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Fraser MacBride

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Converse Predicates and the Interpretation of Second Order Quantification

Fraser MacBride

In this paper I argue that we cannot interpret second-order quantification as quantification over an abundant supply of properties and relations conceived as the referents of predicates. My argument forges a hitherto unexplored connection between debates typically conducted independently, one about whether there are converse relations, the other about the interpretation of second-order quantifiers. I begin from the semantics of converse predicates. Either pairs of mutually converse predicates co-refer or they do not. If they do co-refer, I argue that we lack an understanding of the relevant class of higher-order predicates which are required for second-order quantification over a domain of relations. If they don't co-refer but pick out distinct converse relations then I argue that whilst we may make some abstract sense of the higher-order predicates in question we do so only at the cost of having to impute implausible readings to lower-order constructions. Either way, I conclude that second-order quantification should not be interpreted as quantification over relations conceived as the referents of predicates.

1 Introduction

How should we interpret second-order quantifiers? In this paper I argue that we cannot interpret second-order quantifiers as ranging over relations—not if second-order existential introduction is taken to be a straightforward generalization of first-order existential introduction.

My primary argument takes the form of a dilemma. Either pairs of mutually converse predicates, such as "ξ is on top of ζ" and "ξ is underneath ζ", refer to the same underlying relation or they refer to distinct converse relations. If they refer to the same relation then we lack the supply of higher-order predicates required to interpret second-order quantifiers as ranging over a domain of relations. The higher-order predicates required for such an interpretation of second-order quantifiers are predicates true or false of the referents of lower-order predicates—that is, true or false independently of how the referents of those lower-order predicates are specified. But if mutually converse predicates co-refer then we lack the supply of higher-order predicates required for such an interpretation. If, by contrast, mutually converse predicates refer to distinct converse relations then whilst we can at least make abstract sense of the higher-order predicates required to interpret quantifiers as ranging over a domain of relations, the implausible consequences for the content of lower-order constructions render this interpretation of higher-order quantifiers a deeply implausible semantic hypothesis.

There has been a great deal of recent discussion both about whether or not there are converse relations and about whether we should interpret second-order quantification in terms of a range of properties and relations or otherwise. But these two debates have been conducted separately and independently of one another. Here I seek to show that there are important connections between them.

Some preliminaries. For brevity I state my argument in terms of binary relations but it is intended to generalize to relations of greater arity. By a second-order language I will mean one in which the second-order quantifier rules are a straightforward generalization of the first-order quantifiers rules, allowing for the introduction of the second-order existential quantifier into predicate position, and where these rules are supplemented with the Axiom Scheme of Comprehension according to which, roughly speaking, every predicate determines a relation.[1] What are mutually converse predicates? For present purposes, I take any two binary predicates U and V to be mutual converses iff, for any terms t, t', it is guaranteed by the rules of the language that $t[U].underline t'$ is true iff, $t'Vt$ is true.[2] Similarly, R and R are mutual converse relations iff, for any particulars x, y, xRy iff yRx, not as a matter of accident but metaphysical necessity.

1 See (**Shapiro1991?**), (**Fine2002?**) and (**Williamson2013?**).

2 Further refinements will be required to accommodate the phenomenon of inflected pronouns in English. For other natural and formal languages which place the predicate in prenex position and for natural languages, such as Latin and Hebrew, which rely more heavily upon case, prepositions and particles rather than the mere arrangement of terms, "mutual converses" will require accordingly different definitions.

Finally, what is a second-order predicate? A first-order predicate (say of the form "$F\xi$") results from the extraction of one or more names ("a") from a closed sentence ("Fa") in which it occurs and inserting a variable in the resulting gap. A second-order predicate (say of the form "$\exists x \Phi x$") results from the extraction of a first-order predicate ("$F\xi$") from a closed sentence ("$\exists x Fx$") and inserting a variable into the resulting gap.[3] Our focus here will be binary first-order predicates ("$\xi R \zeta$") which result from the extraction of two names from a closed sentence ("aRb") and unary second-order predicates ("$a\Phi b$") which result from the extraction of a binary first-order predicate from a closed sentence.

2 Converse Predicates and Co-Reference

Whatever is true of an object picked out by a singular term is true of something. That's the primordial idea that justifies the operation of first-order existential introduction. But if converse predicates co-refer the operation of second-order existential introduction cannot be justified along such lines. To present my argument for this claim I begin by describing one semantic motivation for supposing that converse predicates co-refer.[4]

It may appear that we are up to our necks in ontological commitment to converse relations because in English, but not only in English, we have the active and passive voice for many verbs and an abundance of adjectives, adverbs and so on whose reciprocal behavior is readily modeled by converse relations: "above" and "below", "before" and "after", "greater" and "less", *etcetera*. But there's no need to posit converse relations to explain the reciprocal behavior of converse predicates. This is because the behavior of converse predicates can be explained more parsimoniously in terms of converse rules for their employment. The rules in question map the contexts in which pairs of mutually converse predicates occur onto the same configuration of things-

3 See (**Dummett?**) 1981a [38–39]. Note that "predicates" as conceived here are interpreted signs or strings of signs which are true or false of the referents of the expressions to which they are applied and their variables are bindable. The class of predicates of a given type $n+1$ will include complex predicates or open sentences generated from closed sentences with n type terms replaced by variables, as well as including primitive signs of that type. Here I follow, for example, the usage of (**Shapiro1990?**) and (**Shapiro-Weir2000?**).

4 The proposal that mutually converse predicates should be conceived as co-referring can be traced back (at least) to (**Russell1913?**). For alternative metaphysical and semantic motivations for so conceiving converse predicates see (**Evans1958?**), (**Sprigge1970?**), (**Armstrong1978b?**), (**Williamson1985?**) and (**Fine2000?**).

in-relation, so there is no need to posit separate configurations of things-in-relation.

The matter can be considered from a more general perspective on representation. In order to systematically represent things-in-relation we use signs-in-relation; we encode information about how things are related by how we relate signs together.[5] Invariably there is more than one way of configuring signs to encode the same information about how things are related and we can switch between them so long as we keep track of the different means whereby different configurations of signs encode the relevant information.

