dialectica

Vol 76 : 1
March 2022

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doi:10.48106/dial.v76.i1.03

Noah Stemeroff. 2022. "Structural Realism and the Interpretation of Mathematical Structure." *Dialectica* 76(1): 1–1. doi:10.48106/dial.v76.i1.03.

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Structural realists typically appeal to the explanatory and predictive success of science to suggest that the mathematical structure of scientific theory, which is continuous across theory change, provides an accurate description of some aspect of the structure of the world. In this paper, I present a challenge to this claim that concerns how the relevant structure in nature is identified and represented in the context of a physical theory. I argue that the structures, on which many structural realists base the historical support for their position, can only be taken to represent "physical structures" in the context of a broader theoretical framework and that this framework is not necessarily preserved through theory change.

Structural realism holds that science comes closest to comprehending nature, not in its account of its constituents but in its account of its structure (e.g., see Stein 1989, 58). In its epistemic variant, structural realism suggests that scientific knowledge is limited to a structural description of reality. In its metaphysical variant, it defends a radical structural ontology of science. However, in both cases, the structural realist maintains that the significance of successful scientific theories consists in their ability to provide an accurate description of the structure of the world (e.g., see Ladyman and Ross 2007, 92).[1] This structure can be held to be metaphysically basic, or defined over a set of fundamental objects, but in either case, the scientific account of reality is taken to be essentially structural in nature.

1 Here, I take scientific realism, more generally, to be characterized by the belief that our best scientific theories provide a true, or approximately true, description of some aspect of the natural world (i.e., in both its observable and unobservable features).

There are three major motivations for the structural realist position (e.g., see Votsis 2017).[2] The first, historical, motivation is drawn from a particular response to the problem of scientific theory change (Worrall 1989; Ladyman 1998, 2011; Votsis and Schurz 2012). The history of science has shown that science is fallible. Many, if not all, scientific theories of the past are now considered to be false by the standards of modern science, and our current scientific theories will likely suffer the same fate. This pessimistic induction from the history of science is considered to be one of the strongest arguments against scientific realism (for more, see Laudan 1981). In response, many scientific realists have sought to defend selective forms of realism grounded on the portions of scientific theory that are preserved through theory change. The historical motivation for structural realism is based on the apparent continuity in the formal structure of scientific theory through the progress of science.

The second, epistemological, motivation for structural realism is derived from formal studies of the highly abstract nature of modern physics (e.g., Cao 1997; Morganti 2004, 2011; French 2014).[3] Here, it is argued that physics has become, in part, a study of the abstract mathematical structures that are taken to characterize the fundamental features of the natural world. In particular, the essential role that group-theory now plays in modern physics seems to entail that our knowledge of reality can only be determined up to an isomorphism—i.e., a given class of structure (e.g., see Lyre 2004). Thus, scientific knowledge itself may be formally limited to a description of the general structure of reality.

The third, metaphysical, motivation for structural realism takes the epistemological argument a step further (e.g., French and Ladyman 2003, 2011; Lyre 2004; Ladyman and Ross 2007; Esfeld 2013, 2017; Esfeld and Lam 2009, 2011; French 2014). Ontological structural realists argue that modern physics is not only in tension with, but can be taken to present a challenge to, the traditional object-based ontology of classical physics. For instance, the permutation invariance of quantum theory has been taken to directly undermine the individuality of quantum objects. This, along with a host of other examples of underdetermination drawn from both quantum mechanics and general

2 This is not to mention the additional motivation stemming from recent work on the structuralist methodology of science (e.g., Brading and Landry 2006).

3 In this context, it is also important to note the additional epistemic motivation that has come from a renewed interest in Russell's structuralist epistemology (e.g., Votsis 2005).

relativity, suggests that modern physics should be taken to support a theory of structural metaphysics (e.g., see Ladyman and Ross 2007; French 2014).

The epistemological and metaphysical motivations for structural realism have garnered the lion's share of attention in recent debates (e.g., see Bokulich and Bokulich 2011). But this does not mean that scholars have lost sight of the significance of the historical motivation for the position (e.g., Zahar 2007; Ladyman 2011; Vickers 2013, 2019; Saatsi 2019). Indeed, the historical motivation for structuralism continues to support much of the broader interest in the position as a form of scientific realism (i.e., as opposed to an account of the methodology of modern science). Of course, the historical arguments supporting structural realism have not gone unquestioned (e.g., Saatsi 2005; Wright 2017), but many structural realists continue to feel that the historical motivation for structuralism is sound—the cases of structural continuity in the history of science are clear, and the only remaining question concerns how best to understand this continuity (e.g., Ladyman 2011, 2018; French 2014). However, the structural realists' portrayal of the history of science as a progressive series of structural descriptions of reality, and the broader framework that defines this sense of progress, is often taken for granted. This raises the question of what exactly constitutes structural continuity through the progress of science in the first place.

At the outset, it is important to note that a mere formal continuity of structure through theory change, although necessary, is not sufficient to support a viable realist position. The structural realist must demonstrate that the continuous structure of scientific theory represents some aspect of the world—as opposed to simply providing a convenient language to express observable facts (e.g., see Duhem 1991, 151). The retention of structure must mark a sense in which different theories can be said to accurately represent the same reality, at least in some sense (e.g., see Psillos 1995, 1999; Chakravartty 2007). Otherwise, one could easily argue that the retention of structure is simply "a pragmatic feature of scientific practice" (Psillos 1999, 152).

Putnam (1975, 73) famously wrote that scientific realism "is the only philosophy that doesn't make the success of science a miracle." Successful scientific theories explain and predict the outcome of experiments. It is this ability to explain and predict an empirical phenomenon that leads us to conclude that a successful theory provides a true, or approximately true, description of the world. Worrall (1989, 121) suggests that structural realism offers "the best of both worlds" by charting a middle path between Putnam's "no miracles" argument and the pessimistic induction. However, to apply the "no miracles"

doi: 10.48106/dial.v76.i1.03

argument, the structural realist must demonstrate that the structure of scientific theory is, at least in part, responsible for its explanatory and predictive success. The structural realist position would collapse into instrumentalism if the structure of physical theory cannot be said to have some "grip on reality" [@, 418].[4]

To defend the historical motivation for structural realism, structural realists must show that the structure, which is continuous across theory change, can be taken to represent the same structure in nature, and that this structure can account for the relevant physical phenomena. The concern here is to not mistake a continuity of symbolism for a continuity of representation (Cao 2003, 14). The structural realist must not only demonstrate that there exists a continuity in the formalism of physical theory, but also that this continuity entails a continuity of representation. In this context, it is important to note that in order for a structure to account for a physical phenomenon, it must be accompanied by a suitable interpretation. It is the interpretation that correlates a given structure to the natural world.[5] This may be a trivial point, but it represents a non-trivial problem for the structural realist. If a continuity of structure is not sufficient to establish a continuity of representation, then the structural realist must demonstrate that the relevant structure, along with a suitable interpretation, is maintained across theory change.

