹⁹ There are many reasons for us to believe that 2 + 2 = 4. We have learned it, we have calculated it, we have used it in more complex calculations that confirmed it, and we have intuited it many times. Thus, when one says that she has the intuition that 2 + 2 = 4, it remains unclear whether she is talking about her strong conviction or about her experience that has a distinctive phenomenal character concerning this proposition. Furthermore, since this proposition can be intuited so easily and has been intuited by the same person so many times, one might doubt whether there really is a difference between simply knowing this truth and intuiting it. In section 6, I will illustrate the difference between strong convictions and intuitions by using the example of negative multiplication. In this section, I want to discuss examples in which we can plausibly tell the following story: Initially, when confronted with the a priori truth T, one fails to intuit this proposition. One may know that T obtains as one knows that the Pythagorean theorem obtains without being able to intuit the Pythagorean theorem, i.e., without being able to "see" why the Pythagorean theorem must be true. Then, after a process of contemplation and/or gaining new information, one can intuit T. There are many different ways to know an a priori truth. One may rely on a trustworthy authority such as a textbook or a teacher (justification by testimony), one may have learned

¹⁹ I suspect that opponents of intuitive justification who claim that intuitions cannot be justifiers as they can be reduced to mere beliefs (or inclinations to believe) are guilty of such confusion. For a critical discussion of this "absent intuition challenge," cf. Chudnoff (2013, 52–57).

it by heart, one may have proved it rigorously, one may remember to have proved it rigorously, or one may intuit it. The following examples are intended to highlight these differences and to illuminate what is meant by *intuiting* an a priori truth.

5.1 Prime Number

Consider the true proposition that 2 is the only even prime number. Let us assume that when first confronted with this a priori truth, you could not "see" that it obtains and thus refused to accept it. Now, at t, you are reminded that every even number can be divided by 2 without leaving a remainder. This information helps you to see, i.e., to intuit that 2 is the only even prime number. Before t, you understood the statement that 2 is the only even prime number, but you could not intuit it. At t, you are reminded that every even number can be divided by 2. Now you can intuit the statement that 2 is the only even prime number. You can see why it must be true. Of course, there are different ways to come to know that 2 is the only even prime number. Your knowledge may rest on a trustworthy authority or a rigorous proof. Such knowledge, however, is inferential knowledge. Only by intuiting the theorem are you immediately justified in believing it. Such an intuition may occur immediately when confronted with the theorem or may be the result of intuitional learning; in each case, your intuition is a source of immediate iustification.

The thesis of intuitional learning states that there are possible cases in which an a priori truth T at first is not intuitive to a subject S, although S understands T perfectly well. After some time of contemplation, T becomes intuitive to S. The moment t is the moment when the light goes on, when intuitional learning takes place. Intuitional learning cannot be accounted for in terms of understanding the theorem. Before t, S understood T perfectly well. In our example, you understood the theorem that 2 is the only even prime number perfectly well, even before you could see why it must be true.

Furthermore, intuitional learning cannot be accounted for in terms of believing or being strongly convinced of the theorem. Before t, S may have been strongly convinced of T. Intuitional learning is to be accounted for in terms of a phenomenal change concerning how T is presented to S when contemplating it. After t, T is presented to S as necessarily true, and S sees why T must be true. The mental state presenting T in such a way to S is what I call an intuition. Since intuitional learning cannot be accounted for in terms

of beliefs, convictions, or understanding, my analysis supports the claim that intuitions are sui generis mental states, namely, experiences with a distinctive phenomenal character, that cannot be reduced to other mental states such as beliefs or convictions. Also, intuitions should not be reduced to inclinations or dispositions to believe. Intuitions are no more inclinations to believe than perceptual experiences. When you see a black laptop in front of you, you are inclined to believe that there is a black laptop simply because a black laptop is presented to you as being there. When you intuit that 2 is the only even prime number, you are inclined to believe that 2 is the only even prime number simply because you can see why it must be true.

Finally, we note that the *phenomenal* change that takes place in intuitional learning—T becoming intuitive to S—is accompanied by an *epistemological* change—S becoming immediately justified in believing T. Just like in the case of perceptual learning, this supports the claim that phenomenology grounds epistemology in the sense that perceptual and intuitional experiences justify by virtue of their distinctive presentive phenomenology.