Consider the worldly configuration of things-in-relation, famously depicted in the *Alexander Mosaic,* which consists of Alexander sitting astride Bucephalus at the Battle of Issus. The statement "Alexander is on top of Bucephalus" effectively encodes how Alexander is related to Bucephalus by one arrangement of signs along a horizontal line. The statement "Bucephalus is underneath Alexander" no less effectively encodes the same information by another arrangement of signs. Neither statement constitutes a privileged encoding of how Alexander and Bucephalus are related. One is as good as another because it is a matter of convention how we encode information about the vertical arrangement of Alexander and Bucephalus by placing their names along a horizontal line. There are two conventions one might employ: (a) placing the name of the thing which is on top to the left and the name of the thing underneath to the right; (b) placing the name of the thing underneath on the left and the name of the thing on top to the right. When we use the predicate ξ is on top of ζ" we signal that we are exploiting convention (a) to encode information about how things are related by the spatial relation which "ξ is on top of ζ" stands for, whereas when we use "ξ is underneath ζ" we are exploiting convention (b) to encode information about the obtaining of the same relation. Grasping the rules for "ξ is on top of ζ" and "ξ is underneath ζ" we understand straightaway that "Alexander is on top of Bucephalus" represents the same worldly configuration as "Bucephalus is underneath Alexander". Accordingly, we also understand that what we represent concerning Alexander and Bucephalus using "ξ is on top of ζ" could have been equally well represented using only "ξ is underneath ζ" and *vice versa*—so we could have succeeded in describing how Alexander and Bucephalus are depicted by the *Alexander Mosaic* in relation to one another if we'd been provided with only

5 I take this to be the element of truth in Wittgenstein's picture theory, see his (**Wittgenstein1922?**) and (**MacBride2018?**).

one of this pair of converse predicates. But the primary argument here isn't that we only need one predicate and so only one relation. Nor is the argument that there is only one relation here because one predicate can be defined in terms of the other. It is rather that the contexts "Alexander is on top of Bucephalus" and "Bucephalus is underneath Alexander" are mapped by the correlated conventions of their respective predicates onto the same worldly configuration of Alexander and Bucephalus and so there is no need to posit different configurations involving different relations to correspond separately to them.

A similar story can be told about other pairs of mutually converse predicates in English—for example the active and passive forms of a verb ("ξ kissed ζ", "ξ was kissed by ζ"). They don't stand for different relations but the same relation, albeit relative to contrary conventions about how to exploit the arrangement of prefixed and appended signs to represent how the things for which the signs stand are related by whatever relation is picked out by the predicate the signs prefix and append.[6]

3 Converse Predicates and the* *Division of Semantic Labour

Objectual quantification involves quantification over a domain of entities (whether first-order or second-order). In this section I will argue that the intelligibility of objectual quantification presupposes a principle I will call "The Division of Semantic Labour". For singular constructions the principle can be stated in the following terms. It must be possible to distinguish between, on the one hand, an expression whose semantic role is exhausted by picking something out—which, so to speak, drops away once it has discharged this function—and, on the other hand, the rest of the sentence whose complementary role is to say such-and-such about what has been picked

6 The conventions invoked here apply to configurations of things-in-relations rather than merely individuals. Suppose we adopt the convention for the non-symmetric predicate "ξ loves ζ" that we are to place the name of the thing which is the lover to the left of the verb and the name of the thing which is beloved to the right of the verb. And suppose it is both the case that Romeo loves Juliet and Juliet loves Romeo. Then if we apply the convention to individuals we are left in the dark about how to apply the convention because neither Romeo nor Juliet is "the" lover. But this difficulty is avoided if the convention is applied separately to the configurations (1) Juliet's loving Romeo and (2) Romeo's loving Juliet—because with respect to (1), Juliet is the unique lover, whereas with respect to (2), Romeo is the unique lover

out—independently, that is, of how it was picked out.[7] It is only when the Division of Semantic Labour applies to a context that an expression occurring in it may intelligibly give way to a bound variable.

This division is a prerequisite of objectual quantification for the following reason. If the capacity of the rest of the sentence to say such-and-such is nullified by the extraction of a referential expression—if, so to speak, the significance of the rest of the sentence evaporates when the referring expression is pulled out—then we cannot use the rest of the sentence to say such-and-such about the value of a variable upon an assignment of values to variables by replacing the referring expression with a bound variable. In that case the idea behind the rule of existential introduction will have been undone because we cannot intelligibly say that what is true of a certain item picked out by a given referring expression is true of something, i.e. true of it regardless of whether that expression picks it out.[8]

I will now argue that we cannot quantify into the positions occupied by converse predicates because the contexts in which they occur fail to exhibit the Division of Semantic Labour—assuming that mutually converse predicates co-refer.[9] To see this, first observe that it's a consequence of conceiving mutually converse predicates as co-referential that we also have to recognise

7 The Division of Semantic Labour (in the singular case) was recognized by both (**Quine1960?**) and (**Strawson1961?**) who distinguish, on the one hand, expressions occurring in basic predications whose role is to specify or identify an object and, on the other hand, the rest of the sentence whose role is to be true or false of that object however specified or identified. To cover statements in which plural definite descriptions or lists of names feature the principle would need to be augmented with plural quantifiers and pronouns—to distinguish between expressions whose semantic role is exhausted by picking out *somethings* and the rest of the sentence which says such-and-such about *them* independently of how *they* were picked out. (See **Boolos1984?** for the need to recognize the irreducibility of plural forms.) Since the relevant issues surrounding substitution and quantification into the positions of first-order converse predicates already emerge in the singular case, I concentrate attention there.

8 Famously (**Quine1961?**) provided "Giorgione was so-called because of his size" as an example of a context which is resistant to the substitution of co-referential expressions and to which the rule of existential introduction cannot intelligibly be applied. For a sustained treatment of this and other examples *prima facie* resistant to substitution and quantifying in, see (**Fine1989?**) and (**Forbes1996?**).

9 For present purposes I restrict the Division of Semantic Labour to atomic sentences. Whereas it is integral to the Fregean approach to quantification that complex predicates ("ξ is even and ξ is prime") as much as simple predicates ("ξ is even" and "ξ is prime") are true or false of the referent of a name, a Tarskian account explains away complex predicates in terms of simple predicates, i.e. atomic open sentences, and it is only they that are true or false of the referent of a name. See (**Dummett1981b?**). So Tarskians deny that "2 is even and 2 is prime" can be decomposed into "2" and a single predicate which is true or false of the referent of "2". But since

that the substitution of co-referring predicates cannot be guaranteed to be truth preserving.[10] This is so even though such predicates occur in contexts like,

(1) Alexander is on top of Bucephalus

whose truth-value is functionally determined by the referents of its parts and how they are assembled, contexts, moreover, whose name positions are open to truth-preserving substitution by co-referring terms. Since, for example, “Sikandar” is the Persian name for Alexander, we can infer from (1) that,

(2) Sikandar is on top of Bucephalus.

Nonetheless, even if we conceive of “ξ is underneath ζ” and “ξ is on top of ζ” as co-referring, we cannot substitute the former for the latter in (1) whilst preserving truth, because the result is false,

(3) Alexander is underneath Bucephalus.

