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Introduction to the Special Issue 'The Metaphysics of Relational States'

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Quo Vadis, Metaphysics of Relations?

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A many-faceted beast, the metaphysics of relations can be approached from many angles. One could begin with the various ways in which relational states are expressed in natural language. If a more historical treatment is wanted, one could begin with Plato, Aristotle, or Leibniz.[1] In the following, I will approach the topic by first drawing on Russell's *Principles of Mathematics* (1903) (still a natural-enough starting point), and then turn to a discussion mainly of *positionalism*. The closing section contains an overview of the six contributions to this Special Issue.

1 A Trilemma

Assuming that one goes in for talk of states of affairs (as I shall), the following may be considered a non-negotiable datum (cf., e.g., MacBride 2007, 27):

> D1. The state of affairs that Abelard loves Héloïse is identical with the state of affairs that Héloïse is loved by Abelard.

It also seems *prima facie* hard to deny that

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1 Recent discussions of Plato's views on relations (in a liberal sense) may be found in Scaltsas (2013), Duncombe (2020, chaps. 2–4), and Marmodoro (2021, chap. 6). For Leibniz, see, e.g., Mugnai (2012). Aristotle's *Categories* form the principal starting point for medieval theorizing about relations, on which see, e.g., Martin (2016) and Brower (2018). Two other topics that I shall set aside in this introduction are the debate about realism vs. anti-realism about relations and the internal/external distinction. Introductory discussion of these latter topics can be found in Heil (2009, 2021) and MacBride (2020). For more extensive discussion of Russell's views on relations, see, e.g., Hochberg (1987), Lebens (2017), and MacBride (2018, chap. 8).

D2. ‘Loves’ expresses a relation distinct from the one expressed by ‘is loved by.’

But this last statement might give rise to linguistic qualms; for, given that ‘is loved by’ is not even a complete phrase, it does not look like an appropriate target for the attribution of a semantic value. We can get around this by adopting the notational expedient of λ-expressions. Instead of ‘loves’ and ‘is loved by,’ we might speak of ‘$\lambda x, y\, (x$ loves $y)$’ and ‘$\lambda x, y\, (x$ is loved by $y)$,’ and lay down a semantics of λ-expressions under which $\ulcorner \lambda x, y\, (x\ \varphi\text{s}\ y) \urcorner$ denotes whatever dyadic relation is such that the instantiation of that relation by any entities x and y, in this order, is just the state of affairs that $x\ \varphi$s y.[2] Under such a semantics, ‘$\lambda x, y\, (x$ loves $y)$’ denotes the dyadic relation whose instantiation by any entities x and y (in this order) is the state of affairs that x loves y. Analogously for ‘$\lambda x, y\, (x$ is loved by $y)$,’ which may also be said to denote the *converse* of $\lambda x, y\, (x$ loves $y)$.

Using λ-expressions as names for relations, (D2) becomes:

D2′. The relation $\lambda x, y\, (x$ loves $y)$ is distinct from $\lambda x, y\, (x$ is loved by $y)$.

And this *is* hard to deny. As the argument is both straightforward and tedious, I delegate it to a footnote.[3] (D2) closely reflects what Bertrand Russell implies

2 Here I am *provisionally* taking the locution ‘is an instantiation of … by …, in this order’ as primitive. I also take it to be understood that every instantiation is a state of affairs. The second ellipsis in ‘is an instantiation of … by …, in this order’ is supposed to be filled by a list of two or more arguments, and, relatedly, the ‘and’ in ‘is an instantiation of … by x and y’ should not be read as a term-forming operator but as a delimiter. (Cf. van Inwagen 2006, 461.) Worries about the semantic determinacy of this locution, of the sort raised by Williamson (1985), and concerns about its intelligibility, of the sort raised by van Inwagen (2006), will have to be addressed sooner or later; but for now I will adopt the working hypothesis that they can be answered *somehow*. (For recent discussion of Williamson’s argument, see, e.g., Gaskin and Hill 2012, sec. V; and Trueman 2021, sec. 10.4.2.)

3 By the semantics of λ-expressions adumbrated in the previous paragraph, we have that

(1) The instantiation of $\lambda x, y\, (x$ loves $y)$ by Abelard and Héloïse, in this order, is the state of affairs that Abelard loves Héloïse,

whereas the instantiation of $\lambda x, y\, (x$ is loved by $y)$ by Abelard and Héloïse (again, in this order) is the state of affairs that Abelard is loved by Héloïse. Given that (as seems obvious) the state of affairs that Abelard loves Héloïse is distinct from the state of affairs that Abelard *is loved by* Héloïse, it follows that when he, in his *Principles of Mathematics* (1903), speaks of an "indubitable distinction between *greater* and *less*," adding that

> These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and [what they mean] are certainly relations. (1903, 228)

So far, no problem. (D1) and (D2′) can both be maintained without giving rise to any obvious contradiction. But a problem does arise once we adopt a further assumption, to the effect that

> U. For any two relations R_1 and R_2: any instantiation of R_1 fails to be an instantiation of R_2.

In other words, nothing is an instantiation of two relations. In Kit Fine's seminal "Neutral Relations" (2000), this assumption (formulated using somewhat different terminology) is referred to as 'Uniqueness.' And now—at least assuming that there exists an instantiation of $\lambda x, y$ (x loves y) by Abelard and Héloïse (in this order) as well as an instantiation of $\lambda x, y$ (x is loved by y) by Héloïse and Abelard—we have a problem. For, by the semantics of λ-expressions suggested above, the former instantiation is the state of affairs that Abelard loves Héloïse, just as the latter instantiation is the state of affairs that Héloïse is loved by Abelard. By (D1), these 'two' states of affairs are one and the same. So, by (D2′), we have here a single state of affairs that is an instantiation of two distinct relations. So we have a counter-example to (U). But, at least at first blush, (U) may seem an attractive thesis. For instance, the above-quoted passage from Russell's *Principles* continues as follows:

> Hence if we are to hold that "*a* is greater than *b*" and "*b* is less than *a*" are the same proposition, we shall have to maintain that both *greater* and *less* enter into each of these propositions, **which seems obviously false**; or else we shall have to hold that what really occurs is neither of the two [...]. (1903, 228, boldface emphasis added)

(2) The instantiation of $\lambda x, y$ (x is loved by y) by Abelard and Héloïse, in this order, is *not* the state of affairs that Abelard loves Héloïse.

From (1) and (2) we can conclude, by Leibniz's law, that $\lambda x, y$ (x loves y) is distinct from $\lambda x, y$ (x is loved by y).

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What seems to bother Russell here is (i) the thought that the relation *less* should "enter into" an instantiation of the distinct relation *greater* and (ii) the analogous thought that *greater* should enter into an instantiation of *less*. According to MacBride (2020, sec. 4), adherents of (U) may offer the following motivation (cf. also Fine 2000, 4):

> States are often conceived as complexes of things, properties and relations. They are, so to speak, metaphysical molecules built up from their constituents, so states built up from different things or properties or relations cannot be identical. Hence it cannot be the case that the holding of two distinct relations give rise to the same state. (MacBride 2020, sec. 4)

However, the picture of a relational state (i.e., of an instantiation of a relation) as a "metaphysical molecule," admitting only a *single* way in which such a state is put together from its constituents, can seem slightly naïve or at least under-motivated. A possible way to motivate it may be to hold, on the one hand, that, if one and the same relational state is an instantiation of two relations, then there needs to be some explanation of how this can be (cf. Fine 2000, 15; MacBride 2007, 55; 2014, 4; Ostertag 2019, 1482), and, on the other hand, that it is not easy to see what such an explanation might look like. But this argument will be persuasive only as long as no plausible candidate explanation has been produced. So it seems appropriate to take a skeptical attitude towards (U), as MacBride does at the end of his (2007). More recently, David Liebesman notes that *prima facie* "the motivation for Uniqueness looks suspect" (2014, 412) and that "the intuitions elicited by Fine fail to establish Uniqueness" (2014, 413).

Given that the case for (U) looks fairly weak, and given how blatantly this thesis conflicts with (D1) and (D2′), one may naturally expect that the literature on relations would have come down rather strongly against (U). However, this is not what we find.