In what follows, I will argue that this concern presents a challenge to the historical motivation for the structural realist position. This challenge concerns the way in which the mathematical structure of a physical theory is interpreted as a description of the structure of the natural world. In particular, I will argue that the structural realist faces a problem in specifying how a mathematical structure is correlated to nature across the progress of science. To support this criticism, I will present two case studies concerning two of the most prominent articulations of the historical motivation for the structural realist position—i.e., Worrall (1989) and Ladyman and Ross (2007).

Against Worrall's structural realism, I will argue that the mathematical structure, on which he bases his realism, cannot provide a description of the relevant physical structure in nature—at least in the context of an actual experiment—as it requires a theoretical interpretation. To defend this claim,

4 Here, instrumentalism is characterized by the belief that our best scientific theories provide an accurate description of the observable features of the natural world, but nothing more.

5 In structuralist circles, this notion of correlation is typically cashed out in terms of a structural isomorphism or a similar representation relation (e.g., see Pincock 2005, 2012; Brown 2012; Bueno and French 2018).

I will present a detailed re-examination of Worrall's seminal historical case study concerning the transition from Fresnel's optical theory to Maxwell's. Through this case study, I will show that the holistic nature of the interpretation of the mathematical structure of a physical theory threatens to undermine the continuity of structure supporting Worrall's structural realism. The problem is that to correlate a mathematical structure to a physical structure in the world, Worrall's structural realist needs to specify the formal and theoretical framework required to characterize the physical structure and the system of which it is a part. The question is, then, to what extent is this framework maintained, or suitably translated, in the transition to a new theory, and can we still claim that the two theories describe the same physical structure in the world?

In contrast, Ladyman and Ross (2007) do not seem to fall prey to this concern, as their structural realism is based on a more general appeal to the modal structure of reality. On this account, the modal structure of reality is identified with "real patterns" in observational data rather than physical structures in nature. However, against Ladyman and Ross, I will argue that the holistic nature of the interpretation of the mathematical structure of a physical theory may still present a challenge to their modal structural realism in the context of the history of modern particle physics, which is one of the key case studies they take to support their position. Once again, the concern relates to how the abstract mathematical structure of modern physical theory is interpreted as a representation of the modal structure of nature, which they argue is identified in an experiment.

1 Interpreting Mathematical Structures

Ladyman and Ross (2007, 67) define structural realism as "the view that our best scientific theories describe the structure of reality, where this is more than saving the phenomena, but less than providing a true description of the natures of the unobservable entities that cause the phenomena." But what is this "structure" that is "described" by a scientific theory? In the context of the historical argument for structural realism, the structure of reality is often taken to be described by certain aspects of the mathematical formalism of a scientific theory. However, it is not always clear in what sense we should interpret a given mathematical structure as a description of a given structure in nature, or when we should interpret a given mathematical structure as continuous across theory change. On the one hand, it is clear that past and present scientific

doi: 10.48106/dial.v76.i1.03

theory adhered to an entirely different theoretical and experimental practices, not to mention methodologies. On the other hand, the interpretation of the structure of past science must be explained in light of the success of current scientific theory, which is taken to provide an accurate description of the structure of the world—at least approximately. The question is then: how much of the broader framework of modern scientific theory do we need to project back onto past scientific theory to interpret parts of its mathematical structure as a continuous across theory change?

An initial problem relates to the general interpretation of the mathematical formalism of past science. Here, the concern is that we need to specify a formal framework to even determine the meaning of a mathematical structure. This is a fairly general concern—to be applicable, mathematical structures must be definable. We cannot speak coherently of a mathematical structure divorced from the formalism that gives it meaning. Take, for example, a dynamical equation that is taken to provide a description of the evolution of some system, say a ball thrown in the air. It would be meaningless to say that the dynamics of the system can be represented by a solution to a given equation without specifying the underlying formal framework in which the equation is defined. This framework delimits the manifold and metrical structure required to ensure the differential structure of a dynamical equation is well-defined, and the constraints on its domain of application. It is only within the context of this formal framework that the equation can be taken to characterize a path in a geometrical space. It is this path that is actually taken to describe the evolution of the system.[6] However, the successful theories of the past often lacked what we would now consider to be a proper formal framework, and it is not always clear how we should interpret their mathematical structure.

A subsequent problem relates to the manner in which we interpret a given mathematical structure as a description, or representation, of nature. To interpret a mathematical structure in an empirical setting, we need to specify how the structure is to be situated within the context of a physical system or experimental result. The problem is that it is the theoretical framework of a physical theory that is responsible for delimiting its domain of application. Returning to the case of a dynamical equation, it is clear that in order to say that the evolution of the system is characterized by a solution to a given dynamical equation, we need to specify how the equation is to be understood

6 This may be slightly pedantic, but it is important to note that a given dynamical equation may define entirely different paths depending on the mathematical framework in which it is formulated.

in the context of a given empirical setting. It is the theoretical framework of a physical theory, e.g., classical mechanics, that provides an account of the physical space through which a given object moves, the vantage point from which the motion is defined, and the constraints that may be present on the system, as they constitute essential features of the physical context in which the structure is taken to apply. The structure that a given dynamical equation is taken to describe cannot be properly situated or understood outside the theoretical framework of a physical theory. However, the scientific theories of the past were formulated within vastly different theoretical frameworks, and it is, once more, not entirely clear how we interpret their mathematical structure as a representation of a physical system from the perspective of modern science.

These two concerns are closely related. They both pertain to the fact that to ground a realist account of mathematical representation, we must first ensure that the representation is well-defined—i.e., that the relevant mathematics is well-defined and applicable (i.e., interpretable) in a physical setting. To delimit its definition, we must provide a formal framework in which the mathematical structure is defined. To delimit its applicability, we need to specify the physical system in the world that it is taken to represent. This must be done prior to any question of the correlation between a given mathematical structure and nature.

The worry here is that if the formal and theoretical framework of past scientific theory is inconsistent with that of today or not entirely well-defined (from the perspective of modern science), then the structural realist may be forced to project too much of the formalism of modern scientific theory onto past science. Otherwise, it might be impossible for the realist to define the sense in which two seemingly identical equations can be taken to represent the same structure in nature. But the structural realist must be able to identify the sense in which past science, on its own, can be taken to describe the same structure in nature. Otherwise, they run the risk of simply imposing continuity rather than identifying it. In what follows, I argue that these concerns pose a distinct challenge to the historical motivation for structural realism.