Why does substituting “ξ is underneath ζ” for “ξ is on top of ζ” take us from truth to falsehood even though, we are granting, “ξ is underneath ζ” and “ξ is on top of ζ” refer to the same relation? In order for this inference to have been valid what (3) says about the referent of “ξ is underneath ζ” would have had to be the same as what (1) says about the referent of “ξ is on top of ζ”. But they don’t and can’t say the same about *it*. This is because what the rest of (1) says about the referent of “ξ is on top of ζ” cannot survive the extraction of “ξ is on top of ζ”. The rest of (1), which is what results from extracting “ξ is on top of ζ” from (1), is the second-order predicate “Alexander Φ Bucephalus”. When the variable “Φ” in “Alexander Φ Bucephalus” is replaced by “ξ is on top of ζ” the result is a sentence that says Alexander is on top of Bucephalus. But when “Φ” is replaced by “ξ is underneath ζ” the result is a sentence that says Bucephalus is on top of Alexander. Hence, so far from being the same, what (1) says about the referent of “ξ is on top of ζ” is incompatible with what (3) says about the referent of “ξ is underneath ζ” even though “ξ is underneath ζ” and “ξ is on top of ζ” refer to the same relation.

What emerges from this line of reflection is that the significance of “Alexander Φ Bucephalus” isn’t freestanding but varies depending upon which first-

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Fregean and Tarskian accounts agree that simple predicates or atomic open sentences are true or false of the referents of names, their differences over complex predicates may be set aside.

10 See (**Williamson1985?**), (**MacBride2006?**) and (**MacBride2011?**).

order predicate is inserted in place of its variable. The failure of "Alexander Φ Bucephalus" to have a freestanding significance is a consequence of the fact that the rules which we understand when we grasp converse predicates rely upon different conventions about how to interpret the significance of the arrangement of corresponding signs ("Alexander", "Bucephalus"). What it means to prefix an occurrence of one predicate, say "ξ is on top of ζ", with a token of "Alexander" whilst appending a token of "Bucephalus" is different from what it means to prefix an occurrence of a mutually converse predicate, say "ξ is underneath ζ" with a token of "Alexander" whilst appending a token of "Bucephalus". Without a first level predicate to furnish the conventions required to interpret the significance of prefixing "Alexander" and appending "Bucephalus", the second-order predicate "Alexander Φ Bucephalus" means nothing at all—its significance evaporates as soon as a first-order predicate filling its argument place is extracted.

So the strategic situation is this—assuming that mutually converse predicates co-refer. In order for objectual quantification into the position occupied by "ξ is on top of ζ" in (1) to be intelligible, i.e. for

(4) (∃Φ)(AlexanderΦBucephalus)

to be meaningful, (1) must admit of a semantic analysis into two discrete components, the first level predicate "ξ is on top of ζ" and the rest of the sentence, the second-level predicate "Alexander Φ Bucephalus" which, were (4) meaningful, (4) would affirm to be true of some relation in the domain. But (1) fails to satisfy the Division of Semantic Labour. The second level predicate left over once "ξ is on top of ζ" is extracted lacks self-standing significance. It isn't true or false of the referent of "ξ is on top of ζ" independently of how that relation is picked out. Since "Alexander Φ Bucephalus" lacks freestanding significance we cannot intelligibly affirm it of the value of a second-order variable, *i.e.* affirm it of a relation independently of how that relation is picked out by a first-level predicate.[11] Hence we cannot quantify into (1), and (4) is

11 Whilst Williamson, in his (**Williamson1985?**), recognized that the substitution of co-referential converse predicates isn't guaranteed to be truth-preserving, he did not address the consequent difficulties, explained here, for quantifying into the positions of converse predicates. In his more recent *Modal Logic as Metaphysics* (**Williamson2013?**) Williamson recommends higher-order-quantification into predicate position because of its theoretical virtues ("maximizing strong, simple generalizations consistently with what we know," **Williamson2013?**) but he does not mention the issue of substitution failures for converse predicates. The line of argument I advance here shows that Williamson's views cannot be straightforwardly packaged together because

meaningless. The rule of existential introduction into the position of converse predicates, understood as a generalization of existential introduction into the positions of names, is thereby undone.

4 Another Meaning for "Alexander* Φ *Bucephalus"

I have argued against the intelligibility of higher-order quantification (objectually conceived) on the grounds that "Alexander Φ Bucephalus" lacks meaning in isolation, *i.e.* independently of the insertion of a first-level predicate into its argument position—because otherwise there's nothing to settle how to interpret the significance of the prefixed and appended terms. It may be thought that this is going too far. One can envisage an objector granting that "Alexander Φ Bucephalus" lacks a *determinate* significance—because what the significance will be of prefixing "Alexander" and appending "Bucephalus" to a given occurrence of a predicate depends upon the rules governing the predicate that happens to occur between them. Nevertheless, this objector continues, this doesn't rule out "Alexander Φ Bucephalus" having a *determinable* significance, i.e. its being a second-level predicate which is true of the referent R of a first level predicate (when inserted into its argument position) just in case R relates Alexander to Bucephalus in *some manner or other* but without settling any determinate arrangement for them.

The immediate difficulty with this objection is that if "Alexander Φ Bucephalus" is granted the kind of determinable significance proposed, then other sentences get assigned the wrong truth conditions. From (1) follows,

(5) ¬ (Alexander is underneath Bucephalus).

Now according to the semantic hypothesis under consideration, (5) has a higher-order parsing according to which (5) says that it's not the case that the relation which is the referent of "ξ is underneath ζ" satisfies "Alexander Φ Bucephalus", i.e. it's not the case that that relation has the determinable property of relating Alexander to Bucephalus in some manner or other. But this makes (5) incompatible with (1) which says that the same relation, i.e. the referent of "ξ is on top of ζ", *does* relate Alexander to Bucephalus in some manner or other, specifically relating Alexander to Bucephalus so that Alexander is on top of Bucephalus. But (5) isn't incompatible with (1) but entailed by it. So

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there is an irreconcilable tension between conceiving of converse predicates as co-referential and quantifying into their positions (assuming the quantification to be objectual).

the semantic hypothesis that "Alexander Φ Bucephalus" has self-standing but determinable significance results in faulty assignments of truth-conditions.