In the *Principles*, Russell's way out of the conflict between (U) on the one hand and (D1) and (D2′) on the other was in effect to opt for the denial of (D1). Using Peirce's notation for the converse of a relation, he concluded that "R and $\breve{R}$ must be distinct, and 'aRb implies $b\breve{R}a$' must be a genuine inference"

(1903, 229).[4] This last remark suggests that the state of affairs that Abelard loves Héloïse would on Russell's view be distinct from the state of affairs that Héloïse is loved by Abelard. A decade later, however, we find him endorsing the existence of entities that, following Fine, have become known as *neutral relations*. The text in question is his manuscript on the *Theory of Knowledge* (1984), which is worth quoting from at some length:

> The subject of "sense" in relations is rendered difficult by the fact that the words or symbols by which we express a dual complex always have a time-order or a space-order, and that this order is an essential element in their meaning. When we point out, for example, that "*x* precedes *y*" is different from "*y* precedes *x*", we are making use of the order of *x* and *y* in the two complex symbols by which we symbolize our two complexes. [...] Nevertheless, we decided that there are not two different relations, one called *before* and the other called *after*, but only one relation, for which two words are required because it gives rise to two possible complexes with the same terms. (1984, 86)

A few paragraphs further down, the terms '*before*' and '*after*' are recycled for the purpose of naming two special relations that Russell refers to as *positions*:

> Let us suppose an *a* and a *b* given, and let us suppose it known that *a* is before *b*. Of the two possible complexes, one is realized in this case. Given another case of sequence, between *x* and *y*, how are we to know whether *x* and *y* have the same time-order as *a* and *b*, or the opposite time-order?
>
> To solve this problem, we require the notion of *position* in a complex with respect to the relating relation. With respect to time-sequence, for example, two terms which have the relation of sequence have recognizably two different positions, in the way that makes us call one of them *before* and the other *after*. Thus if, starting from a given sequence, we have recognized the two positions, we can recognize them again in another case of sequence, and say again that the term in one position is *before* while the term in the

4 Peirce introduced the '$\breve{R}$' notation in his "Algebra of Logic" (1880, 50). It has subsequently also been used by Schröder (1895), from whom Russell borrowed it in the *Principles* (1903, 25). That aRb is distinct from $b\breve{R}a$ has also been held by Hochberg (1999, 161; 2000, 47).

> other position is *after*. That is, generalizing, if we are given any relation *R*, there are two relations, both functions of *R*, such that, if *x* and *y* are terms in a dual complex whose relating relation is *R*, *x* will have one of these relations to the complex, while *y* will have the other. The other complex with the same constituents reverses these relations. (1984, 87–88)

In this relatively brief passage, Russell introduces a member of what has become one of the most prominent families of views on the metaphysics of relations, namely *positionalism*. (The term is due to Fine, who coined it in his "Neutral Relations"; but I here use it in a slightly relaxed sense, on which a form of positionalism need not involve a commitment to what Fine calls 'neutral relations.') It has received more or less tacit endorsements by Segelberg (1947, 190), Armstrong (1978, 1997), Williamson (1985), Svenonius (1987, sec. 4), Barwise (1989, 180–181), Grossmann (1992, 57), Paul (2012, 251), Gilmore (2013), and Dixon (2018), among others. Where Russell speaks of 'positions,' these other authors speak in related senses of 'sides,' 'relation places,' 'gaps,' 'empty places,' 'argument places,' 'slots,' 'ends,' or 'pockets' of, or in, a relation.[5] Castañeda (1972, 1975, 1982) attributes a form of positionalism to both Plato and Leibniz.[6] More recently, Francesco Orilia (2008, 2011, 2014, 2019a, 2019b) has defended a form of positionalism under which positions, referred to as 'onto-thematic roles,' are widely shared among relations. These 'roles' are thought of as ontological counterparts of the *thematic roles* known from linguistics.

2 Positionalism

Most of the positionalists just cited conceive of relations as unordered or—using Fine's term—'neutral,' i.e., as not imposing any order on the positions with which the respective relations are associated. (The only clear exceptions seem to be Gilmore and Dixon.) Nor has the appeal of unordered relations been limited to positionalists. The so-called *antipositionalist* views defended by Fine (2000, 2007) and Leo (2008a, 2008b, 2010, 2013, 2014, 2016) also

5 Armstrong uses the term 'relation place' in his (1997, 121–122), but not in his (1978). In the latter work, he instead only speaks of the "roles" that particulars can play in a given "relational situation" (1978, 94). This use of 'role' is similar to that found in Sprigge (1970, 69–70).

6 For some discussion critical of Castañeda's interpretation of Plato, see Scaltsas (2013, 34–35).

conceive of relations as unordered, as does the 'primitivist' view proposed by MacBride (2014).[7]

Let us now look back at (D2′). What would a proponent of unordered relations make of that thesis?

According to Williamson (1985), any relation R is identical with its converse, so that we have the equation '$R = \breve{R}$.'[8] But, he says, in this equation 'R' functions as a singular term, whereas, in 'Rxy,' it instead functions as a *relational expression*, and this is supposed to block the inference from 'Rxy' to '$\breve{R}xy$' which one might otherwise have felt entitled to on the strength of '$R = \breve{R}$.' Crucially, while 'R' "stands for the relation R, this does not exhaust its semantic significance: it stands for R *with a particular convention as to which flanking name corresponds to which gap in R*" (italics in the original). He adds that " 'Ř' as a relational expression uses the opposite convention" (1985, 257). On a certain flat-footed way of applying this treatment to the case of $\lambda x, y\,(x \text{ loves } y)$, one would say that this relation is in fact *identical* with its converse $\lambda x, y\,(x \text{ is loved by } y)$ and that (D2′) is therefore false. But this would be to ignore the stipulatively specified semantics of λ-expressions on which that thesis was based (and with the help of which it was justified in footnote 3). What the Williamsonian positionalist should really say is that (D2′) is not false but *meaningless*, due to a crippling mistake in the underlying semantics of λ-expressions. For under that semantics, " '$\lambda x, y\,(x \text{ loves } y)$' denotes the dyadic relation whose instantiation by any entities x and y (in this order) is the state of affairs that x loves y." To the Williamsonian positionalist, this talk of instantiation can make no sense, because it can make no sense, by his lights, to speak of a *relation* as having an instantiation by some entities x and y in a given order. After all, the Williamsonian positionalist conceives of relations as unordered. Mention to someone a certain unordered relation R, together with some entities x and y and an ordering of x and y: the receiver of this information cannot possibly deduce *which* of the two positions of R (or 'gaps,' in Williamson's terminology) is supposed to be filled with x and which with y. Any information about an ordering of x and y is simply irrelevant. What is needed is not a function from some set of ordinals to x and y, but rather a function from *the set of R's positions* to x and y.[9]

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7 Something like the primitivist view seems to have also been held by Armstrong (1993, 430–431) before he reverted to a form of positionalism in his later book (1997) with the same title.

8 For conformity of notation, I use italics where Williamson uses upright letters.

9 By similar reasoning, it can be seen that Williamson's own definition of 'converse' at the outset of his paper ("for x to have one [of a relation and its converse] to y is for y to have the other to

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We have now encountered one way in which the conflict between (D1), (D2′), and (U) might be resolved while holding onto (U): namely, to treat (D2′) as meaningless. Another option, which does *not* require the positing of unordered relations, would be to deny that relations have converses, so that, e.g., there only exists the relation $\lambda x, y\,(x \text{ loves } y)$ or the relation $\lambda x, y\,(x \text{ is loved by } y)$, but not both.[10] There is also a third way, which requires that 'relation' may be said in at least two ways. Thus it might be thought that, in one of its senses, the term 'relation' applies to unordered relations while, in another sense, it applies to what one might call 'ordered' or (using another phrase coined by Fine) 'biased' relations. One might then go on to suggest that this latter sense is operative in (D2′) and the former in (U). In this way the conflict between the three theses would be resolved through the power of equivocation, as it were, without having to abandon any of the three. But now there arises a question: How exactly should the believer in *un*ordered relations conceive of *ordered* relations? We might be content with thinking of unordered relations as unanalyzable metaphysical whatnots, but the question of how ordered relations come by their peculiar directedness still deserves an answer.

According to one such answer, suggested by Fine, the positionalist might

> think of each biased relation as the result of imposing an order on the argument-places [i.e. positions] of an unbiased relation. Thus, each biased relation may be identified with an ordered pair (R, O) consisting of an unbiased relation R and an ordering O of its argument-places. *Loves*, for example, might be identified with the ordered pair of the neutral amatory relation and the ordering of its argument-places in which *Lover* comes first and

x") must also be considered meaningless by the lights of the Williamsonian positionalist. For it cannot make any more sense to speak of an entity x as 'having' an unordered relation 'to' another entity y than to say that an unordered relation is instantiated by x and y 'in that order.' (It is worth noting that positionalistic tendencies are absent from Williamson's more recent metaphysical work.)