One may object that the reminder that every even number can be divided by 2 does not help you to "see" that 2 is the only even prime number but helps you to form an argument and (unconsciously) infer that 2 is the only even prime number. However, even if it turned out that intuition necessarily involves unconscious inference, this would not imply that intuition cannot be a source of immediate justification. Recent investigations concerning perception reveal that even perception involves unconscious inferences. Thus, "we now see that the existence of unconscious inferences is no objection to a process's being perceptual, since perception typically involves unconscious inferences" (Legg and Franklin 2017, 332). So how can perceptual experience be a source of immediate justification despite involving unconscious inferences? According to the picture I aim to establish in this work, the answer is straightforward: It all depends on the experience's phenomenal character. If a perceptual experience presents a table being in front of you, you are justified in believing that there is a table in front of you simply because your experience has a "presentive" character with respect to this object/content. It does not matter why the perceptual experience has such a phenomenal character. Analogously, if an intuition not only presents a statement as true but makes you see why it must be true, you are justified in believing this proposition simply due to how it is presented within intuition. It does not matter why the

intuitional experience has such a phenomenal character.²⁰ I would even say that an intuition can be a source of immediate justification, even if it results from a *conscious* inference. If a simple proof, where you can grasp all steps simultaneously, makes you see why a theorem must be true, you have immediate and inferential justification for this theorem. Immediate justification because you can prove it.

This means that my account of experiential justification is a genuinely internalist one, according to which the genesis or etiology of an experience does not contribute to the experience's justificatory force. Instead, the experience's justificatory force is determined by its phenomenology. This is why I would refer to my account as phenomenological internalism. Of course, the etiology of an experience can play an epistemically important role when the subject knows about it. When I have the perceptual experience of a pink elephant, this perceptual experience provides me with immediate prima facie justification for believing that there is a pink elephant. However, this experiential justification may be defeated by my knowledge that I took a drug that causes hallucinations. This defeating justification is inferential justification. The *immediate* experiential justification provided by my elephant experience is not diminished or reduced; it remains unchanged but is defeated. Although highly controversial, my claim that an experience's justificatory force is not affected by its etiology is not uncommon for internalists. Huemer, for instance, says: "When the subject is unaware of an appearance's etiology, that etiology is irrelevant to what it is rational for the subject to believe" (2013, 344). For a defense of this claim and an elaboration of my phenomenological internalism, cf. Berghofer (2020b).

5.2 Pythagoras

Roger Nelsen, author of the three volumes of *Proofs without Words*, characterizes proofs without words as "pictures or diagrams that help the observer *see why a particular statement may be true*, and also to see how one might begin to go about proving it true" (1993, vi, my emphasis). Assume you are familiar with the Pythagorean theorem $c^2 = a^2 + b^2$, but you cannot intuit it. You cannot see why it must be true. You are told that the picture above is a

²⁰ For similar approaches to intuitional justification, cf. Chudnoff's account of "hard-won intuitions" in (2020) and Koksvik's argument that "contrary to popular opinion, intuition can result from conscious reasoning" (2013, 710).



Figure 3: Pythagoras

proof without words for the Pythagorean theorem, but you cannot see why. You can see that the big left square and the big right square have the same area, $(a + b) \cdot (a + b)$. You can also see that the four right-angled triangles appearing in both big squares have the same area, $2 \cdot (a \cdot b)$. You can see that the square appearing in the left big square equals c^2 , and you can see that this square is equal to the two squares appearing in the big right square. Still, you fail to intuit the Pythagorean theorem and why this picture is supposed to prove it. Now, you are told that the two squares appearing in the right big square have the area of a^2 and b^2 , respectively. By gaining this information, you can now intuit the Pythagorean theorem and see why this picture proves it. Your intuition seems to reveal why the Pythagorean theorem must be true.

Of course, you knew that the Pythagorean theorem obtains even before intuitional learning took place, but only now are you immediately justified in believing it. You are, simply because you can see why it must obtain. *Note the similarity to the case of the ambiguous image* we have discussed in section 1.2. In the case of the ambiguous image, you failed to see that the picture also shows a duck. After being told that the rabbit's ears are the duck's beak, you could see the duck and thus were immediately justified in believing that this picture also shows a duck. In the case of the Pythagorean theorem, the additional information that the two squares appearing in the right big square have the area of a^2 and b^2 helped you to see that the Pythagorean theorem obtains. Importantly, the picture did not help you to understand the terms involved. Even before the picture helped you to intuit the Pythagorean theorem, you understood all the terms involved perfectly well. Similarly, with respect to your beliefs, convictions, and dispositions to believe: The picture proof did not help you to form new beliefs or dispositions to believe. Instead, it helped you to form an intuition that presented to you the theorem in a way such that you could see why it must be true.