2 Worrall, and the Problem of Physical Structure

Worrall suggests that structures are preserved through theory change because they play an essential role in accounting for physical phenomena. For example, he (1989, 116) suggests that there "was an important element of continuity in

doi: 10.48106/dial.v76.i1.03

the shift from Fresnel to Maxwell— and this was much more than a simple question of carrying over the successful *empirical* content into the new theory [...] the continuity is one of *form* or *structure*." The continuity, in this case, is in description of the phenomena of diffraction and the reflection and refraction of polarized light. Worrall continues, "it is no miracle that [Fresnel's] theory enjoyed the empirical predictive success that it did; it is no miracle because Fresnel's theory, as science later saw it, attributed to light the right *structure*" (1989, 117). However, it is not entirely clear how Fresnel's equations actually characterize the structure of light. Worrall assumes that a continuity of mathematical structure entails a continuity of representation—that the two are coextensive (1989, 117–119). However, an argument is needed.

Worrall bases his defence of structural realism on a detailed historical case study concerning the transition from Fresnel's ether-based theory of light to Maxwell's theory of the electromagnetic field. This case study is meant to demonstrate that "if we restrict ourselves to the level of mathematical equations—*not* notice the phenomenal level—there is in fact complete continuity between Fresnel's and Maxwell's theories" (Worrall 1989, 119). Worrall suggests that this continuity in the mathematical structure of scientific theory represents a continuity in the description of the structure of the world (1989, 116). In Worrall's view, this structure was responsible for the empirical success of Fresnel's theory and was retained in the transition to Maxwell's theory of light. However, I will argue that it is not entirely clear that this case study actually supports Worrall's conclusion.

2.1 *Fresnel's Theory of Light*

Fresnel championed the wave theory of light over the corpuscular, or emissionist, ray theory that was dominant at the time. His work on diffraction and the reflection/refraction of polarized light is often credited with bringing about the widespread acceptance of the wave theory of light (Buchwald 1989, 291–310). However, the development and success of Fresnel's mathematical description of light can only be understood within its historical context, as this context determined its interpretation. Worrall's case study focuses on the mathematical structure of Fresnel's theory, but in order to understand how these equations were empirically interpreted, we must first address the specific experiments that Fresnel's equations were taken to describe.

Fresnel's defence of the wave theory of light began with a wave-theoretic account of the phenomenon of diffraction—that is, the bending of light around an obstructing object. The phenomenon of diffraction was first observed by Grimaldi in the seventeenth century. In Fresnel's time, diffraction was easily explained within the context of the ray theory of light, which developed out of Newton's corpuscular theory of light. Newton held that white light is composed of a collection of particles of different size, shape, and velocity. In Newton's view, the size and velocity of the particles accounted for their colour. The primary advantage of the corpuscular theory over the 1991awave theory was the ease through which it accounted for the rectilinear propagation of light. Newton held that the fundamental flaw in the wave theory of light was its failure to account for this fact (Darrigol 2012, 104). Within the corpuscular ray theory, the phenomenon of diffraction was explained by the existence of a localized force in the neighbourhood of the boundary of a diffracting object. This force accounted for the observed inflection of the corpuscles of light at the boundary. In light of the success of the corpuscular ray theory, Fresnel had to show that the wave theory could account for the inflection of light and its rectilinear propagation. Famously, Fresnel was able to account for both diffraction and the rectilinear prorogation of light through an application of Huygens' principle and the principle of interference (Buchwald 1989, 160–162).

Huygens' principle states that each element of a wavefront of light serves as the source of a new outgoing wave. The waveform at any given point in space and time can be determined through the principle of interference. Fresnel simply needed to sum the contributions from each outgoing wave that reaches a given point at a given time. Fresnel's mathematical treatment of diffraction identifies the source of diffraction in the wavefront that passes unimpeded around the diffracting object. He accounted for the interference pattern that is observed in the shadow of a diffracting object by applying Huygens' principle and the principle of interference to sum the outgoing waves from each element of the unobstructed wavefront.

Fresnel was able to integrate over the unobstructed wavefront, and found that the resulting oscillation at any point P beyond the diffracting body is proportional to:

$$\int \cos(\omega t + kz^2)dz, \tag{1}$$

where z is the distance from the source point on the unimpeded wavefront to the point P, ω is the angular frequency, k is the wave number, and t is the

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time (Darrigol 2012, 204). Fresnel defined the amplitude of the wave at the point P to be proportional to $\sqrt{U_c^2 + U_s^2}$, with:

$$U_c = \int \cos(kz^2)dz, \tag{2}$$

$$U_s = \int \sin(kz^2)dz. \tag{3}$$

These equations are collectively known as Fresnel's integrals and constitute the essence of Fresnel's prize-winning paper to the French Academy of Sciences in 1818.[7] If you apply Fresnel's integrals to account for the diffraction of light through a slit in a screen and take the limit as the width of the slit tends to infinity, then you observe that the light's propagation beyond the slit is rectilinear. This is simply due to the effects of destructive interference. This result established the first formal proof of the rectilinear propagation of light within the burgeoning wave theory. In conjunction with Fresnel's mathematical description of diffraction, this result established the formal viability of the wave theory of light.

However, to establish the veridicality of the wave theory, Fresnel had to show that it could successfully account for the observable phenomenon of diffraction. To observe diffraction, we need a source of light, an object or surface, and a screen upon which to cast a shadow. As a simple example, we can consider a variant of the famous diffraction experiment that Poisson devised to test the predictions of Fresnel's prize-winning paper. In this experiment, light is cast on a circular disk, and the diffraction pattern is observed on a screen. Poisson recognized that Fresnel's wave theory of light predicted that

7 In its modern form, the three-dimensional equation states that:

$$U(x,y,z) = \frac{k}{2\pi i}\int_{x_0'}^{x_1'}\int_{y_0'}^{y_1'}\int_{z_0'}^{z_1'} U_1(x',y',z')\frac{e^{ikr}}{r^2}(\hat{n}\cdot\mathbf{r})dx'dy'dz'. \tag{4}$$

Where $U(x,y,z)$ denotes the amplitude of the displacement of the wave at the location (x,y,z) (neglecting polarization for the sake of simplicity), $x_0' \to x_1'$ are the x components, $y_0' \to y_1'$ are the y components, and $z_0' \to z_1'$ are the z components of the wavefront that passes unimpeded around the obstructing object, $U_1(x',y',z')$ is the amplitude of each surface element of the wavefront, and e^{ikr}/r^2 is the amplitude of each propagating wavefront, k is the wavenumber, $\hat{n}$ and $\mathbf{r}$ are the vectors that define the normal of the incoming wavefront and distance to the point under consideration, and $r = \|\mathbf{r}\|$. It is important to note that Kirchoff was the first to provide the formal basis for this mathematical description of diffraction. Before Kirchoff, the mathematics of Huygen's principle was not well-formulated or even well-defined (Buchwald 1989, 188).

a bright spot would appear behind the diffracting disk at the centre of the screen. This central spot was indeed observed, and this experiment served as an important early verification of the wave theory of light.