10 For recent discussion of such a view, see Bacon (2023). Bacon adopts a "broadly Fregean picture of properties and relations as unsaturated propositions," which may be thought of "as propositions with holes poked into some of the argument places" (2023, sec. 2). While these "unsaturated propositions" may *prima facie* seem to be properties and *un*ordered relations, Bacon holds that "there is a language independent ordering of the constituents a and b" in a given proposition Rab (2023, sec. 2). The assumption of such a language-independent ordering is also a component of Hochberg's theory of relational facts. For critical discussion of Hochberg's view, see MacBride (2012).

> *Beloved* second; and similarly for *is loved by*, though with the argument-places reversed. (2000, 11, original italics)

If we let $\mathcal{A}$ be the "neutral amatory relation" and understand an "ordering of its argument-places in which *Lover* comes first and *Beloved* second" to be the ordered pair (*Lover*, *Beloved*), then this amounts to the suggestion that the ordered relation *loves* is the ordered pair ($\mathcal{A}$, (*Lover*, *Beloved*)) while its converse is the ordered pair ($\mathcal{A}$, (*Beloved*, *Lover*)). On a common construal of ordered *triples*, one might also put this by saying that *loves* is the ordered triple ($\mathcal{A}$, *Lover*, *Beloved*) while its converse is the ordered triple ($\mathcal{A}$, *Beloved*, *Lover*).

On this proposal, then, ordered relations are certain set-theoretic constructions. Such a proposal is apt to provoke resistance in anyone who is used to conceiving of ordered relations as the objectively determined semantic values of such verbs as 'loves' or 'stabs,' which these latter verbs stand for "without need of philosophical stipulation" (Williamson 1985, 254). It is also apt to provoke resistance in anyone who conceives of relations as "*fundamental* entities, not mere projections onto the world of idiosyncratic facts about human language" [Dorr (2004), 187; emphasis in the original]. However, the thesis that transitive verbs have determinate semantic values, outside of any more or less arbitrary assignment scheme, is a strong assumption that it is not *a priori* easy to see how to defend. And the idea that relations, whatever they are, can only be "fundamental" entities looks far from incontrovertible in light of the fact that it was once not unusual to conceive of relations as mere *entia rationis* (see, e.g., Brower 2018, sec. 5.2).

Once we have reached a point at which we are prepared to take seriously the identification of *loves* with ($\mathcal{A}$, *Lover*, *Beloved*), it becomes natural to ask whether we might not, in the interest of both ontological and ideological parsimony, get rid of unordered relations altogether and take ordered *n*-adic relations to be simply ordered *n*-tuples of *positions*. On this view, *loves* would be the ordered pair (*Lover*, *Beloved*) and its converse would be (*Beloved*, *Lover*). In the case of certain symmetric relations, one might even make do with a single position. Thus the dyadic relation of adjacency might be construed as the ordered pair (*Next*, *Next*).[11] A great advantage of this construction lies in the fact that it immediately reveals this relation to be identical with its converse and thereby offers a satisfying explanation of *why* adjacency is symmetric.

11 Some positionalistically-minded theorists, such as Yi (1999), would regard adjacency not as a relation at all but as a property that has 'plural' bearers. However, cf. Pruss and Rasmussen (2015).

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However, presumably not *every* ordered pair of positions should count as a relation; and it might be argued that here is where unordered relations earn their keep. For instance, it might be thought that the pair (*Lover*, *Giver*) should not count as an ordered relation because there are no states of affairs in which both *Lover* and *Giver* are occupied; and the non-existence of such states may in turn be thought to be due to the putative fact that *Lover* and *Giver* do not belong to the same unordered relation.[12] Thus, more generally, unordered relations may be thought of as organizing positions into groups such that only members of the same group can have occupants in the same states of affairs. But again one might wonder why the work that is thus ascribed to unordered relations cannot be done more cheaply. After all, together with the category of unordered relations, we would need to have in our conceptual inventory a non-symmetric relational notion of 'belonging' that applies to unordered relations and their respective positions. Yet if unordered relations merely serve to 'collect together' certain sets of positions, then why not adopt instead a symmetric notion of *connectedness* that holds directly between positions? Rather than to say that *Lover* and *Beloved* are the only two positions that 'belong' to a certain unordered relation, we might then, for example, say that *Lover* and *Beloved* form a maximal clique of connected positions. Some other options will be mentioned in section 4.

3 The Instantiation Problem

Whether one keeps unordered relations in the picture or not, the task of working out the details of a positionalist theory of relations is not trivial. Above all, the positionalist will have to specify what exactly is required for a given ordered relation to be instantiated by some entities $x_1, \dots, x_n$, in this order. While it may *in principle* be open to the positionalist to leave the concept of *being instantiated by … (in this order)* unanalyzed, this would be profoundly unsatisfactory. After all, on the positionalist view, at least of the sort now under discussion, ordered relations are fairly artificial set-theoretic constructs,

12 In an Orilia-style positionalism, unordered relations also perform a vital additional role in the individuation of relational states. For example, since the relations of *loving* and *admiring* are in Orilia's metaphysic both associated with the roles of *Agent* and *Patient*, there would in his system be no way to distinguish Antony's *loving* Cleopatra from Antony's *admiring* Cleopatra if there did not exist an unordered amatory relation that in some sense 'enters into' the first state but not into the second or an unordered *admiratory* relation that enters into the second state but not the first.

and one would not expect that any metaphysically fundamental notion, other than the 'formal' notions of set-membership and identity (and perhaps mereological notions, if one follows Lewis (1991) in thinking of sets as fusions of singletons), would apply directly to ordered relations, any more than one would expect a set to have mass or charge other than in a derivative sense.[13] Consequently the notion of instantiation, given that it *does* apply directly to ordered relations, would not plausibly be thought of as metaphysically fundamental. What we would like to have, then, is an account of what it takes for a given ordered relation to be instantiated by such-and-such entities in a given order.[14]

Can this *instantiation problem*, to give it a name, be avoided by abjuring (with Williamson, for example) all talk of ordered relations and acknowledging only *un*ordered ones? Strictly speaking, yes. But the believer in unordered relations will then still be faced with the problem—which I shall call the *contribution problem*—of explaining what metaphysical work those unordered relations are supposed to do; and since their only reasonably clear hope for employment lies in contributing to the truth-conditions of relational predications, our theorist will thus be confronted with the task of specifying just what that contribution consists in. For example, someone who posits a 'neutral amatory relation' will need to tell some story, in the terms of her favored metaphysic, of what it takes for it to be the case that Abelard loves Héloïse; and that amatory relation had better play a prominent part in that story. (Or

13 McDaniel (2004, 145) makes a similar point.

14 An argument for the view that the notion of *being instantiated by … (in this order)*—call it '$\mathcal{I}$'—fails to be metaphysically fundamental can also be found in Dorr (2004, sec. 3–4). An important intermediate result that Dorr seeks to establish in the course of his argument is the claim that, if $\mathcal{I}$ *were* fundamental, then the following thesis would be neither metaphysically necessary nor knowable with *a priori* certainty:

> C. For any dyadic relation R_1 there exists a relation R_2 such that, for any x and y: R_1 is instantiated by x and y (in this order) iff R_2 is instantiated by y and x (in this order).

(I have adapted Dorr's thesis to the terminology of the present essay. For the original version, see Dorr 2004, 161.) Dorr thinks that we have good *a priori* reason to think that (C) expresses a metaphysical necessity: if we took it to be possibly false, we would have to expect there to be "spurious structural distinctions between possible worlds" (2004, 167). Hence, in light of the aforementioned intermediate result, we have (according to Dorr) good *a priori* reason to think that $\mathcal{I}$ is not metaphysically fundamental.

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at least, so one may argue.[15]) Moreover, since for it to be the case that Abelard loves Héloïse is patently not the same as for it to be the case that Héloïse loves Abelard, the unordered-relations theorist will need to be able to tell a *different* story of what it takes for it to be the case that Héloïse loves Abelard, or at the very least allow that the relational state of Abelard's loving Héloïse is distinct from that of Héloïse's loving Abelard.