In the case of perceptual learning, once you have seen that the picture also shows a duck, it becomes easier for you to spot the duck the next time you look at this ambiguous image. Similarly, for intuitional learning, once you have understood the proof without words, the next time you look at the picture, it is easier for you to intuit the Pythagorean theorem simply by looking at the picture. One may say that it is not such a surprise that a picture proof works for a theorem of *geometry*. However, the next picture proof we discuss is intended to help you intuit a theorem of number theory.





This picture is a proof without words for the theorem: $1+3+5+...+(2n-1) = n^2$. Assume you understand the theorem but fail to intuit it and fail to see how the picture might prove it. You start with n = 2. $2^2 = 4 = 1+3$. You see that in the picture, 2^2 is the area of one black square +3 white squares, which means that the picture gets it right for n = 2. Still, you fail to see how this proves

²¹ This picture stems from Wikimedia Commons, licensed under: <u>https://creativecommons.org/li-censes/by-sa/3.0/deed.en</u>. No changes were made.

the theorem. You proceed with n = 3. $3^2 = 9 = 1 + 3 + 5$. You see that in the picture, 3^2 is the area of one black square +3 white squares +5 black squares, which means that the picture gets it right for n = 3. Still, you fail to see how this proves the theorem. You proceed with n = 4. $4^2 = 16 = 1 + 3 + 5 + 7$. You see that in the picture, 4^2 is the area of one black square +3 white squares +5 black squares +7 white squares, which means that the picture gets it right for n = 4. Now, suddenly, you can intuit the theorem and see how the picture proves it. Importantly, your insight does not rest on or consist in some kind of empirical induction. The point is not that you have realized that the picture works for the three cases discussed so far, and now you are convinced that it will work for any number. The point is that now you can *see why it must work for any number*.²²

5.4 Summary

One may object that proofs without words are not real proofs since everything depends on the intuitions of the subject who is looking at the picture. For more details on the relation between picture proofs, intuitions, and justification, cf. Berghofer (2020a). Now, let us recapitulate the results of the present section. By discussing concrete examples, I argued that TIL obtains. Our discussion supports the following three theses (analogous to the results P1-P3 of section 1):

I1. Intuitional learning is fundamentally linked to a *phenomenal* change in the subject's contemplating.

I2. Intuitional learning can have an influence on the contemplating's justificatory force, such that before the learning process, your contemplating did not immediately justify you in believing that p, but after the learning process, it does—by the contemplating resulting in an intuitional experience.

²² Concerning the differences and similarities between perceptual and intuitional learning, an anonymous referee of this journal pointed out "that there are fundamental differences between perceptual processes and the processes resulting in intuition. And if intuitional learning is similar to perceptual learning, then it can only be in the results, not the causes." I agree. And as it was put in an editorial comment of this journal, the point is that perceptual and intuitional learning "both result in a phenomenology that has epistemological import." This captures precisely what I take them to have epistemologically in common.

13. Intuitional experiences gain their justificatory force precisely by virtue of their distinctive justification-conferring phenomenal character.

I1 and I2 should be plausible.²³ All this suggests I3, since I3 offers the most natural explanation of the link between intuitional learning and intuitional justification. All examples had the following structure: Before t (i.e., the moment when intuitional learning takes place), you were thinking about T, you were contemplating T, and you understood T, but you were not immediately justified in believing T. After t, due to intuitional learning, the phenomenal character of your thinking about T has changed. You can now see why T must obtain. Your contemplating T has resulted in an intuition of T. Now you are immediately justified in believing that T. You are immediately justified simply because you can see why T must obtain. Thus, in each of these cases, a shift in phenomenal consciousness is accompanied by a shift from not being immediately justified in believing T to being immediately justified in believing T. I3 delivers the most natural explanation: intuitional justification is linked to phenomenal consciousness in the sense that certain experiences gain their justificatory force simply by virtue of their distinctive phenomenal character.