At this point in the story, everything seems to be going according to plan for Worrall's structural realist. Fresnel's diffraction integrals appear to correctly predict the outcome of this novel experiment, and the reason why could very well be that the mathematical structure of Fresnel's equations accurately describes the structure of light. However, the problem remains to show how Fresnel's diffraction integrals can be interpreted to provide a well-defined representation of the structure of light. Fresnel's integrals are not well-defined when separated from the mathematical framework of Fresnel's optical theory. This framework is required to define the underlying fixed spatial and temporal structure through which wave propagation is defined. This framework is required to not only effectively solve Fresnel's integrals, but to ensure that they actually define the structure of wave propagation—i.e., to interpret the mathematical structure. If the structural realist wants to argue that Fresnel's diffraction equation can account for the structure of light, then the mathematical formalism that is required to interpret Fresnel's integrals must be written out explicitly and included in the set of equations that define Fresnel's account of light. This formalism must then be maintained, or at least suitably translated, in the transition to Maxwell's theory.

In addition, it is not clear whether Fresnel's integrals, on their own, can be interpreted to provide a prediction of the outcome of a diffraction experiment, or to situate the structure of light within it. The integrals only describe light in free propagation, but we never observe light in free propagation; we can only observe light when it interacts with matter. In fact, there is nothing in Fresnel's integrals that makes reference to matter or the condition of observation. Although the integrals are thought to describe the propagation of light and the interference pattern that results from the propagation of the unobstructed wavefront, they cannot take into consideration the actual physical setup of the experiment. What is lacking is an account that serves to correlate the observable outcome of the experiment to the structure of Fresnel's equations—i.e., to show that the observed result is a consequence of this structure. But this would require an account that locates this structure within the experimental setup, which can only be defined by certain aspects of Fresnel's optical theory that account for the initial emission, reflection, and observation of light. To define a continuity of interpretation across theory change, this framework

doi: 10.48106/dial.v76.i1.03

must also be maintained, or at least suitably translated, in the transition to Maxwell's theory.

Unfortunately, the structural realist seems to run into similar problems in the case of Fresnel's equations for the reflection and refraction of light—that is, the reflection of light off of a surface boundary and the bending of light as it passes through the surface boundary into a medium with a higher or lower refractive index. After his initial success with diffraction, Fresnel turned his attention to the newly discovered phenomenon of polarization. Although the phenomena of reflection and refraction had been studied since antiquity, the phenomenon of polarization was first observed by Malus in the early nineteenth century. Initially, the phenomenon of polarization was easily accounted for by the corpuscular theory under the umbrella of the ray theory of light.[8] Ray theorists held that light consists of a bundle of rays and that each light ray possessed an inherent asymmetry—Buchwald (1989, 50–51) uses the analogy of a broom handle with a nail hammered in it (the broom handle represents the direction of the ray and the nail its asymmetry). Ray theory offered a theoretical means to treat polarization as a property of a bundle of rays. The theory suggested that, under normal circumstances, the distribution of the asymmetries in a given bundle of rays is entirely random and unobservable. However, under rare circumstances, they held that the rays with a particular asymmetry could be preferentially selected, thus resulting in a skewed distribution in a given bundle. To the ray theorist, polarization was nothing but a prevalence of a certain asymmetry within a given bundle. The ray bundle theory of polarization successfully explained a number of early polarization experiments. Unfortunately, this all changed with Arago's discovery of chromatic polarization and Fresnel's discovery of rectilinear, circular, and elliptical polarization (Buchwald 1989, 67–85, 222–231).

In contrast to the static ray theory of polarization, Fresnel formulated a dynamical transverse wave theory of light. He suggested that light waves oscillate in time perpendicular to the normal of the wavefront. This dynamical conception of polarization marked a profound reconceptualization of the structure of light. Fresnel's theory took polarization to be a local feature of every element of a wavefront. This meant that Fresnel had to trace the dynamical propagation of every single element of the wavefront in order to

8 Within the ray theory, light was taken to be constituted out of luminous corpuscles that formed rays. The rays were assumed to be countable and were taken to correspond to the ray tracks in geometric optics.

explain the observed behaviour of light. Despite this challenge, Fresnel was able to devise a successful account of the reflection and refraction of light.

Fresnel's equations define the polarization-dependent angle of reflection and refraction at the interface between two transparent substances. To derive these equations, Fresnel made two assumptions. First, he assumed conservation of energy across the surface that defines the boundary between the two substances. Second, he assumed that the amplitude of the transverse polarization is continuous across the surface. Given these conditions, the law of reflection, and Snell's law, Fresnel was able to derive his reflection/refraction equations.[9] Fresnel's equations state:

$$\frac{U_y^{reflected}}{U_y^{Incident}} = \frac{\sin(i-r)}{\sin(i+r)}, \tag{5}$$

$$\frac{U_x^{Reflected}}{U_x^{Incident}} = \frac{\tan(i-r)}{\tan(i+r)}, \tag{6}$$

$$\frac{U_x^{Refracted}}{U_x^{Incident}} = \frac{2\sin(r)\cos(i)}{\sin(i+r)\cos(i-r)}, \tag{7}$$

$$\frac{U_y^{Refracted}}{U_y^{Incident}} = \frac{2\sin(r)\cos(i)}{\sin(i+r)}. \tag{8}$$

Where U denotes the transverse amplitude of the displacement of the light wave at the interface, the subscripts x and y refer to the orthogonal components in the plane of polarization, i refers to the angle of the incident and reflected waves, and r the angle of the refracted wave (both measured relative to the normal to the surface).

To establish the veridicality of the wave theory of polarization, Fresnel had to show that it accounts for the observable phenomena of reflection and refraction. To observe reflection and refraction, we need a source of light, a block of a homogeneous transparent substance (e.g., glass), and two screens—one to observe the reflected light and one to observe the refracted

9 The law of reflection states that the angle of incidence equals the angle of reflection $\theta_{incident} = \theta_{reflected}$, both measured relative to the normal of the surface. Snell's law, or the sine law for refraction, states that $n_1 \sin\theta_1 = n_2 \sin\theta_2$, where n_1 is the index of refraction of the first substance (e.g., air), n_2 is the index of refraction of the second substance (e.g., glass), θ_1 is the angle of the incident light, and θ_2 the angle of the refracted light (both measured relative to the normal to the surface).