Arguably, however, mere numerical distinctness is not quite sufficient. Consider a 'minimalist' view that takes any two states *Rab* and *Rba* (for distinct a and b) to be merely numerically distinct 'completions' of some unordered relation R: "two indiscernible 'atoms' within the space of states," in Fine's memorable phrase. If such a view were correct, it would be more perspicuous to write '$(R\{a,b\})_1$' and '$(R\{a,b\})_2$' instead of '*Rab*' and '*Rba*,' using the subscripts '1' and '2' as nothing more than arbitrary tags. With the help of this amended notation, the minimalist view can be seen to suffer from the following difficulty: Suppose we have three particulars a, b, and c, giving rise to six possible instantiations of R, namely $(R\{a,b\})_1$, $(R\{a,b\})_2$, $(R\{b,c\})_1$, $(R\{b,c\})_2$, $(R\{a,c\})_1$, and $(R\{a,c\})_2$. Suppose further that, of these six states, only the following three obtain: $(R\{a,b\})_1$, $(R\{b,c\})_1$, and $(R\{a,c\})_2$. Question: Is R transitive on the set $\{a,b,c\}$? There appears to be no fact of the matter, or maybe one should say that the question is ill-posed. In either case, the minimalist has no ready way of capturing the distinction between transitive and non-transitive relations.[16]

How might the Finean antipositionalist address the contribution problem? A crucial feature of antipositionalism, as developed towards the end of "Neutral Relations," is that it conceives of the 'completions' of neutral relations as interrelated by substitution, where the relevant notion of substitution is taken as primitive. Positions and ordered relations do not enter the picture at

15 Put quite simply: If the amatory relation were to play no part in the metaphysics of Abelard's loving Héloïse (or Antony's loving Cleopatra, say), it would *prima facie* be hard to see what point there could be in positing such a relation in the first place.

16 At first blush the view that has here been called 'minimalism' might be thought to be similar to the one recommended at the end of MacBride (2014), which is to the effect that "we should just take the difference between *aRb* and *bRa* as primitive" (2014, 14). However, this identification would be a mistake, for MacBride holds that the difference between *aRb* and *bRa* is not mere numerical distinctness but a difference "which arises from how the constituents of these states are arranged, where how they are arranged is a primitive matter" (2014, 14), and he also explicitly allows that "[s]*ometimes* it may be helpful to appeal to the notion of an *agent* or *patient* to elucidate the distinction between (for instance) *loves* applying one way rather than another" (2014, 15). (Thanks to Fraser MacBride for alerting me to this point and for valuable additional discussion.)

the ground level (as it were) but are rather conceived of as abstractions and set-theoretic constructions. While the antipositionalist is able—unlike the minimalist—to distinguish between transitive and non-transitive relations, she is *un*able to characterize the difference between, say, Abelard's loving Héloïse and Héloïse's loving Abelard without appeal to a reference state, such as as that of Antony's loving Cleopatra (cf. Fine 2000, 29–30). As a result, the antipositionalist is unable to say what it takes for it to be the case that Abelard loves Héloïse *independently* of who else loves whom. This need not by itself constitute a problem. The antipositionalist might maintain that in fact there is nothing interesting to be said in response to the question of what it takes for Abelard to love Héloïse: she might regard Abelard's loving Héloïse as a "basic relational fact (at least in the relevant respect)," as Fine (2007, 62) puts it. However, this view still leaves us in a curious position: plausibly there exist precisely two completions (or *possible* completions) of the neutral amatory relation in which Abelard and Héloïse function as relata. But antipositionalism offers no explanation as to *why* there should be exactly two such completions, rather than only one (as in the case of the adjacency relation), or three, or a hundred. Under antipositionalism, the fact that, for any given pair of distinct entities, there are exactly two completions of the amatory relation with those two entities as relata appears to be effectively treated as brute.[17]

While there is certainly more to be said about antipositionalism, I will have to leave the matter here.

4 Positionalism Developed

Let us now return to the positionalist's instantiation problem, which (as may be recalled) was to provide "an account of what it takes for a given ordered relation to be instantiated by such-and-such entities in a given order." This problem is inseparable from the question of how facts concerning positions—and, where applicable, unordered relations—determine what ordered relations there are. In addition, it is inextricably linked to the positionalist's selection of basic notions and to the question of what role positions play in the individ-

17 Gaskin and Hill (2012) make essentially the same point with regard to the adjacency relation. They also claim, however, that *positionalism* has to "concede that whether a relation is symmetric or not is a brute fact" (2012, 185). This seems to me mistaken; cf. the previous section's example of (*Next*, *Next*). Additional discussion of *anti*positionalism may be found in §IV of Gaskin and Hill's paper, as well as in MacBride (2007, 44–53; 2014, 14). For responses to MacBride, see Fine (2007) and Leo (2014, sec. 6).

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uation of relational states (where a relational state is just an instantiation of a relation). The menu of available options is marked by at least five noteworthy choice points.

Choice point #1: The occupation predicate. Arguably the central notion in the positionalist's ideology is that of *occupation*, which in its simplest form applies to an entity, a position, and a relational state. While more complicated notions of occupation are conceivable, in the following we will only be discussing forms of positionalism that operate with this simple triadic concept, expressed by the predicate 'occupies … in …'.

Choice point #2: Unordered relations. As already noted, positionalists have traditionally assumed that there are such things as unordered or 'neutral' relations with which positions are in some sense associated. However, at least in those forms of positionalism that (unlike the view put forward by Orilia) do not allow for positions to be shared among relations, the only theoretically significant work performed by unordered relations seems to lie in organizing positions into different 'groups,' where the theoretical role of these groups in turn lies in determining what relational states there are. Thus it might be said that it is because *Lover* does not 'belong' to the same unordered relation as *Giver* that there does not exist a state in which Antony occupies *Lover* and Cleopatra occupies *Giver*. To the positionalist who rejects unordered relations, by contrast, it is open to dispense with the concept of an unordered relation as well as with that of 'belonging,' and to work instead with a concept of *connectedness* that applies directly to positions (cf. section 2 above). She will then be able to say that it is simply because *Lover* is not *connected* to *Giver* that there does not exist a state in which Antony occupies *Lover* and Cleopatra occupies *Giver*.[18]

In following this route, the positionalist can further choose among several options. For example, she might assume that connectedness is transitive. But likewise she might hold that it isn't, and allow that there are positions p, q, and r such that p is connected to q and q to r, but p is not connected to r, and that, correspondingly, there exist relational states in which both p and q are

18 An important question that arises at this point is how best to understand this 'because.' (Is there some form of 'metaphysical necessity' afoot? Are we dealing with a case of 'metaphysical grounding'?) According to Dorr (2004, sec. 7), the positionalist is in this connection committed to 'brute necessities,' which Dorr regards as a serious liability of the view. It is not clear, however, that the positionalist is under any pressure to posit 'necessities' rather than merely general truths—such as a principle to the effect that no two (fundamental) positions are occupied in the same state of affairs unless they are connected.

occupied, and also states in which both q and r are occupied, but *no* states in which both p and r are occupied. Another possibility would be to hold that what matters for the question of whether there exists a state in which two given positions p and q are occupied is not whether p and q are *directly* connected but rather whether they are directly *or indirectly* connected, i.e., whether there exist any positions $p_1, \ldots, p_n$ such that (i) $p = p_1$, (ii) $q = p_n$, and (iii) for each i with $1 \leq i < n$, p_i is connected to p_{i+1}. Or again, she might hold that what matters is whether p and q are both members of the same maximal clique of connected positions.

Another interesting option would be to understand *being connected* as a *multigrade* notion, i.e., as a relational concept that can apply to different numbers of arguments. Equipped with such a concept, the positionalist might propose that the question of whether there exists a relational state in which some given positions $p_1, p_2, \ldots$, and no others, are occupied depends on whether $p_1, p_2, \ldots$ are connected, where this is *not* analyzable in terms of whether any two of them are connected.

Choice point #3: Non-obtaining states. The third choice point we have to consider concerns the question of whether to allow for non-obtaining relational states. Let us use the term *state-positivism* for the view that every state of affairs obtains (or in other words: for the view that every state of affairs is a *fact*).[19] According to the state-positivist, there is no distinction to be drawn between obtainment and existence: Abelard loves Héloïse if and only if the state of Abelard's loving Héloïse exists. The state-*anti*positivist, by contrast, will allow that this latter state exists even if Abelard does not love Héloïse.

Choice point #4: Multiply occupiable positions. To see how the positionalist might address the instantiation problem, let us focus on that form of positionalism that (i) employs a simple triadic notion of occupation, (ii) dispenses with unordered relations in favor of a multigrade notion of connectedness, and (iii) rejects state-positivism. On such a view, the question of how facts about positions determine what relations there are may be answered as follows:

19 A corollary of this view is that no state of affairs is a *negation* of another, since in that case both the former and the latter (of which the former is a negation) would have to obtain, which would be absurd. So it might be said that, on this view, every state of affairs is 'positive,' which provides the motivation for the second part of the proposed label (viz., 'positivism'). A concise statement of state-*anti*positivism—i.e., the denial of state-positivism—may be found in Pollock (1967, sec. 2).