Denying that "Alexander Φ Bucephalus" has the self-standing significance required for quantifying into the position of converse predicates is consistent with allowing that "Alexander Φ Bucephalus" has some weaker kind of significance. After all, when "Alexander Φ Bucephalus" is completed with a given first level predicate the result is a statement with a certain content, or, to speak more generally, a certain semantic value. So, *prima facie*, we can assign it the derived syntactic category S/(S/NN).[12] But we cannot interpret "Alexander Φ Bucephalus" as having as a semantic value a function from the referents of binary predicates to the semantic values of sentences. Since, we are supposing, mutually converse predicates have the same referent, such a function will map the semantic value of "ξ is underneath ζ" to the same semantic value (of the kind appropriate to a sentence), as it maps the semantic value of "ξ is on top of ζ". But the result of substituting a co-referential but converse predicate, "ξ is underneath ζ" for "ξ is on top of ζ" in a sentence in which "Alexander Φ Bucephalus" occurs, is not guaranteed to preserve the semantic value of the sentence upon which the substitution is performed. Nor does it follow even if it is conceded that "Alexander Φ Bucephalus" belongs to a derived syntactic category, that "Alexander Φ Bucephalus" has content in the sense relevant to sustaining the intelligibility of second-order quantification, i.e. has content in the sense of itself having the capacity to be true or false of a relation independently of how that relation is specified by a first level predicate.

5 Relations and the Axiom Scheme of Comprehension

I have taken us along a route from point (a) supposing that converse predicates co-refer to point (b) the unintelligibility of second-order quantification conceived as quantification over the referents of binary predicates, the connecting link being that if converse predicates co-refer then there's a lack of extractable higher-order predicates capable of being true or false of their referents independently of how they are picked out. But are there other routes between these two points?

To suppose that mutually converse predicates co-refer is to adopt a (relatively) sparse view of our ontological commitments. The view is (relatively)

12 See Ajdukiewicz's categorial grammar (**Ajdukiewicz1967?**).

sparse insofar as "ξ is on top of ζ" and "ξ is underneath ζ" are conceived as equally good predicates for referring to one and the same relation—so less abundant than a view according to which our use of "ξ is on top of ζ" and "ξ is underneath ζ" commit us to distinct converse relations. But the Axiom Scheme of Comprehension for second order logic,

Comp. $\exists R^n \forall x_1 ... x_n (R^n x_1 ... x_n \leftrightarrow \Phi x_1 ... x_n)$

where R^n is an n-ary relation variable which does not occur free in Φ, is typically conceived as embodying an abundant conception of relations because, taken together, the instances of Comp tell us that every formula determines a relation. Doesn't this already establish that embracing second order logic is incompatible—because Comp is abundant—with the sparseness of supposing converse predicates to co-refer?

Now it is certainly true that Comp is straight out incompatible with certain sparse conceptions of relations. Comp says that every formula determines a relation even if the formula in question isn't satisfied by anything. So embracing Comp forces the admission of uninstantiated relations where the corresponding formulae are unsatisfied. This means that if we admit only instantiated relations, what's often called an "Aristotelian" conception of relations, or universals more generally, then we must reject Comp.[13] To bring second-order logic in line with this "Aristotelian" stricture, Comp needs to be restricted to recognise only relations that correspond to formulas that are true of something:

Aristotelian Comp. $\exists x_1 ... \exists x_n \Phi x_1 ... x_n \rightarrow \exists R^n \forall x_1 ... \forall x_n (R^n x_1 ... x_n \leftrightarrow \Phi x_1 ... x_n)$.

Further restrictions along these lines can be envisaged. Aristotelian Comp still requires a relation for every polyadic predicate that's satisfied. But this won't be sparse enough for us if, for example, we're doubtful that there are relations corresponding to disjunctive predicates even if they're satisfied.

By contrast to an Aristotelian approach which requires relations to be instantiated, the (relatively) sparse doctrine that mutually converse predicates are vehicles for referring to one and the same relation does not conflict with the existential requirements of Comp. This is because (a), unlike the Aris-

13 See (**Armstrong1978a?**) for "Aristotelian realism". See (**Shapiro-Weir2000?**) for the suggestion of an aristotelian second order logic and the proposed restriction on Comp.

totelian approach, the doctrine that mutually converse predicates co-refer does not require that the relations to which they refer are instantiated. Moreover, (b) Comp does not require that each formula determines a unique relation but only that each formula determines a relation—which is consistent with different formulas having the same referent. So whilst Comp requires that $\xi$is on top of ζ" and "ξ is underneath ζ" both pick something out this requirement does not by itself force us towards a more abundant conception of relations according to which "ξ is on top of ζ" and "ξ is underneath ζ" pick out distinct converses.

Williamson, in his treatment of higher-order logic, argues against the restriction of Comp to natural properties and relations, e.g. the universals which, according to Armstrong, are only to be recognized *a posteriori* on the basis of total science. Rather, according to Williamson, Comp is the "most obvious example of a logical principle of higher-order logic that depends on unnatural properties and relations".[14] Williamson advances his case on the grounds that the extensive literature on naturalness has failed to supply a fruitful logic of natural properties and relations. By contrast, Williamson maintains, Comp is an informative logical principle which depends "on the absence of any naturalness restriction" because it allows us quantify into the position of formulae, however unnatural the conditions they define, e.g. not smoking or being everything bad. But this line of reflection doesn't establish that the existence of converse relations can be settled by appeal to Comp alone. Comp only tells us that to every formula there corresponds a property or relation. Comp taken by itself does not tell us that there is a 1-1 correspondence between formulas on the one hand and properties and relations on the other, however unnatural.

Nonetheless, it can be shown in short order that supposing mutually converse predicates to co-refer conflicts with the application of second-order generalization to atomic formulae—even without relying upon the full strength of Comp which applies to formulae of arbitrary complexity. From

(1) Alexander is on top of Bucephalus

it follows that

(5) ¬Alexander is underneath Bucephalus.

Applying the operation of existential generalisation to (1) and (5) it follows that

14 See (**Williamson2013?**).

(6) (∃Φ)(AlexanderΦBucephalus)

and

(7) (∃Φ)¬(AlexanderΦBucephalus).

There's no formal contradiction here because the variables in (6) and (7) aren't bound by the same initial quantifier. But we cannot coherently suppose that the open sentences which occur in (6) and (7) are both satisfied under the same assignment of a relation to "Φ" because the higher-order predicates "Alexander Φ Bucephalus" and "¬ (Alexander Φ Bucephalus)" express contradictory properties of relations. But if both (1) and (5) are interpreted as saying something about the same relation, picked out by "ξ is on top of ζ" and "ξ is underneath ζ" respectively, then their existential entailments should be compatible with the open sentences which occur in (6) and (7) both being satisfied on the same assignment of a relation to "Φ".