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light. Fresnel's equations are able to accurately describe the observed location of the reflected and refracted light in a diffraction experiment.

However, once again, it is not clear whether Fresnel's equations for the reflection and refraction of polarized light, on their own, can be interpreted to provide a well-defined representation of the structure of light. To reiterate, Fresnel's equations for the reflection and refraction of light are not well-defined when separated from the mathematical formalism of Fresnel's optical theory. This framework serves to define the fixed spatial and temporal structure through which the waves propagate, and the very distinction between transverse and longitudinal wave propagation. Just as in the case of diffraction, if the structural realist wants to argue that Fresnel's equations can account for the physical structure of reflection and refraction, then the mathematical formalism that is required to interpret Fresnel's equations must be written out explicitly and shown to be suitably maintained in the transition to Maxwell's theory.

Similarly, it is not clear whether Fresnel's equations, on their own, can be interpreted to provide a prediction of the outcome of a reflection and refraction experiment or situate the structure of light within it. Fresnel's equations only describe the structure of light at the interface between the air and the refractive substance, but that is not what we observe. The structural realist needs to clarify the sense in which this result is due to the structure of the phenomenon that these equations are taken to describe. The problem is that these equations, on their own, are not able to locate this structure in the world. There is nothing in Fresnel's equations that makes reference to the condition of observation, and they cannot take into consideration the actual physical setup of the experiment. What is lacking, once again, is an account that relates the observable outcome of this experiment to the structure of Fresnel's equations—i.e., to show that the observed result is a consequence of this structure. Again, this account must be maintained, or appropriately translated, in the transition to Maxwell's theory.

In response to these concerns, the structural realist might simply acknowledge that the relevant structures require an interpretation to be formally defined and correlated to the appropriate structure in nature. Of course, this will be done differently in each theory, and some features of these interpretations may be abandoned through theory change, but the underlying structure remains and can still be effectively correlated to the relevant phenomena. The problem is that it is not at all clear that Fresnel's equations will describe the same structure in this case. We need to be very careful to not mistake a continuity of symbolism for a continuity of structure. Worrall's structural realist needs to show that despite the change in the theory, the relevant mathematical structure still depicts the same structure in the world.

It is important to note that there is no question here of the instrumental value of Fresnel's equations. The question is whether they can be taken to depict the structure of light, and whether this structure is responsible for the explanatory and predictive success of the theory. Worrall's structural realist needs to show that the explanatory and predictive success of the theory is a consequence, at least partially, of the accurate description of the structure of light. To do this, they need to clarify the sense in which this structure is responsible for the outcome of the relevant experiments—i.e., they need to directly correlate this structure to the relevant observable phenomena. In the case of Fresnel equations, they need to present an account of the structure of the wave propagation throughout the physical system and correlate this structure to experimental observation. This requires that they situate Fresnel's equations within a framework to account for how light interacts with the experimental setup. Once this is done, the question is to what extent this account is maintained, or suitably translated, in the transition to Maxwell and whether we can still claim that the two theories describe the same structure in the world.

2.2 *The Transition to Maxwell's Theory*

It is clear that within Maxwell's theory of the electromagnetic field, one finds equations in the symbolism of Maxwell's theory that appear to be formally similar to Fresnel's equations for the diffraction and reflection and refraction of light. This continuity is not in question. The challenge is to determine whether this continuity is merely a symbolic continuity, or whether it represents a continuity of description. This is a question of the interpretation of the shared mathematical structure of the theories. Worrall's structural realist needs to show that it is the shared structure that is responsible for the shared success of the theories. However, both theories describe the structure of light in terms of a transverse wave equation, and both theories seem to refer to this structure to explain the phenomena of diffraction, reflection, and refraction of light, so there may not be much of a problem. The only worry is that they correlate this structure to observable phenomena in slightly different ways. This concern primarily involves the way in which light is taken to interact with matter.

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Fresnel initially attempted to address the interaction between light and matter through an account of the phenomenon of dispersion—that is, the wavelength-dependent refraction of light. He knew that the effects of dispersion had to be taken into account and that his neglect of dispersion in the case of diffraction and reflection/refraction meant that his results were only approximate in nature (Buchwald 1989, 306–310). In fact, Fresnel put forward an intriguing idea for the development of a dynamical theory of dispersion (e.g., see Buchwald 2012). He suggested that dispersion might be the result of the coarse-grained nature of matter. He assumed that matter is composed of many "atoms" with a certain characteristic spacing. Fresnel thought that each "atom" would place a stress on the ether, which he considered to be an elastic medium. Fresnel suggested that we could use this periodic loading of the ether to account for the phenomena of dispersion. In Fresnel's theory, dispersion was taken to be dependent upon the ratio of the wavelength of light to the characteristic spacing of the "atoms" in a substance.

Sadly, Fresnel passed away at the age of thirty-nine, before he was able to complete an account of the interaction between light and matter (Buchwald 1989, 307–308). However, three years after Fresnel's death, Cauchy took up Fresnel's suggestion for a theory of dispersion. By applying Navier's theory of elastic solids and point-centres of force, he was able to derive a modified wave equation for the propagation of light within a dispersive substance. Cauchy's modified wave equation states:

$$\frac{\partial^2 \eta}{\partial t^2} = \alpha \left(\frac{\partial^2 \eta}{\partial x^2} \right) + \beta \left(\frac{\partial^4 \eta}{\partial x^4} \right) + \gamma \left(\frac{\partial^6 \eta}{\partial x^6} \right) + \dots, \tag{9}$$

where α, β, and γ are constants, η is the displacement of the ether, and x is the direction of propagation [@, 165]. Substituting in the solution $\eta = e^{2\pi i(x-c_1 t)/\lambda}$, Cauchy then solved for the velocity of light in a dispersive medium:

$$c_1^2 = \alpha - \beta(\frac{2\pi}{\lambda})^2 + \gamma(\frac{2\pi}{\lambda})^4 + \dots, \tag{10}$$

where c_1 is the phase velocity of light, and λ is the wavelength of light. This expression shows that in a dispersive medium, the velocity of light is wavelength-dependent, as we would expect. The index of refraction for a dispersive substance is then given as:

$$\mu^2 = \frac{\alpha}{c^2} - \frac{\beta}{c^2}(\frac{2\pi}{\lambda})^2 + \frac{\gamma}{c^2}(\frac{2\pi}{\lambda})^4 + ..., \tag{11}$$

The essential feature of Cauchy's account is the use of a modified wave equation to represent the effects of dispersion. Since Fresnel placed the locus of polarization on the wavefront itself, a structural account of an optical experiment requires that we trace the propagation of a wavefront throughout the experimental setup. Cauchy's modified wave equation would have allowed Fresnel to do exactly that.