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R. An entity x is an (ordered) *relation* iff there exist some positions $p_1, \dots, p_n$ (for some $n > 1$) such that (i) $p_1, \dots, p_n$ are connected and (ii) $x = (p_1, \dots, p_n)$.[20]

It may further be natural to adopt the following uniqueness claim for relational states:

US. For any $n > 1$, any positions $p_1, \dots, p_n$, and any entities $x_1, \dots, x_n$: if $p_1, \dots, p_n$ are connected, then there exists at most one state of affairs s that is such that, for each i with $1 \leq i \leq n$: x_i occupies p_i in s.[21]

However, if the positionalist wishes to allow for positions to be *multiply occupiable*, a weaker claim is needed:

US′. For any $n > 1$, any positions $p_1, \dots, p_n$, and any entities $x_1, \dots, x_n$: if $p_1, \dots, p_n$ are connected, then there exists at most one state of affairs s that is such that, for each i with $1 \leq i \leq n$ and any x: x occupies p_i in s iff $x = x_j$ for some j with $1 \leq j \leq n$ and $p_j = p_i$.

Finally, the instantiation problem may be addressed in two steps. In the first and main step, the positionalist may adopt a thesis that characterizes instantiations of ordered relations:

I1. For any $n, m > 1$, any positions $p_1, \dots, p_n$, any entities $x_1, \dots, x_m$, and any y: y is an *instantiation* of $(p_1, \dots, p_n)$ by $x_1, \dots, x_m$, in this order, iff (i) $m = n$, (ii) $p_1, \dots, p_n$ are connected, and (iii) y is a state of affairs such that, for each i with $1 \leq i \leq n$ and any x: x occupies p_i in y iff $x = x_j$ for some j with $1 \leq j \leq n$ and $p_i = p_j$.

Note that, together with (R) and (US′), it follows from this that any ordered relation has only at most one instantiation by a given sequence of entities. One

20 For simplicity's sake, I will be ignoring the question of how to accommodate infinitary relations.

21 To see the need for the antecedent ("$p_1, \dots, p_n$ are connected"), suppose that there are three positions *Giver*, *Gift*, and *Recipient*, and suppose moreover that these three are connected (in that irreducibly multigrade sense) while *Giver* and *Recipient* are not connected. Thanks to the antecedent, (US) does then *not* have the consequence that, for any entities x_1 and x_2, there exists at most one state of affairs s that is such that x_1 and x_2 respectively occupy in s the positions of *Giver* and *Recipient*.

can now specify what it takes for a given ordered relation to be *instantiated* by some such sequence:

> I2. For any $n > 1$, any ordered relation R, and any entities $x_1, \dots, x_n$: R is *instantiated* by $x_1, \dots, x_n$, in this order, iff there exists an obtaining instantiation of R by $x_1, \dots, x_n$, in this order.

This solves the instantiation problem for the form of positionalism that we have here been considering.

Choice point #5: The place of relations in the world. So far it has been left largely implicit what thesis positionalism amounts to: just what it is that positionalists want us to believe about the world. To remedy this situation, one could employ the concept of a *relational phenomenon*. For present purposes, a relational phenomenon may be understood to be simply any state of affairs that can be felicitously expressed with the help of 'relational' vocabulary—notably, transitive verbs and prepositions, as in 'the cat is on the mat' or 'Abelard loves Héloïse.' Unlike the concept of a relational *state* (i.e., of an instantiation of a relation), the concept of a relational phenomenon is not directly tied to that of a relation. Once we settle on a specific conception of relations, and also clarify the notion of an *instantiation* of a relation, we will have specified what a relational state is; but we will not thereby have specified how relational states relate to relational *phenomena*. Among the options that the positionalist is presented with in this regard, we can usefully identify two extremes, which might be called the *strong* and the *weak* thesis, respectively:

> ST. Every relational phenomenon is a relational state.

> WT. At least one relational phenomenon is 'partially grounded' in a relational state (or the negation of such a state).[22]

22 For present purposes, we may understand a state of affairs s_1 to be *partially grounded in* a state of affairs s_2 iff s_1 obtains and s_2 is a member of the smallest class C that satisfies the following four conditions:

(i) $s_1 \in C$.
(ii) For any $s \in C$ and any state of affairs s': if s is a conjunction of two or more states of affairs, and s' is one of the conjuncts of s, then $s' \in C$.
(iii) For any $s \in C$ and any state of affairs s': if s is a disjunction of two or more states of affairs, and s' is one of the *obtaining* disjuncts of s, then $s' \in C$.
(iv) For any $s \in C$ and any state of affairs s': if s is an existential quantification and s' one of its obtaining instances, then $s' \in C$.

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Of course, neither (ST) nor (WT) by itself amounts to a form of positionalism. However, we obtain a form of positionalism if we combine either (ST) or (WT) with a positionalistic conception of relations and relational states; and one such conception is given by (R) and (I1) above. A form of positionalism that entails (ST) may be called 'strong positionalism,' while a theory that entails only (WT) may be called 'weak positionalism.' Unlike the strong positionalist, the weak positionalist may well deny that the sentence 'Abelard loves Héloïse' expresses a relational state (although she will presumably agree that it expresses a relational *phenomenon*) and, correspondingly, that there exists such a thing as the relation $\lambda x\,(x \text{ loves } y)$. For the sake of the example, however, I will in the following continue to assume that there is such a relation.

On the background of the above solution to the instantiation problem, let us now return one last time to the conflict observed in section 1 between (D1), (D2′), and (U). To recapitulate, (D2′) states that the (ordered) relation $\lambda x, y\,(x \text{ loves } y)$ is distinct from $\lambda x, y\,(x \text{ is loved by } y)$. The positionalist who wishes to analyze relational states like that of Abelard's loving Héloïse in terms of the occupation of two positions *Lover* and *Beloved* will, if she also accepts (R), identify the relations $\lambda x, y\,(x \text{ loves } y)$ and $\lambda x, y\,(x \text{ is loved by } y)$ with, respectively, the ordered pairs (*Lover*, *Beloved*) and (*Beloved*, *Lover*). That these are distinct follows straightforwardly from the assumed distinctness of *Lover* and *Beloved*. So (D2′) holds true. By contrast, (U)—the thesis that nothing is an instantiation of two relations—looks now more questionable than ever. For if one thinks of an ordered relation as an ordered tuple of positions, one will hardly be inclined to think of its instantiations as 'metaphysical molecules' in which it figures as a constituent. But then it becomes difficult to see the intuitive appeal of (U). With (U) accordingly given up, nothing prevents us from accepting (D1), i.e., the thesis that Abelard's loving Héloïse is the same state as that of Héloïse's being loved by Abelard. And indeed, if one identifies $\lambda x, y\,(x \text{ loves } y)$ with (*Lover*, *Beloved*) and $\lambda x, y\,(x \text{ is loved by } y)$ with (*Beloved*, *Lover*), then (D1) can be seen to follow from (US′) and (I1).[23]

Clauses (ii)–(iv) correspond to commonly accepted 'introduction' rules for grounding claims. (Cf. Fine 2012, 58–59) The concept of partial ground thus defined differs from more traditional ones (like Fine's notion of 'strict partial' ground) by the fact that it does *not* require a state of affairs to be distinct from its grounds. This constitutes a simplification that seems, at least for present purposes, to be harmless.

23 In particular, by the semantics of λ-expressions hinted at in section 1, the instantiation of $\lambda x, y\,(x \text{ loves } y)$ by Abelard and Héloïse, in this order, is the state of affairs that Abelard loves Héloïse. Given the identification of $\lambda x, y\,(x \text{ loves } y)$ with (*Lover*, *Beloved*), this same state is, by (US′) and (I1), the unique state in which *Lover* and *Beloved* are only occupied by

5 Potential Objections

Still, it is not all smooth sailing for the positionalist. A first worry is akin to 'Bradley's regress.' As we have seen, the positionalist (at least of the sort considered in this essay) characterizes relational states in terms of what positions are occupied in them by what entities. If now *s* is the state of Abelard's loving Héloïse, shouldn't there also be a further state of affairs to the effect that, in *s*, the position *Lover* is occupied by Abelard—as well as a state of affairs to the effect that the position *Beloved* is in *s* occupied by Héloïse? If the positionalist is to apply her approach to these further states, she has to introduce three additional positions, of *State*, *Occupant*, and *Position*.[24] With their help the state of Abelard's occupying *Lover* in *s*—call it s'—can be characterized as a state in which *s* occupies the position of *State*, *Lover* occupies *Position*, and Abelard occupies *Occupant*. (See figure 1.) But now we seem to have three further states on our hands, one of which may be characterized by saying that s' occupies in it the position of *State*, *s* the position of *Occupant*, and *State* the position of *Position*. And so the regress takes its course.[25] It is not obvious, however, that this regress is vicious. For it is not as if the state of Abelard's loving Héloïse is in any sense *grounded in* (or 'explained by') the fact that Abelard occupies in it the role of *Lover*; rather, the former state is merely (in some suitable sense) "characterized" by the latter. We thus have a "regress of characterization," not of grounding or explanation.