One notable difference between perceptual learning and intuitional learning is that the former is more of a gradual process where it is often not possible to identify a moment t at which the phenomenal character shifts from not presenting p within experience to presenting p within experience (and thereby from not having a justification-conferring phenomenology concerning p to having a justification-conferring phenomenology).²⁴ In the case of intuitional learning, there often is a eureka moment, an aha moment,²⁵ at which phe-

24 However, as two anonymous referees of this journal have pointed out to me, this is not true about perceptual learning in general. There are borderline cases in which no gradual process takes place but that involve an aha or eureka moment, similar to cases of intuitional learning. One such example is described in Ahissar and Hochstein (2004, 457–458). In fact, they call this the "Eureka effect." Similar cases are bistable figures such as the duck-rabbit image discussed in section 1.2. All this strengthens the analogy between perceptual and intuitional learning.

25 An anonymous referee of this journal called my attention to psychological studies of so-called insight problems, e.g., Metcalfe (1986), in which problem solvers undergo similar aha moments as in my examples of intuitional learning. This aha- or eureka-phenomenology manifests experimentally such that "no gradual rationalization process precedes the correct response to insight problems" (1986, 623). What I take to be particularly interesting about Metcalfe's findings is that

²³ You may still doubt I1 in the sense that you are not convinced that intuitions have a distinctive phenomenal character. In section 6, I will provide a further example that is intended to reveal more clearly the phenomenal difference between strongly believing and intuiting.

nomenal consciousness shifts from merely thinking about T or understanding T to intuiting T, i.e., seeing why T must obtain.²⁶

6 Intuition vs. Strong Conviction: Negative Multiplication²⁷

Assume you are still not convinced that intuitions have a distinctive phenomenal character. In your opinion, they are strong convictions and nothing else. By contemplating the Pythagorean theorem, for instance, the picture proof helps you to form an argument in favor of the Pythagorean theorem such that you simply know that it is true. There is no distinctive phenomenal character involved. In this section, I shall provide an example that clearly shows that strong convictions, firm knowledge, or gut feelings can be phenomenologically distinguished from intuitions. This example is different from the foregoing ones in that I do not provide an example of an intuition of which I argue that it has a distinctive phenomenal character. Instead, I will provide an example in which you believe that you have an intuition. However, I will argue that what you take to be an intuition merely is a strong conviction and that there is a clear *phenomenal contrast* to the cases of intuitions we have discussed so far.

This example concerns negative multiplication. We all know that $(-3) \cdot (-2) = 6$. Let us assume that you are just as strongly convinced in believing that $(-3) \cdot (-2) = 6$ as you are in believing that $3 \cdot 2 = 6$. I claim that although you are equally strongly convinced that the respective statement is true, you can only intuit the latter but not the former. You say there is no phenomenal difference in your grasping of both statements, which shows that there is no distinctive phenomenal character involved. But now I ask you: Do you see *why* it is true that $(-3) \cdot (-2) = 6$? Concerning *my* intuition that $3 \cdot 2 = 6$, I can see that this statement must be true because I can see that 3 + 3 = 6 and

if an answer was preceded by gradual rationalization processes (and thus a less pronounced ahaphenomenology), this "indicated that the answer would be incorrect" (1986, 623). I consider this experimental support for my claim that intuitional learning involves such an aha-phenomenology (and evidence for the significance of intuitional experiences).

²⁶ In this section, we discussed three examples of intuitional learning in some detail. Of course, there are many more plausible examples, and only a few of them involve picture proofs. Further mathematical statements that could be used to demonstrate intuitional learning would be "there is no greatest natural number" or the axiom of extensionality.

²⁷ This example is based on Chudnoff's talk, "Mature Intuition," delivered at the University of Graz on May 13, 2016.

could not be different. But what about your "intuition" that $(-3) \cdot (-2) = 6$? Can you tell me why this statement must be true?