The Fresnel-Cauchy theory of dispersion was eventually shown to be fundamentally flawed by the discovery of anomalous dispersion by Leroux in 1862. Leroux observed that a prism filled with iodine gas refracted red light more than blue light. This contradicted the Fresnel-Cauchy theory of dispersion, which predicted that the refractive index increases with the frequency of light. Stokes pointed out that effects of anomalous dispersion could be explained if we simply posit that every substance possesses certain natural frequencies of vibration. He suggested that matter itself is a dynamical system that possesses natural vibratory frequencies that interact with the incident vibrations of light. Stokes also noted that the effects of anomalous dispersion could account for the surface colour of objects.

Maxwell devised a theory of dispersion that took account of the crucial discovery that every substance possesses a set of natural dispersive frequencies [@, 262–265]. Maxwell suggested that material bodies are formed out of an immense number of "atoms," which occupy holes in the ether. He thought that each "atom" consists of a number of shells, where the outermost shell is in contact with the ether. In Maxwell's view, dispersion was a result of the natural vibrational character of the shells within each "atom." The idea is that as light propagates through a material substance, it can set the atoms in motion. Since each "atom" has certain allowable oscillatory frequencies, each frequency represents a natural basis for dispersion.

Maxwell derived a modified wave equation for the propagation of light in a dispersive substance by specifying the kinetic and potential energy of the ether between the "atoms" of a substance. Assuming that the system conserves energy, he was able to derive the equation of motion for light propagation in a dispersive medium. He found that the propagation of light in a dispersive substance with a single natural vibrational frequency is given by the following equation:

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$$\left(1+\frac{\sigma}{\rho}\right)\frac{\partial^2\eta}{\partial t^2}-c^2\left(\frac{\partial^2\eta}{\partial x^2}\right)+\frac{1}{p^2}\left(\frac{\partial^4\eta}{\partial t^4}\right)-\frac{c^2}{p^2}\left(\frac{\partial^4\eta}{\partial x^2\partial t^2}\right)=0, \qquad (12)$$

where η is the displacement of the ether, σ is the mass of the atomic particles per unit volume, ρ is the ethereal density, and p is the vibrational frequency of the "atom" (Whittaker 1951, 263).[10] Assuming that the substance through which the light propagates has a natural frequency of vibration, n, Maxwell found that the index of refraction, μ, in a dispersive substance—within the limits of the visible spectrum—is given as:

$$\mu^2 = 1+\frac{\sigma}{\rho}\left(1+\frac{n^2}{p^2}+\frac{n^4}{p^4}+\ldots\right), \qquad (13)$$

Maxwell then expanded his dispersion relation to allow for a possibly infinite series of natural oscillatory frequencies and determined the refractive index for a substance through the following relation:

$$\mu^2 = 1+\frac{c_1}{p_1^2-n^2}+\frac{c_2}{p_2^2-n^2}+\ldots, \qquad (14)$$

where c_1 refers to the velocity of a light wave of frequency p_1, c_2 to the velocity of a light wave of frequency p_2, and so on.[11] Through this relation, Maxwell was able to determine the dispersion of light in any substance once the natural oscillatory frequencies of the atomic constituents had been found. Maxwell's theory of dispersion was confirmed at the end of the nineteenth century by Rubens (Whittaker 1951, 265).

Comparing Cauchy's and Maxwell's modified wave equations, we can see that Cauchy's structural description of dispersion is not maintained across the transition to Maxwell's theory. This is not a surprise, since Cauchy and Maxwell had a different understanding of the structure of both matter and the ether. To Cauchy, dispersion was a result of the coarse-grained nature of matter, whereas to Maxwell, it was the result of the interaction between the light and the natural oscillatory frequencies of matter. The question is whether this apparent discontinuity poses any real challenge to the structural realist. It is clear that both Cauchy and Maxwell agree that light will satisfy a modified

10 Note that either one of the last two terms in the above relation is sufficient to produce dispersion within the substance.

11 This result is based on an account given by Whittaker (1951). A slightly different account concerning dispersion in prisms is given in Maxwell (n.d.).

wave equation with additional derivative terms accounting for the dispersive nature of the substance. Despite the fact that Cauchy and Maxwell disagree about the nature of the dispersive terms, there is a sense in which they agree as to the nature of dispersion. That is, that dispersion should be represented by a modified wave equation.[12] It seems that this subtle discontinuity should not pose a real challenge to the structural realist.

However, it is important to recall that Worrall argues that the success of Fresnel's theory is due to the fact that he "attributed to light the right *structure*" (1989, 117). Given that both theories defend a wave theory of light in which the key structural features propagate on the wavefront itself, this entails that they must be able to locate this wave structure throughout an experiment to correlate it to the observable outcome. Otherwise, one could easily argue that the equations are of mere instrumental value. The problem is that the observable phenomena, in the case of the diffraction, reflection, and refraction of light, are not correlated directly to the shared wave structure. Due to their differing accounts of the interaction between light and matter, the specific mathematical structures picked out by Fresnel and Maxwell as responsible for the observable phenomena are actually subtly different. The structures picked out by Fresnel and Maxwell differ because of their disparate accounts of the emission and dispersion of the propagating wave. It is not the actual description of the physical structure of light that is continuous, but rather the more general wave-like nature of this structure. In both cases, we are detecting something that has the mark of a transverse wave, but not the same physical structure. However, this may not pose a significant problem for the structural realist, as one could argue that the shared type of structure is responsible for a key part of the explanatory and predictive success of the theory.[13]

In the end, this challenge may be manageable. All that Worrall needs to show is that, from the perspective of modern science, we can continue to reinterpret both Fresnel's and Maxwell's theories to account for the right general structure, instead of a specific physical structure. In this case, it would seem that we can easily mitigate the challenge posed by the holistic nature of the

12 In addition, it can also be shown that Cauchy's formula converges to Maxwell's when n, the natural vibratory frequency of matter, is taken to fall within a specific range (Whittaker 1951, 264).

13 In addition, in the transition from Fresnel to Maxwell, the structural realist may be able to mitigate some of these concerns if they can identify an appropriate notion of approximate structural representation or approximate continuity. However, Saatsi's (2005) appeal to explanatory approximate truth may not be of much help in this case, as it also seems to run afoul of the holistic nature of mathematical representation in the physical sciences.