To be sure, the positionalist should presumably allow that

(1) There exists an obtaining state of affairs in which Abelard, and nothing else, occupies *Lover* and in which Héloïse, and nothing else, occupies *Beloved*

is in a certain sense a more perspicuous representation of Abelard's loving Héloïse than the simpler and more familiar 'Abelard loves Héloïse': because

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Abelard and Héloïse, respectively. And by parallel reasoning, this state is also the instantiation of $\lambda x, y\,(x \text{ is loved by } y)$ by Héloïse and Abelard, in this order, and is hence the state of affairs that Héloïse is loved by Abelard.

24 In the following, I will assume that the positionalist has to introduce these positions as primitive posits. An alternative approach (which I will not explore here) might be to construe them as 'abstractions' of some sort, in a sense more or less analogous to lambda-abstraction.

25 Cf. MacBride (2005, 585–586; 2012, 99; 2014, 12). A similar regress has been discussed by Russell (1984, 111–112). Orilia (2014, sec. 9) offers a reply to MacBride in the terms of Orilia's own brand of positionalism. For an introduction to Bradley's regress, see Perovic (2017). Also cf., e.g., Eklund (2019) and Heil (2021, sec. 6).

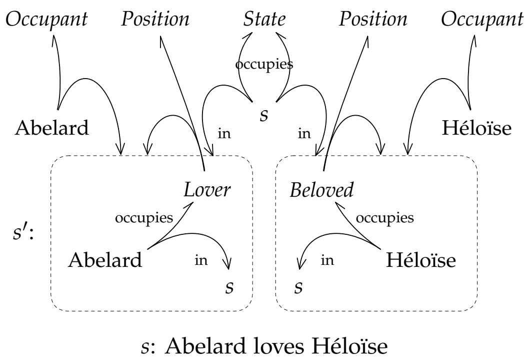

Figure 1: Various states related to Abelard's loving Héloïse. (See text for details.)

(1), but not 'Abelard loves Héloïse,' lets us know about the existence of the two positions of *Lover* and *Beloved*. By the same token, a positionalist who posits the aforementioned positions of *State*, *Occupant*, and *Position* should presumably allow that

(2) There exist three obtaining states of affairs s, s', and s'' such that: (i) s' is the only obtaining state in which s occupies *State* and *Lover* occupies *Position*; (ii) in s', nothing other than s occupies *State*, nothing other than *Lover* occupies *Position*, and only Abelard occupies *Occupant*; (iii) s'' is the only obtaining state in which s occupies *State* and *Beloved* occupies *Position*; and (iv) in s'', nothing other than s occupies *State*, nothing other than *Beloved* occupies *Position*, and only Héloïse occupies *Occupant*

is more perspicuous than (1); but this is *only* because from (2)—and not from (1)—we can infer the existence of those three positions. Hence it is *not* the case that the positionalist has now embarked on some infinite 'regress of perspicuity.' Nor has she embarked on an infinite regress of *analysis*, in the form of some incompletable attempt at providing a metaphysical analysis of the 'occupies … in …' locution. To think that she has would be to presuppose that (2) is put forward as an attempt at such an analysis; but this would be highly uncharitable, given that (2) itself is rife with instances of that locution. The positionalist, at least of the stripe considered here, is 'stuck' with that locution in the same way in which a more traditional proponent of universals is stuck with 'instantiates' or 'is an instantiation of … by ….' But this in itself is not an objection.

So much for potential worries about a vicious regress. In his "Neutral Relations," Fine has raised a number of additional concerns about positionalism. According to one of his objections, positionalism is guilty of "ontological excesses" (2000, 16–17). This objection, however, appears to rest largely on the claim that "surely we would not […] wish to be committed to the existence of argument-places [a.k.a. positions] as the intermediaries through which the exemplification of the relations was effected" (2000, 16–17).

Fine has also maintained that positionalism is unable to accommodate strictly symmetric or multigrade ('variably polyadic') unordered relations (2000, 17, 22), where " [a]n unbiased binary relation R is said to be *strictly symmetric* if its completion by the objects a and b is always the same regardless of the argument-places to which they are assigned" (2000, 17). This claim relies on a special feature of the particular form of positionalism discussed by Fine, namely that no position is ever occupied by more than one entity in the same state. There seems to be nothing incoherent, however, in embracing an alternative form of positionalism that *does* allow for multiple occupancy.[26]

26 Cf. (US′) in the previous section. For an explicit defense of a view that admits multiply occupiable positions, see Orilia (2011) or Dixon (2018). The view that Donnelly (2016) refers to as 'Naïve Positionalism' is also of this kind. The possibility of allowing positions to be multiply occupiable has first (to my knowledge) been considered by Fine (2000, fn10). His celebrated objection to this approach will be discussed in the next section.

It is further worth noting that, by allowing for multiply occupiable positions, the positionalist is (at least in principle) able to address a problem that has been raised by Joop Leo (2008a, 2008b, 2010) for a certain way of "modelling relations." Leo considers a relation $\mathfrak{R}$ "in which $\mathfrak{R}abc$ represents the state that a loves b and b loves c" (2008a, 374). In present terminology, this may be understood as referring to a triadic relation R whose instantiation by any entities x, y, and z (in this order) is the conjunction of x's loving y and y's loving z. At first blush, a positionalistic treatment of this relation requires three positions p_1, p_2, p_3 such that an instantiation of R by any entities x, y, z is the unique state in which p_1 is occupied only by x, p_2 is occupied only by y, and p_3 is occupied only by z. However, as a consequence of this treatment, for any entities a and b, the state $Raba$ is distinct from $Rbab$. This is arguably implausible, for, on an intuitively reasonable, at least moderately coarse-grained conception of relational states, 'both' $Raba$ and $Rbab$ are just the state of affairs that a and b *love each other*. Multiply occupiable positions may be thought to solve this problem. In particular, positing only two positions p_1 and p_2, the positionalist can say that the instantiation of R by any three entities x, y, z is the unique state in

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Admittedly, a positionalist who, contrary to the form of positionalism discussed by Fine, *does not admit any unordered relations* will *a fortiori* not be able to accommodate unordered relations that are strictly symmetric or multigrade. However, the idea that there are strictly symmetric or multigrade unordered relations is less of a datum than a metaphysical hypothesis. A theorist might be drawn to the idea that there are *strictly symmetric* unordered relations because it helps to accommodate certain intuitive identities between relational phenomena, such as the identity of *a*'s being next to *b* with *b*'s being next to *a*. And a theorist might be drawn to the idea that there are *multigrade* unordered relations because it helps to accommodate certain analogies between relational phenomena, such as the analogy between, on the one hand, the state of affairs that *a* and *b* jointly support *c* and, on the other hand, the state of affairs that *a*, *b*, and *c* jointly support *d*. But neither of these considerations constitutes a compelling argument for invoking unordered relations. The first intuition—that *a*'s being next to *b* is the same state of affairs as *b*'s being next to *a*—can be accommodated by adopting a form of positionalism under which *a*'s being next to *b* and *b*'s being next to *a* are 'both' characterized as a state in which a certain position *Next* is occupied by both *a* and *b*. And the intuitive analogy between the state of affairs that *a* and *b* jointly support *c* and the state of affairs that *a*, *b*, and *c* jointly support *d* can be accommodated by positing two connected positions, *Supporter* and *Supportee*, of which at least the first is multiply occupiable (cf. Marmodoro 2021, 173).

which p_1 is occupied only by x and y and in which p_2 is occupied only by y and z. As a result, the state $Raba$ turns out to be the unique state in which both p_1 and p_2 are occupied only by a and b; and exactly the same description is given of $Rbab$. In this way $Raba$ and $Rbab$ come out identical, as desired.