If you are like most people, you cannot. Your seeming that $(-3) \cdot (-2) = 6$ is simply a strong conviction. You have learned it many years ago, and since then, this belief has turned out to be consistent with many other beliefs, which reinforced its evidential status. But, despite the fact that you "simply know" it to be true in the sense that you do not have to think about or need to actively infer it, you cannot intuit it. There is a clear phenomenological difference between intuiting that $3 \cdot 2 = 6$ and knowing that $(-3) \cdot (-2) = 6$. The difference is that $3 \cdot 2 = 3 + 3 = 6$ is presented to me as being necessarily true, such that I can see that it could not be different. 3 and 3 add up to 6, which is why $3 \cdot 2 = 6$. With respect to $(-3) \cdot (-2) = 6$, you would not even know how to express $(-3) \cdot (-2)$ in terms of "+." You believe, are strongly convinced, and know that $(-3) \cdot (-2) = 6$, but this truth is not presented to you in any distinctively intuitive way. You may respond that $(-3) \cdot (-2) = 6$ because two negatives, of course, make a positive. This is true, but obviously, this just leads to the question of why two negatives make a positive. Do you intuit that two negatives make a positive? Most likely, you do not (and neither do I).

The point of this example is that what you initially took to be intuitively clear turned out to be nothing but a strong conviction that is phenomenologically clearly different from intuiting that, e.g., $3 \cdot 2 = 6$. Hence, intuiting involves a distinctive phenomenal character. The foregoing suggests that only an intuition can *immediately* justify an a priori truth, and that intuiting T means that your intuition has its distinctive phenomenal character with respect to T.

7 Epistemological Lessons from Intuitional Learning

Questions concerning the nature and epistemic status of intuitions rank among the most controversial and most widely discussed problems in the history of philosophy. Often, it has been claimed that there is an astonishing parallel between perceptual experiences and intuitions. But apart from arguing that both perceptual experiences and intuitions can be a source of immediate justification, the details of this parallel often remain unclear. To address this issue more thoroughly, one needs to answer the following questions: Q1: What is it that makes perceptual experiences a source of immediate justification?

Q2: What is it that makes intuitions a source of immediate justification?

Q3: In what sense are perceptual and intuitional justification related?

This paper engaged with these questions by first discussing concrete examples of perceptual learning and then proceeding to show that there are parallel cases of intuitional learning. With respect to perceptual learning, I put forward the thesis TPL. By discussing concrete examples, I showed that TPL obtains and that these examples support the following three claims:

P1. Perceptual learning is fundamentally linked to a change in the experience's phenomenal character.

P2. Perceptual learning can have an influence on the experience's justificatory force, such that before the learning process, your experience did not immediately justify you in believing that p, but after the learning process, it does.

P₃. Perceptual experiences gain their justificatory force precisely by virtue of their distinctive justification-conferring phenomenal character.

This highlights that the phenomenal character of perceptual experiences should be *the* focus of investigations concerning perceptual justification, which implies a close connection between epistemology and philosophy of mind. Furthermore, we are provided with an answer to Q1: Perceptual experiences gain their justificatory force from their distinctive phenomenal character.

Concerning intuitional learning, I proposed the thesis TIL. By discussing concrete examples, I showed that TIL obtains and that these examples support the following three claims:

I1. Intuitional learning is fundamentally linked to a *phenomenal* change in the subject's contemplating.

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I2. Intuitional learning can have an influence on the contemplating's justificatory force, such that before the learning process, your contemplating did not immediately justify you in believing that p, but after the learning process, it does—by the contemplating resulting in an intuitional experience.

13. Intuitional experiences gain their justificatory force precisely by virtue of their distinctive justification-conferring phenomenal character.

This highlights that the phenomenal character of intuitional experiences should be *the* focus of investigations concerning intuitional justification, which implies a close connection between epistemology and philosophy of mind. Furthermore, we are provided with an answer to Q₂: Intuitional experiences gain their justificatory force from their distinctive phenomenal character. This, of course, also provides an answer to Q₃: Perceptual experiences and intuitional experiences are parallel in the sense that they both gain their justificatory force by virtue of their distinctive phenomenal character. This may be specified as follows: A perceptual experience E or an intuitional experience I immediately justifies believing that *p* if and only if E/I has its distinctive phenomenal character with respect to *p*.