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interpretation of a mathematical structure in the context of an experiment. The real problem is that this challenge has only become worse with time. The past two centuries have witnessed a dramatic shift in the structural account of matter and the dynamics of light. In particular, the transition to quantum theory and quantum electrodynamics has redefined our fundamental understanding of the constitution of matter and the structure of light. In the transition to quantum theory, there can be no question that there has been a large-scale change in the account of the nature of light, not to mention the interaction between light and matter and the nature of observation. The structure of light depicted by these theories can still be defined in terms of a transverse wave equation, and one can find analogues of Fresnel's equations in many cases. However, the actual structures picked out by these equations are so different, given their theoretical setting, that they simply no longer constitute a description of the same physical structure in the world.

In addition, it is hard to see how the general structure described by these equations could account for any physical phenomena or support a robust realism in this case—at least in anything but a vacuous sense. It is for this reason that Ladyman and Ross (2007) frame their structural realism in terms of an account of modal structure, rather than the structure of a specific phenomenon. But even in this case, as I will argue in the next section, lingering concerns remain about whether they can account for the holistic nature of mathematical representation.

3 Ladyman and Ross, and the Problem of Situating Structures

Ladyman and Ross (2007) provide a compelling structuralist interpretation of the epistemology and metaphysics of modern science. Their structural realism is based on a model-theoretic account of scientific representation. On this view, scientific theories are taken to present a family of formal, i.e., mathematical, models, and these models are assumed to relate to nature through a structural similarity. Specifically, Ladyman and Ross defend a modal structural realism, through which parts of the mathematical structure of successful scientific theories are held to map onto the modal structure of reality.[14] In response to the pessimistic meta-induction, they argue (2007, 123)

14 Here, I should highlight the radical ontological nature of their view. As Ladyman and Ross (2007, 130) characterize it: "Ontic Structural Realism (OSR) is the view that the world has an objective

that the "idea that science describes the objective modal structure of the world is not undermined by theory change in the history of science, since all the well-confirmed modal relations expressed by old theories are approximately recovered in their successors." In addition, to account for the no-miracles argument, they note (2007, 153) that if "science tells us about objective modal relations among the phenomena (both possible and actual), then occasional novel predictive success is not miraculous but to be expected."

In response to the concerns presented in the previous section, Ladyman and Ross can simply acknowledge that the mathematical structure that Worrall highlights fails to map onto the physical structure of light. The problem was Worrall's narrow focus on the physical structure, rather than the modal structure, of nature. This modal structure is defined in terms of their account of "real patterns" in nature (e.g., see Dennett 1991; Ladyman and Ross 2007, 190–257). Following Dennett (1991), a pattern is termed "real" when we can make successful predictions concerning it. These "real patterns" are often identified through data models, which are taken to represent the underlying phenomena. The patterns within these models are real, in this sense, when they can be taken as a basis for predictions.

In the transition from Fresnel to Maxwell, we are no longer concerned with identifying the relevant physical structure that is responsible for the observed phenomena. Rather, we need to explicate the manner in which the patterns in the observable phenomena—i.e., the location of the diffracted, reflected, and refracted light—are accounted for in terms of the relevant modal structure, where for "modal" one could read "nomological" (Ladyman and Ross 2007, 130). The laws governing Fresnel and Maxwell's accounts of diffraction, reflection, and refraction, are, indeed, formally similar. They are expressed through the same mathematical structures, and these structures are both derived through an appeal to a similar set of principles (e.g., the conservation of energy, Huygen's principle, etc.). The remaining question is whether there is sufficient continuity in the broader formal and theoretical framework to actually show that the same modal structure in the world is identified and represented in the transition from Fresnel to Maxwell.

In response to the challenge posed by the holistic nature of the interpretation of mathematical structure in the previous section, Ladyman and Ross can simply accept that, to a certain extent, we need to be more careful to

modal structure that is ontologically fundamental, in the sense of not supervening on the intrinsic properties of a set of individuals."

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articulate the structures that we presuppose in characterizing a mathematical representation. Of course, these presuppositions will certainly be weaker when we are only concerned with representing the general modal structure of reality. Here, we no longer face the challenge of situating a structure in a physical setting, but rather, situating a structure in a set of experimental data. Mathematical modality simply needs to represent the physical modality in a given domain. All that is required, then, is that both the data model and the mathematical structure can be defined within the same basic theoretical formalism. Given that one can formulate both Fresnel's theory and Maxwell's theory within the context of classical mechanics, we can ensure that their mathematical structures are well-defined and can be formally related to one another, and, on the assumption that the data model is well-understood, the structural realist can simply define a mapping between the shared structure and the "real pattern" in nature.[15]

However, this solution does not entirely alleviate the challenge for the structural realist. There remains a concern with how we interpret the data produced in an experiment in the context of the mathematical structure of a physical theory. Recall that a data pattern is termed "real" when it can serve as the basis for successful predictions, but to make a prediction in a novel situation, we need to know the sense in which a given pattern is to be both located in nature and interpreted.

In response to the question of how "real patterns" are located in nature, prior to their representation, Ladyman and Ross (2007, 121) suggest that "[o]ne picks out a real pattern independently of its structural description by an ostensive operation—that is, by 'pointing at it'." But here we should "think of 'pointing' as meaning 'directing a measurement apparatus'." In this context, they (2007, 122) are quick to point out that they "are not suggesting that one begins by locating real patterns and then discovers their structural descriptions." Rather,

> Location is a recursive practice, and generally goes on against the background of some already developed structure. In practice, then, a locator will be a partial interpretation of a structure in the context of another, presupposed, structure. (2007, 122)

15 The hope would be that one could do something similar in the transition to quantum mechanics as well.

Here, Ladyman and Ross suggest that as theory progresses, it specifies the location of a "real pattern" with greater precision within the context of a "presupposed" structure that is developed through the refinement of empirical theory.

This is an important point. In the context of, say, modern particle physics, it is not sufficient to simply state the energy range in which one might encounter some novel structure—i.e., of where a real pattern may be located. To even understand the sense in which a given experiment provides a probe of a certain energy range, one must presuppose a vast theoretical framework to account for the operation of the measurement apparatus. Thus, in modern particle physics, one needs to be careful to clearly specify the relevant theoretical structure that is presupposed and its role in locating and interpreting the "real patterns" in nature. The challenge, in this case, is to delineate the sense in which the broader theoretical and formal framework of particle physics determines which patterns are real, i.e., detectable, and the manner in which they can serve as the basis for prediction. This is essential to both locate a real pattern and correlate it to the nomological structure of the standard model of particle physics.