It's important to appreciate how the fact that the open sentences which occur in (6) and (7) cannot be true upon the same assignment of values to variables conflicts with supposing both that converse predicates co-refer and that second-order existential generalisation is the analogue of first-order generalisation. Why? Because it's mysterious how, if (1) and (5) incorporate reference to only *one* relation, applying the operation of second-order existential generalization to them can result in statements, (6) and (7), which taken together are ontologically committed to *two* relations. The idea behind the operations of second-order existential generalisation—conceived as an analogue of the operation of first-order quantification—is that whatever is true of the referent of a first-order predicate is true of (second-order) something. But this inference loses its justification if whatever is said to be true of something cannot be true of the referent of the first-order predicate. Since the open sentences which occur in (6) and (7) cannot be true upon the same assignment of values to variables, the application of existential generalisation to (1) and (5), assuming their first-level predicates co-refer, must take us from saying things true of one and the same relation to saying things which can only be true of at least one other relation. But then it is unclear how existential generalisation is guaranteed to preserve truth—because we have undertaken a passage from talking about one relation to committing ourselves to at least two. So we have an unstable package of commitments: (a) that the predicates of (1) and (5) refer to one and the same relation, (b) that (6) and (7) taken together are committed to the existence of two relations, and, (c) the rule of second-order existential introduction is guaranteed to preserve truth when understood as an analogue of first-order existential generalisation.

In light of preceding sections, we can appreciate how the failure of sentences like (1) and (5) to exhibit the requisite Division of Semantic Labour (assuming their first-order predicates co-refer) contributes to this unstable package of views. What (1) affirms of the referent of "ξ is on top of ζ" isn't the negation of what (5) denies of "ξ is underneath ζ" because the respective rules governing the use of "ξ is on top of ζ" and "ξ is underneath ζ" reverse the semantic significance of their prefixed and appended terms. But because quantifying into (1) and (5) extrudes these rules about how to interpret the significance of their flanking terms—by replacing the first-order predicates which carry these rules with bound variables which don't—we are left with the bare statements (6) and (7), whose constituent open sentences cannot be true upon the same assignment of values to variables.

6 Converse Relations

What have we learnt about the possible interpretation of second-order quantifiers? Earlier I argued that if mutually converse predicates co-refer then we cannot intelligibly objectually quantify into the positions they occupy for lack of the requisite higher-order predicates. I have also argued that the operation of second order existential generalization cannot be intelligibly combined with such commonplace truths about mutual converses as (1) and (5) whilst supposing that mutually converse predicates co-refer. This was the first horn of the dilemma envisaged in the introduction.

Prima facie it would not be unreasonable to conclude that second-order languages are committed to converse relations after all—because these problems can be made to go away by assuming that mutually converse predicates pick out distinct converse relations. But even if pairs of mutually converse relations are admitted, thus avoiding the difficulties that arose from dispensing with them, higher-order predicates of the form "$a\Phi b$" are still required for the intelligibility of quantification into the positions of converse predicates, i.e. higher-order predicates capable of being true or false of a relation belonging to the domain independently of how that relation is specified. So the question still remains even if it is granted that mutually converse predicates pick out distinct converse relations: do we have an understanding of higher-order predicates of the form "$a\Phi b$" which will enable us to interpret second-order quantification as quantification over a domain of relations? I will argue that we don't. This is the second horn of the dilemma envisaged in the introduction.

We have already considered the proposal that predicates of the form "$a\Phi b$" have a purely determinable significance—so that, for example, "Alexander Φ Bucephalus" stands for a property of a relation, *viz.* the property of holding between Alexander and Bucephalus in some manner or other, a property which is indifferent to the order in which Alexander and Bucephalus are related by whatever relation has the property. The problem identified earlier with this proposal was that it gets the truth-conditions of (1) and (5) wrong if mutually converse predicates co-refer. But the problem of conceiving "Alexander Φ Bucephalus" as having this kind of determinable significance is a problem for non-symmetric relations *per se* regardless of whether they are accompanied by converses. Consider,

(1) Alexander is on top of Bucephalus

and one of its consequences,

(8) ¬ Bucephalus is on top of Alexander.

If "Alexander Φ Bucephalus" has purely determinable significance then "Bucephalus Φ Alexander" does too but they will mean the same. The latter will stand for a property that a relation has if it relates Bucephalus and Alexander in some manner or other. But a relation has the property of relating Bucephalus and Alexander in some manner or other iff it has the property of relating Alexander and Bucephalus in some manner or other—because the property of relating some things in some manner or other is order-indifferent. Then (8) will have a higher-order parsing according to which (8) says that it's not the case that the non-symmetric relation that "ξ is on top of ζ" picks out has the order-indifferent property of relating Alexander and Bucephalus in some manner or other. But (1) will have a corresponding parsing according to which (1) says that the relation "ξ is on top of ζ" picks out does have that property and (8) follows from (1). This problem doesn't go away if the relation that "ξ is on top of ζ" has a converse because it's a problem that arises solely by reflection upon that relation without consideration of its converse—the relation that "ξ is underneath ζ" picks out doesn't feature.

We can avoid this problem by interpreting "Alexander Φ Bucephalus" as standing for a property sensitive to the order in which Alexander and Bucephalus are related by whatever relation has this property.[15]

But unless the order in question is explicable independently of how "Alexander Φ Bucephalus" is completed by the insertion into its argument position of a first level predicate standing for a relation, we will still have failed to secure the division of semantic labour which I have argued is required for second-order quantification objectually conceived.

In order for a predicate of the form "$a\Phi b$" to have the required self-standing significance it must stand for a higher-order property which relations have independently of how they are picked out. This requirement is fulfilled if relations hold between the things they relate in an order, where the notion of order in play is absolute in the following sense: for any relation *R* which holds between any two things *a* and *b*, either *R* applies to *a* first and *b* second or b first and *a* second. If that is how relations apply to the things they relate then there is a higher-order property any relation has if it applies to *a* first and *b* second, another higher-order property any relation has if it applies to *b* first and *a* second—properties which relations have independently of how they are picked out by first-level predicates because they are properties relations have solely in virtue of how they apply rather than how they are depicted. If that is indeed the case, then a higher-order predicate of the form "$a\Phi b$" meeting our requirement may be understood as standing for the property that any relation has if it applies to *a* first, *b* second.