Similar results can be found in Bengson (2015b), Chudnoff (2013), Church (2013), and Koksvik (2011). One distinctive feature of my reasoning concerns the way I arrived at my conclusion: beginning with cases of perceptual learning, I proceeded to show that there are parallel cases of intuitional learning. Another distinctive feature concerns the difference between a perceptual experience's justification-conferring phenomenal character and an intuitional experience's phenomenal character. According to the aforementioned authors, there is no real difference. At least none is specified. The twenty-first century roots of such an identical treatment of perceptual and intuitional justification can be found in Michael Huemer's principle of phenomenal conservatism, according to which every seeming is a source of immediate prima facie justification, suggesting that intellectual seemings and perceptual seemings do not differ in phenomenologically significant ways (cf. 2001, 99). From a phenomenological point of view, such a conception is superficial at best. In this paper, I argued that intuitions do not only present their contents as true: they seem to reveal why they must be true. This is a clear phenomenological difference to how perceptual intuitions present their objects/contents.

These results, of course, also have significant implications concerning the question of what it means that an a priori statement is immediately justified. Immediate justification does *not* entail that, when understanding theorem T, you can immediately grasp that T obtains. It might well be that even when understanding the terms involved, you fail to intuit T. Immediate justification only takes place when your contemplating T results in an intuition of T in the sense that you can see why T must obtain. When contemplating T, such an intuition of T may never occur, occur only after a long period of time, or occur only after proving T.²⁸ Thus, immediate justification is *not* linked to how strongly one is pushed towards believing a proposition or to the belief's reliability, *but only to how a content is presented within experience.* Of course, this leads to a distinctively internalist conception of experiential justification.²⁹

Finally, we may address the questions raised in section 2. Intuitions are sui generis mental states, namely experiences that have a distinctive phenomenal character. They are a source of immediate but fallible³⁰ justification and seem to tell us something about a mind-independent reality. However, I may point out a certain limitation of my investigations. In section 2, I mentioned that current analytic philosophy draws heavily on intuitions as evidence, which is one of the reasons why intuitions must be a focus of philosophical considerations. In the present paper, however, I have only discussed cases of mathematical intuitions. One might argue that even if all I have argued for here is true, this may have significant implications for perceptual and

- 28 Of course, not every proof of a theorem T results in an intuition of T. But I take it that a simple proof involving only very few steps can result in an intuition of T. In such a case, you have immediate and inferential justification for T. You have immediate justification because you can intuit that T, and you have inferential justification because you can prove it.
- 29 Internalist in the sense that it is not external factors such as reliability or truth that determine a basic belief's epistemic status but rather the internal factor of how the content is presented within experience.
- 30 In this paper, we have only superficially touched on the topic of fallibility. While traditional rationalists such as Descartes insisted on the infallible character of immediate justification, rationalists now accept the fallible character of intuitions. This is mainly due to well-known historical and everyday cases in which intuitions turn out to be misleading. Our considerations reveal why it is natural to assume that intuitional justification is fallible. Since intuitions gain their justificatory force from their distinctive phenomenal character, there is no reason to assume that intuitions are infallible. It is all about how these experiences present their contents. There is no systematic reason to think that such presentations can never be misleading. Of course, there may be cases of infallible intuitive justification. Your beliefs that 1 + 1 = 2, that 2 > 1, or that 2 is an even number may be cases in which your intuitions never go wrong. 17 + 15 = 32 may be another story.

intuitional justification, but only with respect to mathematical intuitions and not with respect to philosophical intuitions such as, e.g., Gettier intuitions.³¹

I admit that there is a need for further elaborations. It might well be that there are different types of justification-conferring intuitions—different types in the sense of having a different type of justification-conferring phenomenology. Epistemic intuitions concerning hypothetical cases—such as Gettier intuitions—may differ phenomenologically from epistemic intuitions of general epistemological principles, from ethical intuitions, and from mathematical intuitions. The aim of the present paper was only to show that there are cases of intuitional learning and that there are intuitions that are sui generis experiences that have a distinctive, justification-conferring phenomenology. The question of whether "our" intuitions in epistemology, ethics, and other areas of cognition can have the same justification-conferring phenomenal character as our examples of mathematical intuitions remains to be discussed by future phenomenological-epistemological investigations.*

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³¹ Williamson, for instance, states that "mathematical intuition can have a rich phenomenology, even a quasi-perceptual one" (2007, 217), but doubts that this is true for philosophical intuitions. For a critical discussion, cf. Chudnoff (2013).

^{*} I would like to thank Martina Fürst and Philipp Schönegger for important comments on an earlier version of this paper. Also, many thanks to two anonymous referees for helpful clarifications and suggestions. This work was supported by the Austrian Science Fund (FWF) [P 31758].

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