Whether this proposal is ultimately satisfactory is, however, another matter. First of all (though this is not an objection), it is worth noting that the proposal does not sit well with the conception of relations as tuples of positions; instead it appears to favor a conception under which relations are tuples of *sets* of positions. (Thus R might under this proposal be conceived of as the ordered triple $(\{p_1\}, \{p_1, p_2\}, \{p_2\})$, with the previous section's thesis (I1) modified accordingly.) It might also be asked how the proposal can be generalized to higher-adic analogues of Leo's relation. (Thanks to Joop Leo for pressing this point.) For example, let S be the tetradic relation whose instantiation by any entities x, y, z, and w, in this order, is the conjunction of x's loving y, y's loving z, and z's loving w. The positionalist might then postulate two positions q_1 and q_2 such that an instantiation of S by any entities x, y, z, w is a state in which q_1 is occupied only by x, y, and z, while q_2 is occupied only by y, z, and w. On this approach, the state $Sabca$ would be given exactly the same characterization as the distinct state $Sacba$, but this need not be seen as a fatal problem. A more pressing concern would be the question of how to formulate a general principle that would lead to the particular positionalistic treatment of the relations in question.

6 Symmetries

Nonetheless, at least under a sufficiently 'abundant' view as to what (ordered) relations there are, some of them—in particular ones that exhibit a 'cyclical' symmetry—do not easily lend themselves to the positionalist approach.[27] To elaborate this point, we first have to go over some technical preliminaries.

Let us say that a function f is a *symmetry* of an n-adic ordered relation R iff f is a permutation of the set $\{1, \dots, n\}$ such that, for any sequence of entities $x_1, \dots, x_n$ and any y: y is an instantiation of R by $x_1, \dots, x_n$, in this order, iff y is an instantiation of R by $x_{f(1)}, \dots, x_{f(n)}$, in this order.[28] It is easy to verify that, for any n-adic unigrade ordered relation R, the symmetries of R form a *group* with respect to function composition. That is to say, where S_R is the set of R's symmetries, the following three conditions are satisfied:

(i) For any permutations $f, g \in S_R$, S_R also contains the permutation $g \circ f$ that applies g to the result of applying f.
(ii) S_R contains the function id_n that maps each member of $\{1, \dots, n\}$ to itself (and which therefore acts as an identity element within S_R).
(iii) For any permutation $f \in S_R$, S_R also contains the unique permutation g that is such that $f \circ g = g \circ f = id_n$ (i.e., the inverse of f).

This set S_R is also called the *symmetry group* of R.[29] Further, for any group G of functions defined on a common set, let us say that the latter is the *domain* of G. For example, if a given group consists of permutations of the set $\{1, \dots, n\}$ (for some $n > 0$), then this set is the domain of that group.

Consider now an n-adic ordered relation R (for some $n > 2$) whose symmetry group satisfies the following condition:

> C. It contains a permutation f such that, for some k in its domain: (i) $k \neq f(k)$, and (ii) it contains no permutation that merely transposes k and $f(k)$ and maps all other members of the domain to themselves.

27 The previous footnote describes a related difficulty.

28 An adherent of the view that has above been called 'state-positivism' (which rejects non-obtaining states of affairs) might criticize this definition for giving rise to 'spurious symmetries.' For example, if R happens to be uninstantiated, it has no instantiations (by the state-positivist's lights); and as a result any permutation of $\{1, \dots, n\}$ will under the present definition be classified as a symmetry of R. A possible solution would be to insert a 'necessarily' after the 'such that.' Another definition, which also appeals to modal notions, can be found in Svenonius (1987, 37–38).

29 Leo (2008b, 344) speaks in a similar case of 'permutation groups.'

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A well-known example of such a relation is due to Fine (2000, 17, n.10): "the relation R that holds of a, b, c, d when a, b, c, d are arranged in a circle (in that very order)". Fine goes on to say that "the following represent the very same state *s*: (i) *Rabcd*; (ii) *Rbcda*; (iii) *Rcdab*; (iv) *Rdabc*." If this list is supposed to be exhaustive, then the relation in question will have to be understood as a relation of circular arrangement that is either clockwise or counter-clockwise *relative to some vantage point*; for otherwise the state *s* may also be represented as (v) *Rdcba*, (vi) *Rcbad*, (vii) *Rbadc*, and (viii) *Radcb*.[30] Given that Fine specifies neither a vantage point nor a direction (clockwise or counter-clockwise), let us take R to be 'direction invariant' in this latter sense, i.e., so that the state *Rabcd* is identical not only with *Rbcda* (etc.), but also with *Rdcba*. R's symmetry group will then have eight members, which may be respectively represented as (i) id_4, (ii) (1 4 3 2), (iii) (1 3)(2 4), (iv) (1 2 3 4), (v) (1 4)(2 3), (vi) (1 3)(2)(4), (vii) (1 2)(3 4), and (viii) (1)(2 4)(3).[31]

This set is also known as a 'dihedral group of order eight.' To verify that it satisfies (C), it is enough to note that it, on the one hand, contains the permutation (1 4 3 2), which for instance maps 1 to 4, but on the other hand does *not* contain the permutation (1 4)(2)(3) that merely transposes 1 and 4. As Maureen Donnelly (2016, 88–89) points out, relations whose symmetry groups are of this kind—i.e., such as to satisfy (C)—tend to pose a problem for positionalism. More specifically, they pose a problem for the sort of positionalism that operates with a simple triadic occupation predicate and individuates relational states exclusively in terms of what entities occupy in them which positions. To see this, let us focus on the particular form of positionalism that conceives of relations in accordance with the statement (R) in section 4 above, and which conceives of *instantiations* of relations in accordance with the statements (US′) and (I1) in the same section.

To begin with, we can note that the question of what position(s) an entity a occupies in the instantiation of R by some given sequence of entities $x_1, \dots, x_4$ (at least one of which is a itself) depends, apart from R, only on where a

30 For example, if a, b, c, and d are four cups arranged in a circle on a glass table, they might be said to be arranged in the clockwise order a, b, c, d as seen from *above* the table; but, seen from *below* the table, they will appear to be arranged in the clockwise order a, d, c, b. The expressions '*Rabcd*,' '*Rbcda*,' etc., should here be understood in the obvious way as names of instantiations of R.

31 In this representation scheme, non-trivial permutations are represented by their 'orbits.' For example, the permutation (1 3)(2)(4) has three orbits: one consisting of 1 and 3, and the other two consisting of, respectively, 2 and 4. It accordingly transposes 1 and 3 and maps 2 and 4 to themselves.

appears in this sequence.[32] From this it follows that a has to occupy exactly the same position(s) in *Radbc* as it does in *Rabcd*. Further, since the former state is identical with *Rdbca* (as is reflected in the fact that *R*'s symmetry group contains the permutation (1 2 3 4)), it follows that a occupies exactly the same position(s) in *Rdbca* as it does in *Radbc*. Putting the previous two statements together, we have that a occupies the same position(s) in *Rdbca* as it does in *Rabcd*. By analogous reasoning, it can be shown that d occupies the same position(s) in *Rdbca* as it does in *Rabcd*. Hence, the two states *Rabcd* and *Rdbca* cannot differ with respect to which positions are in them respectively occupied by a and d. And clearly they cannot differ, either, with respect to which positions are in them respectively occupied by b and c. Accordingly, since, under the form of positionalism now in question, relational states are characterizable up to uniqueness in terms of what entities occupy in them which positions, it follows that the two states are identical. But they aren't, as is reflected in the fact that *R*'s symmetry group fails to contain the permutation (1 4)(2)(3). So we have a contradiction.

To have a name for this difficulty, let us refer to it as the *symmetry problem*. How might a positionalist respond to it? The first thing to note is that it is not obviously a problem for what has above (in section 4) been called *weak* positionalism. This is because—as has in essence already been pointed out by MacBride (2007, 41)—it is open to the weak positionalist to deny the existence of relations whose symmetry groups satisfy (C).[33] In the particular case of Fine's example, the weak positionalist may maintain that, for any entities a, b, c, and d, the state of affairs that a, b, c, and d, in this order, are arranged in a circle is only a relational phenomenon rather than a relational *state*: in other words, that it is not an instantiation of a relation. (It is compatible with this claim that the state of affairs in question is grounded in, or analyzable in terms of, states of affairs that *are* relational states.) Thus the positionalist may hope to obviate the symmetry problem by retreating to some form of weak positionalism and, with it, to a 'sparse' ontology of relations. Admittedly,

32 More formally: for any entities $x_1, \ldots, x_4$ and $y_1, \ldots, y_4$: if the set $\{i \mid x_i = a\}$ is identical with $\{i \mid y_i = a\}$, then a occupies in $Rx_1x_2x_3x_4$ (i.e., in the instantiation of R by x_1, x_2, x_3, and x_4, in this order) exactly the same position(s) as it does in $Ry_1y_2y_3y_4$. This can be seen to follow from (R) and (I1).