7 The Untoward Semantic Consequences for Atomic Statements

What is important for present purposes is to appreciate the untoward consequences of so interpreting higher-order predicates of the form "$a\Phi b$". These

15 One way to sidestep all these problems would be to restrict the rule of second-order existential introduction to the positions of symmetric predicates, i.e. contexts where it makes no semantic difference which left-right flanking arrangement of names are used, or, more radically, to quantification over monadic predicates. But this restriction is unappealing because a second-order language without quantification into the positions of non-symmetric predicates would be unable to codify categorical versions of key mathematical principles, one of the key attractions of higher-order languages. Consider Cantor's Theorem construed as the claim that no binary relation can represent the collection of all subsets of its domain ($\forall R \exists X \forall x \exists y[(Rxy \& \neg Xy) \vee (\neg Rxy \& Xy)]$). For further examples, including the Continuum Hypothesis and the Well-Ordering Principle, see (**Shapiro1991?**).

include consequences for our understanding of atomic statements which entail second-order generalizations. Why so? Applying existential generalization to a statement of the form "*aRb*" whose first-order predicate picks out a relation yields a statement of the quantified form "$\exists\Phi a\Phi b$". If a higher-order predicate of the form "$a\Phi b$" expresses the higher-order property that a relation has when it applies to *a* first and *b* second then what a statement of the form "$\exists\Phi a\Phi b$" says is that some relation has that property. But in order for existential generalization to have its usual justification this is a property the entailing statement of the form "*aRb*" must already have affirmed of the relation picked out by its first-order predicate. In other words, it's a consequence of the proposed interpretation of higher-order predicates of the form "$a\Phi b$" that a statement of the form "*aRb*" already says that the referent of a first-order predicate has the property of applying to *a* first and *b* second.

It follows that we can test the proposed interpretation of predicates of the form "$a\Phi b$" by checking whether atomic constructions which entail existential generalizations of the form "$\exists\Phi a\Phi b$" can be interpreted as saying that a relation has the property of applying to *a* first and *b* second. I will argue that the proposed interpretation fails this test for both symmetric and non-symmetric atomic constructions.

Since second-order logic permits existential quantification into the positions of symmetric predicates, it follows—assuming the proposed interpretation of higher-order predicates—that atomic statements in which symmetric predicates occur attribute to symmetric relations the property of applying to the things they relate in an order. But it is far from plausible that they do. Consider, for example,

(9) Darius differs from Alexander

and,

(10) Alexander differs from Darius.

If predicates of the form "$a\Phi b$" mean what they're proposed to mean then (9) says that the relation picked out by "ξ differs from ζ" applies to Darius first and Alexander second, whereas (10) says that it applies to Alexander first and Darius second. But, as both linguists and philosophers have reflected, *prima facie* statements like (9) and (10) don't say different things but are distinguished solely by the linguistic arrangement of their terms.[16] So *prima facie* interpreting higher-order predicates of the form "$a\Phi b$" as standing for a property that a relation has if it applies to a first and b second imports ordinal notions—first, second—into the content of atomic constructions expressing symmetric relations, ordinal notions which are alien to our ordinary understanding of statements like (9) and (10).[17]

Second-order logic also permits existential quantification into the positions of non-symmetric predicates. Is it at all realistic to interpret a statement in which a non-symmetric predicate occurs as saying of a non-symmetric relation that it has the property of applying to things it relates in an order? Certainly there is a significant class of non-symmetric constructions, paradigmatically action sentences, in which the arrangement of terms may be felt to depict an order imposed upon the things they pick out. Consider, for example,

(11) Bucephalus kicks Oxyathres

which might be conceived as representing a kind of "energy flow" from the agent (Bucephalus) to the patient (Oxyathres).[18] In this kind of case it is perhaps relatively natural to say that the relation which "ξ kicks ζ" stands for is represented as applying to Bucephalus first and Oxyathres second. But there are what linguists sometimes describe as "static" cases which aren't comfortably described in such terms, for example,

(12) Alexander has lighter hair than Darius,

and,

16 See, for example, (**Langacker1990?**), (**Dowty1991?**) and (**RappaportHovav-Levin2015?**) where it is suggested that (9) and (10) have the same content or have arguments whose roles cannot be distinguished. In the *Principles of Mathematics* Russell famously advocated the view that statements like (9) and (10) express distinct propositions (**Russell1903?**). For a more recent endorsement of this view about symmetric relations, see (**Hochberg1980?**). But Russell had earlier maintained, in his "Fundamental Ideas and Axioms of Mathematics" (**Russell1899?**), that statements like (9) and (10) say the same and would later revert to this view in his *Theory of Knowledge* (**Russell1913?**). See (**MacBride2012a?**) and (**MacBride2018?**) for discussion of Russell's evolving views and (**MacBride2012b?**) for an examination of Hochberg's treatment of relations.

17 A similar point applies to constructions incorporating partially symmetric predicates like "ξ is between ζ and η" where, for example, "Oxyathres is between Alexander and Darius" and "Oxyathres is between Darius and Alexander" *prima facie* differ only by the linguistic arrangement of the terms "Darius" and "Alexander" rather than differing because of the way the relation is described as applying to them.

18 See (**Langacker1990?**).

(13) Alexander is to the left of Darius.

With regard to neither statement does there seem to be a sense in which one participant is described as the "agent" rather the "patient"; neither is identified as the "energetic partner". So there's nothing corresponding to "energy flow" between Alexander and Darius here. Indeed there seems nothing to distinguish Alexander and Darius in how they are described except that they are the things that stand in the relation identified by the predicate—as one thing lighter haired than another, as one thing to the left of another.[19]

Of course, the term "Alexander" occurs first in (12) in the sense that it is the first term that we encounter as readers of English when we scan the sentence from left to right. But it's only an accidental feature of English that we read left to right and it's a further accidental feature that we describe something as being lighter haired than something else by writing its name to the left of the verb. There are actual languages, such as Hebrew or Arabic, as well as possible ones, which don't have these accidental features but different ones.

What is nonetheless essential for depicting states that result from the application of non-symmetric relations, hence common to different languages whose features may otherwise vary, is that for each n-ary predicate in a language there be some rule for assigning a distinguished significance to each occurrence of a term in a closed sentence that results from completing the predicate with terms. In English we employ, for example, the rule that a term which occurs to the left of the predicate "is lighter haired than" in a statement like (12) has the significance of standing for something that is lighter haired than something else which it is the significance of the right-flanking term to stand for. This rule suffices to interpret what (12) says but it doesn't invoke the ordinal notions of "first" and "second" to do so. This shows that it isn't essential for depicting a state that results from the application of a non-symmetric relation that we conceive of the relation as applying to the things it relates to something first and something second—because all that is required to interpret (12) is a rule that settles a distinguished significance for the occurrence of each term and the rule provided does so without invoking "first" and "second". What the rule does is co-ordinate the arrangement of terms in a sentence with the way that the objects corresponding to the terms

19 See (**Huddleston1970?**) and (**MacBride2014?**). Of course, it may be that comprehending these statements a language speaker alights attention upon Alexander and Darius in a given order. But this psychologistic notion of content is evidently different from the objective notion of content at stake which pertains to the content of what is said rather the manner of its grasping.

must be arranged for the sentence to be true. But neither the arrangement of terms, right and left of the verb, nor the arrangement of corresponding objects, lighter-haired to darker-haired, is fundamentally ordinal in character.