In defence of their view, Ladyman and Ross (2007, 130–189) specifically appeal to modern particle physics, which they take to not only undermine the individuality of objects required by traditional scientific realism, but to also motivate their formal account of scientific understanding. It is clear, even to a casual observer, that modern particle physics is now largely based on a study of the abstract mathematical structures that characterize the natural world. Indeed, no pursuit better encapsulates the profound structural nature of modern theoretical physics better than the historical development and conceptual foundation of the standard model. Here, elementary "particles" are defined through the group-theoretic structures that characterize their properties. The standard model is a gauge theory—i.e., a theory through which one appeals to local symmetry structures to derive the relevant fields and their interactions. Thus, the nature of reality is described, at its fundamental level, through the structural relations it obeys.

The potential problem is that the standard model of particle physics has been tested in some of the most elaborate experiments ever devised. To even understand the output, i.e., data pattern, of one of the ATLAS detectors at the high-energy particle accelerator at CERN (the European Centre for Nuclear Research), we need to interpret the results within a broad theoretical framework that includes the standard model itself. The data produced from one of

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these detectors is so vast that it cannot possibly be processed. We must discard the overwhelming majority of it by an initial filtering, which is based upon theoretical expectations. This filtering process is guided by the standard model itself. But, more generally, the data itself cannot even be processed until it is "understood" through a theory that defines the detector function. This theoretical framework includes quantum field theory, non-relativistic quantum mechanics, solid state theory, electromagnetic theory, classical mechanics, chemistry, and computational theory—just to name a few. In the context of this disparate and inconsistent theoretical framework, it is not always clear how exactly we should interpret the structure identified by the detector and the recursive theoretical process through which "real patterns" are precisely located and related to the modal structure of the standard model.

In the move from "physical" to "modal" structure (within the "real patterns" account), the hope was that the problem of situating structures "in nature" would be resolved. But situating a structure in a set of data may be no less problematic, and for the very same reasons. Once again, we can only interpret a data pattern within a theoretical and formal framework, and in the transition to a new theory, one will again face the same concerns relating to how we interpret these "real patterns" across inconsistent frameworks. It is not all clear the sense in which a "real pattern" from classical physics, or even non-relativistic particle physics, is approximately maintained in modern high-energy particle physics, given the vast theoretical change and the deeply theory-laden nature of the measurement procedure.

However, the hope may be that the theoretical assumptions that go into the location and interpretation of the data patterns produced by experiments in modern particle physics are so well-grounded, or general enough, that they will likely survive any future theory change—at least as an approximation. Indeed, there is a tradition in the philosophy of physics that has argued for the necessity of theory-laden experimentation, as an essential feature of scientific enquiry (e.g., Stein 1994; Smith 2014; Curiel 2019). In particular, Koberinski and Smeenk (2020) and Koberinski and Smeenk (2020) have brought attention to the fundamental role that the framework of modern quantum field theory plays in the precision tests of the standard model, and the search for new physics. They highlight the importance that this framework plays in securing theoretical continuity in the search for novel phenomena. But these merits presuppose that quantum field theory is itself on the right track—i.e., in the sense that it will be maintained as a low-energy approximation to whatever future theory replaces it. Thus, the continuity required by the modal "real patterns" account of structural realism may be easily secured, but only within the framework of quantum field theory. The concern is that this would pin structural realism (in the context of modern particle physics) to a particular "assumed structure." The modal structural realist would be required to presuppose the framework of quantum field theory to locate real patterns in nature. But this would sit uneasily with the structural realist response to the pessimistic induction on the history of scientific theory change. At the very least, these problems seem to pose a potential challenge to the structural realism of Ladyman and Ross (2007), and its subsequent defence (e.g., Ladyman 2011, 2017, 2018).[16]

In addition, these issues may reach beyond the historical motivation for structural realism, as they bring into question the manner in which the abstract formal structures of modern physics are related to reality, more generally. Although this paper has focused on only two articulations of the historical motivation for structural realism, the assumptions underwriting these positions are shared by a number of other variants of structural realism (e.g., see Frigg and Votsis 2011). The common assumption is that modern physical theory presents us with a family of models, or formal structures, and that the problem of realism can be solved if we can simply specify how these structures map onto nature. This "mapping" or "model-theoretic" account of structural realism (e.g., French 2014) has led to a profound new understanding of the nature of mathematical representation in physics (e.g., Bueno and French 2018), but it has yet to sufficiently articulate how the "structures" in nature are themselves individuated and identified. Thus, the concerns addressed in this paper may pose a general challenge to the modern structural realist, as they may need to pay closer attention to the practice of how the abstract structure of modern physics is related to the reality that it is taken to describe.

In this context, there lies a further problem concerning the consistency of modern science. Here, the issue is that the broader mathematical framework of high-energy particle physics is, itself, not even consistent.[17] The theory currently lacks a well-defined formulation. Given that the definition of a mathematical structure essentially depends on the formalism of a theory, it is unclear whether a mathematical structure within a poorly defined or

16 This later work has served to further refine the metaphysics and justification of the position, but has largely maintained the "real patterns" account under consideration (e.g., see Ladyman 2018, 105).

17 This is not even to mention the manner in which this framework will be maintained in any subsequent theory.

doi: 10.48106/dial.v76.i1.03

inconsistent formalism can be said to represent a structure in nature. It is interesting to note that this is not a problem in quantum field theory alone. A pertinent example from the case study in the previous section is Fresnel's use of a flawed dynamical formalism (e.g., see Kelvin 1904, 424). In this context, it is also important to note that Kirchoff was actually the first to provide the viable formal foundation for Fresnel's diffraction integrals in the late nineteenth century. Before Kirchoff, the mathematics of Huygen's principle was not even well-defined (e.g., see Buchwald 1989, 188). The structural realist can reformulate Fresnel's theory in the context of modern mathematical physics and relate it to a modern reformulation of Maxwell's theory. But this sort of formal inconsistency has been quite common in the history of science—e.g., one could argue that the entire field of mechanics dealt with poorly-defined structures before the calculus was reformulated and finally placed on a rigorous foundation. The concern is that our current physical theories may suffer the same fate, and we may have to concede that our theories will generally fail to specify well-defined structures from the perspective of future science.

4 Conclusion

The structural realist seems to face a challenge in accounting for the holistic nature of the interpretation of the mathematical structure of physical theory. To provide an interpretation of a mathematical structure, we need to specify the theoretical and formal framework required to give it meaning. The problem is that even when structures are maintained, their broader interpretations are often not. The case studies presented in the paper illustrate the need for a more refined structural realism, one that is able to present a viable account of how we interpret and situate the structures of a physical theory.*

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* I would like to thank Joseph Berkovitz, James Robert Brown, Michael Stuart, Stuart Presnell, Michael Miller, and the anonymous reviewers for their helpful comments on earlier drafts of this paper. I also want to thank the Azrieli Foundation and the Dan David Foundation for generously supporting this research.

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