33 In addition, MacBride argues that the positionalist may question whether Fine's relation, "even if it exists, constitutes any kind of counter-example" (2007, 41). However, see Fine's (2007, 59) reply.

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however, this move is not likely to appeal to a theorist who is unwilling to give up the advantages of an abundant ontology of intensional entities.[34]

Alternatively, the positionalist might opt for giving up the assumption that relational states are characterizable up to uniqueness in terms of what entities occupy in them which positions. She might then for instance allow that the states *Rabcd* and *Rdbca*, although distinct, are both such that a, b, c, and d occupy in them one and the same position p. The idea that all four relata thus occupy the same position can be readily motivated by the symmetry of R. This line of thought is not available, however, in the case of Leo's (2008a, 2008b, 2010) example of a triadic relation S whose instantiation by any entities x, y, and z (in this order) is the state of affairs that x loves y and y loves z. Given that this relation is thoroughly non-symmetric—its symmetry group contains only the identity permutation—the positionalist should find it hard to avoid positing three positions p_1, p_2, and p_3 such that, for any x, y, and z, the instantiation of S by x, y, and z (in this order) is a state in which p_1 is occupied only by x, p_2 only by y, and p_3 only by z. But if she follows this approach, she will not be able to accommodate the idea that, for any x and y, the state *Sxyx* is identical with *Syxy*. Plausibly *Sxyx* and *Syxy* are 'both' the state of affairs that x and y love each other, yet on the approach in question, p_2 is in *Sxyx* occupied only by y, while, in *Syxy*, p_2 is occupied only by x.[35]

A very different view has recently been proposed by Donnelly (2016). According to her *relative* positionalism, there exist unordered relations, associated with which there are 'relative properties.' At least from a formal point of view, these relative properties behave much like ordered relations: just as an ordered relation may be instantiated by some entities $x_1, \ldots, x_n$ (in this order), so a relative property may be instantiated by an entity x_1 "relative to" an entity x_2, ..., "relative to" an entity x_n.[36] Relatedly, Donnelly's view is not limited with regard to the symmetry groups it can accommodate; but this flexibility comes at a steep price in ontological commitment. Suppose R is a tetradic ordered relation whose symmetry group contains only id_4. In place of R, the relative positionalist would posit $4! = 24$ different relative properties. A *non*-relative positionalist, by contrast, would only posit four different positions $p_1, \ldots, p_4$. It is true that, given standard set theory, there would then also exist 24 different tuples (p_i, p_j, p_k, p_l) for pairwise distinct

34 MacBride himself (2007, 41) considers the present maneuver unsatisfactory, criticizing it as "insufficiently systematic to really address the concern Fine has raised."

35 For further discussion of this example, see footnote 26 above.

36 See Donnelly (2021) for discussion of how to understand this locution.

$i, j, k, l \in \{1, \ldots, 4\}$; and, as proposed above, these tuples could play the role of ordered relations. But the ontological commitment to these tuples would be a consequence of set theory, given the existence of $p_1, \ldots, p_4$. They would be 'derivative' entities. By contrast, the 24 relative properties posited by the relative positionalist would presumably have to be regarded as ontologically fundamental; for it is not easy to see (and Donnelly doesn't specify) how they might be derived from anything more basic.[37]

7 The Contributions to this Special Issue

Four of the papers of this Special Issue have first been presented at a workshop on "Properties, Relations, and Relational States" that has taken place in Lugano in October 2020.

Scott Dixon presents an extensive defense of what is often called the 'standard view' of relations, or 'directionalism,' against objections recently raised by Maureen Donnelly. A central thesis of directionalism is to the effect that a relation "applies to its relata in an order, proceeding from one to another." Donnelly (2021, 3592) has criticized this conception as "obscure" and as failing "to connect with ordinary thinking about" the semantic difference between such statements as 'Abelard loves Héloïse' and 'Héloïse loves Abelard.' She also argues that directionalism "does not have the right structure to explain the differential application of partly symmetric relations like *between* or *stand clockwise in a circle*" (2021, 3592). Dixon responds to these criticisms and moreover argues that directionalism has advantages over a number of competing views, including Donnelly's own.

Joop Leo describes a new form of positionalism, dubbed 'thin positionalism,' which can be regarded as a middle ground between traditional forms of positionalism on the one hand and antipositionalism on the other.[38] Thin positionalism, like its more traditional counterparts, accords a central place to

37 Further discussion of Donnelly's view can be found in MacBride (2020, sec. 4). In an interesting objection to positionalism that has not so far been discussed, Ralf Bader (2020) considers the "weak betterness relation" R, which is "the disjunction of the symmetric 'equally as good' relation and the asymmetric 'strictly better than' relation" (2020, 37). He holds that, when a and b are equally good, the state Rab is identical with Rba, due to their 'both' being grounded in the fact that a and b are equally good. The positionalist, by contrast, will have to *distinguish* the two states, due to a's (as well as b's) occupying a different position in Rab than in Rba. To avoid this problem, the positionalist may feel compelled to reject Bader's grounding-theoretic way of individuating states of affairs.

38 Cf. Remark 4.1 in his (2014, 272).

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the notion of a position. But positions are here conceived of as "substitutable places in a structure or form." The substitution of entities for such positions yields *relational complexes*, which are also related among each other by substitution relationships. As in Fine's antipositionalism, the relevant notion of substitution is taken as primitive. And, like Fine's antipositionalism, thin positionalism is immune to the symmetry problem discussed in the previous section.

Fraser MacBride argues that quantification into predicate position, as one finds it in second-order logic, cannot be understood as quantification over "relations conveived of as the referents of predicates." He argues for this thesis by constructing a dilemma. On the one hand, if converse predicates—understood as open sentences, such as 'ξ is on top of ζ' and 'ξ is underneath ζ'—co-refer, then we fail to understand the higher-order predicates that are involved in quantification into relational predicate position: predicates (understood, again, as open sentences) such as 'Alexander Φ Bucephalus.' On the other hand, if converse predicates do *not* co-refer, then we can still not make sense of those higher-order predicates unless we "impute implausible readings to lower-order constructions." For instance, even a symmetric predicate, such as 'ξ differs from ζ,' would have to be read as applying to its relata in a given order, which, MacBride argues, would be implausible.

Francesco Orilia offers a sophisticated form of positionalism, dubbed *dualist role positionalism*, that on the one hand embraces very finely individuated 'biased' relations (and their abundant converses) at the 'semantic' level while, on the other hand, rejecting them "at the truthmaker or ontological level of sparse attributes." At this more fundamental level, Orilia allows only *neutral* relations, whose exemplification he conceives of as being mediated through 'roles' such as *agent* and *patient* or *inferior* and *superior*. For instance, where V is a neutral relation of vertical alignment with respect to the Earth's surface, Orilia would write (in boldface) 'V(superior(a), inferior(b))' to represent the state of affairs of a plane a's being above a bird b.

MacBride and Orilia, in their joint contribution, respond to van Inwagen's (2006) argument for the conclusion that we do not have any "formal and systematic" names for non-symmetric relations. They concede the plausibility of supposing that, if non-symmetric relations had distinct converses, then it would be impossible to introduce such names for them. But they do not follow van Inwagen in holding that non-symmetric relations *do* have distinct converses. They point out that there are alternative conceptions of non-symmetric relations under which the existence of distinct converses—and hence the conclusion of van Inwagen's argument—can be avoided. And they moreover argue, *contra* van Inwagen, that it is possible (either in English or a modest extension of English) to introduce names for non-symmetric relations of an adicity greater than 2.

Finally, Edward Zalta replies to two papers by MacBride. More specifically, he replies (i) to MacBride's argument, in his contribution to the present issue, for the conclusion that second-order quantifiers cannot be interpreted as ranging over relations and (ii) to the argument in MacBride (2014) for the conclusion that (as Zalta puts it) "unwelcome consequences arise if relations and relatedness are *analyzed* rather than taken as *primitive*" (emphases in the original). Both arguments are examined in the light of Zalta's theory of relations, as developed in the context of his object theory.[39] The resources of this theory are brought to bear on the individuation of states of affairs, an issue which Zalta identifies as central to both of MacBride's arguments.

As I hope can be seen from this brief overview, the metaphysics of relations and relational states continues to be a fertile field of inquiry.[*]

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39 On the latter, cf. Zalta (1983, 1988, 1993).

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