Isn't there a straightforward counter to be made to these claims? Surely it is the *raison d'être* of relations to relate things "in an order"—a feature which, for example, distinguishes non-symmetric relations from monadic properties? Constructions like (12) and (13) describe Alexander and Darius as being related by certain non-symmetric relations. Since non-symmetric relations have the distinguishing feature of relating things "in an order", it follows that (12) and (13) describe Alexander and Darius as being related "in an order". So (12) and (13) must presuppose ordinal notions after all!

This counter trades upon the ambiguity of the phrase "in an order", which admits of a weaker and a stronger reading. Once the ambiguity is taken into account it's evident that the conclusion doesn't follow from its premises. The weaker reading of "in that order" is simply that of relating things so that they are arranged one way rather than another—so, for example, that one thing is above another. The stronger reading is that of relating things so that one thing occurs first, the other second. The weaker reading does not imply the stronger reading. From the fact that one thing is above another it doesn't follow that one thing is first, the other second. Note that the weaker reading is consonant with one grammatical use of "order" in ordinary English. When, for example, we describe placing chess pieces in their proper order before the start of a game we don't mean that one piece is placed first, another second. Similarly, when a historian describes how Alexander arranged his men in a certain order before the Battle of Issus this doesn't mean describing which men Alexander put first, second and so on, but rather how he placed the Thessalonian cavalry on the left flank, the Macedonian cavalry on the right flank and so forth.[20] Now we can readily acknowledge that it is the *raison d'être* of non-symmetric relations to relate "in an order" in the weak sense without having to suppose that they do so in the strong sense. We don't thereby compromise our capacity to distinguish non-symmetric relations from properties because properties don't relate the things that bear them in any sense. But if non-symmetric relations only relate "in an order" in a weak sense then it doesn't follow from (12) and (13) describing Alexander and Darius as being related by non-

20 When Defoe described Robinson Crusoe as putting up shelves "to order my Victuals upon", he didn't mean that Crusoe wanted somewhere to arrange coconuts first, ship's biscuits second, mangoes third or anything of the sort (**Defoe1719?**).

symmetric relations that they must also be describing Alexander and Darius as being related first and second, i.e. "in an order" in the strong sense.

Acknowledging order in the weak sense does allow us to admit talk of coming "first" and "second" but only as an eliminable *façon de parler*. So, for example, we can say that Alexander comes first, Darius second in the relation "ξ is to the left of ζ" stands for, meaning by that just that Alexander is to the left of Darius. And we can say that Darius comes first, Alexander second in the relation that "ξ is to the right of ζ" stands for, meaning by that just that Darius is to the right of Alexander. But the notions of "first" and "second" are only defined here relative to the specification of a relation—"first" and "second" relative to the relation that "ξ is to the left of ζ" stands for, "first" and "second" relative to the relation that "ξ is to the right of ζ", and so on. Indeed we might have introduced a different *façon de parler* whereby saying that Darius comes first, Alexander second in the relation "ξ is to the left of ζ" stands for, also just means that Alexander is to the left of Darius. So it doesn't follow from granting order in this weak sense that one thing's being to the left of another makes one thing first or second in some sense that can be expressed without specifying a given relation. So acknowledging order in the weak sense doesn't provide a basis for making sense of one thing coming first, another second in a relation regardless of whether or how the relation is specified, i.e. coming first or second in the absolute sense. And it's order in the strong sense, I've argued, which is required to make sense of objectual quantification into predicate position.

So far we have tested the proposed interpretation of higher-order predicates of the form "$a\Phi b$" by checking whether atomic constructions which entail second-order generalisations of the form "$\exists\Phi a\Phi b$" can be read as saying that a relation has the property of applying to a first and b second (in the strong sense). I've argued that the proposed interpretation fares poorly because neither symmetric constructions ((9) and (10)) nor some non-symmetric atomic constructions ((12) and (13)) plausibly admit of such a reading. Consider now a further consequence of the proposed interpretation of predicates of the form "$a\Phi b$" that if there is a higher-order property of applying to a first and b second (in the strong sense) then any relation can be compared to another with respect to this property.[21] Why should the intelligibility of such comparisons be a consequence of the proposed interpretation? Because if there is such a higher-order property then for any binary relation and two things it relates to

21 See (**MacBride2014?**) and (**MacBride2015?**).

one another, there's a fact of the matter about which of them it applies to first, which second. Hence if any two relations *R* and *S* relate any two things *a* and *b* then there is a fact of the matter about whether (i) *R* and *S* both apply to *a* first, *b* second, or whether (ii) both apply to *a* second and *b* first, or whether (iii) *R* applies to *a* first and *b* second and *S* applies to *a* second and *b* first, or whether (iv) *R* applies to *b* second and *a* first and *S* applies to *a* first and *b* second. But, as I have argued, it isn't part of what we ordinarily mean when we say that one thing has lighter hair than another or that one thing is to the left of another that anything comes first or second (in the absolute sense) in the relations "ξ has lighter hair than ζ" or "ξ is to the left of ζ" stand for. Since coming first or second (in the absolute sense) isn't part of what we ordinarily mean when we use these predicates, it cannot be a further part of what we ordinarily mean that there is a fact of the matter about whether the relations they stand for apply to the same pair of things in the same or a different order.

Accordingly the proposed interpretation of higher-order predicates of the form "$a\Phi b$" fails to mesh with what we mean by what we say using lower-order predicates that serve as arguments to higher-order predicates of this form. If that were what predicates of the form "$a\Phi b$" meant then their application would impose an order (in the strong sense) on the relata of relations. But we have no idea of what the relevance of such an order could be to our ordinary classificatory practices—because our facility with such constructions as (9) and (10) in which "ξ differs from ζ" occurs, or (12) and (13) in which "ξ has lighter hair than ζ" and "ξ is to the left of ζ" occur, don't give a semblance of our relying upon it at all.

This point has significance for the justification of second-order logic itself. Introducing second-order quantifiers brings about a sea change in the expressive capacities of language, so we cannot expect to explain second-order quantifiers before introducing them. So how can we hope to justify the introduction of second-order quantifiers? Williamson maintains that we can account for second-order quantifiers retrospectively by seeking to explain how our understanding of those quantifiers is "rooted in our understanding" of constant predicative expressions of the same category as the quantified variables (**Williamson2013?**). But since we don't understand the predicative expressions in question as standing for relations which apply to the things they relate in an order (in the strong sense) our understanding of second-order quantifiers as ranging over a domain of relations which apply to the things they relate in an order (in the strong sense) can hardly be rooted in our understanding of constant predicative expressions. So we cannot justify the introduction of second-order quantifiers even "retrospectively" if they are interpreted this way.

Might there be an alternative interpretation of higher-order predicates of the form "$a\Phi b$" over which we have more control and which will facilitate an interpretation of second-order quantifiers as ranging over a domain of relations? The ordinary language construction "*–bears—to___*" as it figures in,

(14) Alexander bears a great resemblance to Philip,

might appear to be a promising candidate for a construction in which our understanding of a predicate of the form "$a\Phi b$" might be rooted. Roughly speaking, the idea is that a relation R satisfies the predicate "$a\Phi b$" just in case a bears R to b whereas R satisfies "$b\Phi a$" just in case b bears R to a. Nevertheless the natural language construction "*–bears—to___*" is unsuited to this role.[22]

One obstacle is that "$a\Phi b$" and "*a* bears—*to b*" have different logical forms—hence it is problematic to suppose that our understanding of the one is rooted in the other. The key difference is that whilst "$a\Phi b$" takes a first level predicate as its argument, "*a* bears—*to b*" takes noun phrases rather than predicates in its argument position, for example, the indefinite description "a great resemblance" which occurs in (14). Because they take noun phrases, rather than predicates, constructions like (14) are more naturally formalised in first-order terms as expressing a ternary relation between three first-order entities, one of them a relation. Another difference is that whereas "*a* bears—*to b*" has a converse, *viz*. the passive form "— is borne by *a* to *b*", "$a\Phi b$" does not. Because "$a\Phi b$" and "*a* bears—*to b*" are so logically different, it doesn't follow from the fact that we understand constructions of the form "*a* bears—*to b*"

22 Fine has made the suggestion that a converse relation be conceived as an ordered pair of an underlying neutral relation and an ordering of its argument positions, albeit without endorsing the suggestion because he eschews argument positions (**Fine2000?**). In that case "$a\Phi b$" might be interpreted as standing for a property had by a relation when a figures in its first argument position and b in its second. (Thanks to Jan Plate for pointing out the relevance of Fine's suggestion to the present discussion). But I doubt this proposal fares any better than the interpretation we have been considering. We no more have a grasp of which argument position of (e.g.) the relation picked out by "ξ is to the left of ζ" come first and which second than we have a grasp of which thing the relation applies to first and which second. Moreover, it is just as questionable to suppose that when we understand an atomic construction like (13) we grasp that one of the argument positions (e.g.) *right* figures first in the sequence which constitutes the converse relation in question whilst *left* figures second. Of course there are further, more familiar objections to be raised to invoking argument positions as pieces of our ontology. See (**Fine2000?**), (**Fine2007?**) and (**MacBride2007?**), (**MacBride2014?**) and (**MacBride2020?**).

that we also understand predicates of the form "$a\Phi b$". Nor does it follow that if we don't understand "$a\Phi b$" that we don't understand "*a* bears—*to b*" either.

A further consideration against this proposal is that for a wide range of cases, constructions of the form "*a* bears—*to b*" admit of a deflationary reading in first-order terms.[23] According to this reading, what it means for *a* to bear a relation *R* to *b* is simply that *aRb*. So "*a* bears—*to b*" doesn't furnish a means of understanding how a relation applies independently of the lower order construction to which it reduces when its argument position is completed. In support of this reading witness the equivalence of (14) and

(15) Alexander greatly resembles Philip.

It's not just that (14) entails (15) but the fact that (14) appears to be just a longwinded way of saying what (15) says.

Now it may be acknowledged that there are a limited number of cases in natural language resistant to this deflationary reading, cases where the "bears" construction appears to take quantifier phrases in its argument position, notably

(16) The text bears some relation to the facts

and

(17) The text bears no relation to the facts.

It is arguable that grammatical appearances are misleading here, that in fact there is no genuine quantification over relations going on and really (16) and (17) are more transparently rendered as saying that some of the text is true and none of it is (respectively). Nonetheless even if there is quantification over relations in play in (16) and (17), these statements don't correspond in any straightforward sense with second-order quantificational claims. This is because anything of the form,

(18) $(\exists\Phi)(a\Phi b)$

is a higher-order logical truth and anything of the form,

(19) $\neg(\exists\Phi)(a\Phi b)$

23 See (**MacBride2015?**).

is a higher-order logical falsehood, whereas (16) and (17) are contingent claims. Accordingly *if* (16) and (17) involve genuine quantification, it is more natural to read the constituent quantifiers as first-order. For these reasons, the natural language construction of the form "*a* bears—*to b*" appears unsuited as a basis for understanding what the genuinely higher-order predicate "$a\Phi b$" really means.

8 Conclusion

I have argued that whether mutually converse predicates co-refer or they don't, difficulties arise for the interpretation of higher-order quantifiers as ranging over a domain of relations. If, on the one hand, mutually converse predicates co-refer then the attempt to quantify into the positions they occupy conflicts with the Division of Semantic Labour. If, on the other hand, they pick out distinct relations then we lack a grasp of the higher-order predicates required to characterize relations in a higher-order setting, a grasp which is appropriately rooted in our understanding of atomic statements. We may have other theoretical reasons to hold the metaphysical doctrine that relations apply in an order (in the strong sense) but I have argued that that doctrine isn't credible as a presupposition of higher-order logic.

These arguments don't tell us that second-order quantification *per se* is unintelligible because it remains open that second-order quantifiers may be interpreted along other lines, i.e. other than ranging over a domain in mimicry of the manner in which first-order quantifiers are typically understood to do so. Nevertheless we now have novel and independent reasons to favour alternative interpretations which don't treat second-order existential introduction as a straightforward generalization of first-order existential introduction—whether in terms of quantification over the extensions of predicates, rather than properties and relations conceived as the referents of predicates, or in terms of quantifiers that aren't conceived as having ranges at all.[24] And we now have strong reasons to doubt that second-order logic has a distinguished claim

24 Alternative interpretations vary from Shapiro's relatively conservative proposal that second-order quantifiers range over extensions of predicates conceived as sets to the more radical interpretations inspired by Prior's idea that non-nominal quantifiers lack a range altogether (**Prior1971?**; **MacBride2006?**; **Wright2007?**; and **Sainsbury2018?**). The conclusion of this paper can also be seen as support for Leo's more radical proposal that we require a logic that eschews any kind of artificial ordering altogether (**Leo2014?**; and **Leo2016?**).

to be the logic of relations because of the difficulties that attend quantifying into the positions of converse predicates.

9 References

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Fraser MacBride
University of Manchester
fraser.macbride@manchester.ac.uk

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