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Traditionally, we distinguish between relations and their converses, e.g., *above* and *below* or *before* and *after*. This distinction poses a dilemma. Is a relation really distinct from its converse or are they one and the same? There are contrasting arguments that favor one or the other reply, both of them in Russell, who first opted for the former (in *Principles of Mathematics*) and then for the latter (in *Theory of Knowledge*). Since then, accounts of relations that side with one or the other option have flourished. A hybrid approach to properties and relations (attributes), according to which there are both sparse and abundant attributes, is here offered as a way out of the dilemma: distinct converses are acknowledged at the semantic or propositional level of abundant attributes, and rejected at the truthmaker or ontological level of sparse attributes. A positionalist account of relations is also adopted, *role positionalism*, according to which positions are understood as *roles*, which are ontological or semantic counterparts of the thematic roles invoked in linguistics. In this way, distinct abundant converses differ because of the different roles involved in them, but they are intimately connected in that they correspond to a single sparse relation.

Traditionally, we distinguish between relations and their converses, e.g., *above* and *below*, *before* and *after*, *giving* and *receiving*. This poses a dilemma. Is a relation really distinct from its converse or are they one and the same? To put it otherwise: should we admit, *pro-converses option*, that relations have distinct converses, or should we rather, *anti-converses option*, deny that? There are two contrasting arguments that favor one or the other alternative. Both of them can be found in Russell's *Principles of Mathematics* (POM, 1903). One is a *semantic* argument; in a nutshell, pairs of *converse predicates* such as "is above" and "is below," appear to have different meanings and thus must stand for distinct relations. The other is an *ontic* argument; if, e.g., an airplane flies over a bird, even though at some point we can describe how they are mutually

situated with two different converse predicates, “the airplane is above the bird” or “the bird is below the airplane,” surely there is just one relational state of affairs or fact that we are describing, which suggests that only one relation is involved.

In *POM*, Russell privileged the semantic argument and thus opted for the pro-converses option. He did this by buying a *directionalist* approach to relations, the *standard view*, according to Fine (2000). Later on, however, in the 1913 manuscript *Theory of Knowledge (TK, 1984)*, he came to privilege the ontic argument and shifted to the anti-converses option. He thus endorsed a *positionalist* account of relations. Since then, many philosophers have opted for one of the options while rejecting the other. Followers of the pro-converses option include Grossmann (1983), Wilson (1995), van Inwagen (2006). Moreover, this route seems implicit in first-order logic with its standard model-theoretic semantics, where relational predicates are interpreted as sets of ordered sets. Among the supporters of the anti-converses option, there are Castañeda (1975), Williamson (1985), Hochberg (1987), Fine (2000), Dorr (2004), MacBride (2014), Paolini Paoletti (2021b). I myself have defended an account that seems to leave no room for converse relations (Orilia 2008, 2011, 2014).

However, both the semantic and the ontic arguments make reasonable demands on a theory of relations, and thus these “exclusivist” approaches do not fully release the tension that the dilemma generates. I shall thus offer a way out that tries to do justice to both of its horns. Following Bealer (1982) and Lewis (1983, 1986), it is common to distinguish between a *sparse* and an *abundant* conception of properties and relations (in short, *attributes*) (see Orilia and Paolini Paoletti 2020, sec. 3.2). The way out takes advantage of a *dualist* view, which admits both sparse and abundant attributes. In essence, at the ontological, or truthmaker, level, where attributes are sparse, there are no distinct converses, whereas at the semantic, or propositional, level, where attributes are abundant, there are distinct converses. At both levels the proposed approach is *role positionalist*, that is, it takes positions to be *roles*, such as *agent*, *patient*, *source*, *destination*, *location*, etc. [whether *o-roles* or *c-roles*, as we shall see; Orilia (2010), 6]. The motivations for, and the implications of, this move will be clarified in the following.

Here is a preview of the paper. In section 1, I consider the two arguments offered by Russell in *POM* and briefly illustrate the *directionalist* approach of *POM* and the *positionalist* approach of *TK*. In section 2, I focus on the ontic argument and show how it can be accommodated at the truthmaker level

by a role positionalism that buys the anti-converses option. In section 3, I elaborate on the semantic argument and show how we can do justice to it by invoking abundant relations with a role positionalism that makes room for the pro-converses option. In section 4, I discuss how sparse and abundant attributes can co-exist in a dualist view of attributes that reconciles the pro-converses and the anti-converses options. In section 5, I briefly consider some possible objections and close the paper.

1 Russell's Two Arguments, Directionalism and Positionalism

In *POM*, Russell hints at the ontic argument that later will lead him to positionalism, but he sets it aside, while giving greater weight to the semantic argument, based on the different meanings of pairs of converse predicates such as “greater” and “less.” Here is the relevant passage (in 1903, sec. 219):

It may be said that, owing to the exigencies of speech and writing, we are compelled to mention either *a* or *b* first, and that this gives a seeming difference between “*a* is greater than *b*” and “*b* is less than *a*”; but that, [*ontic argument*] in reality, these two propositions are identical. But [*semantic argument*] if we take this view we shall find it hard to explain the indubitable distinction between *greater* and *less*. These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and are certainly relations.

In an effort to accommodate the semantic argument, in *POM* Russell develops an approach according to which relations have an intrinsic *sense* or *direction*.¹ It can thus be aptly called *directionalism*. Russell (*POM*, 1903, sec. 94) puts it thus: “it is characteristic of a relation of two terms that it proceeds, so to speak, *from one to the other*. This is what may be called the *sense* of the relation [...]” The idea is that, since relations are endowed with a sense or direction, they are exemplified by *relata* as given in an appropriate order. And there can be relations that differ from one another merely in their direction and otherwise have, one might suggest, an identical *content* (Fine 2000, 11); such relations are mutual converses. In this way, Russell makes room for the

¹ Russell uses the term “direction” in *TK* but not in *POM*, as far as I can tell.

pro-converses option. For example, *above* and *below* differ merely in their respective directions, say d_1 and d_2 , and otherwise have the same content, say C . Hence, we could represent them as " C_{d_1} " and " C_{d_2} ," respectively. They are such that, necessarily, if C_{d_1} is exemplified by two objects in a certain order, then C_{d_2} is exemplified by the same objects in the opposite order. In effect, the approach is telling us that a relation is exemplified not simply by some objects but by an ordered set of objects (Castañeda 1975, 239).

To illustrate, suppose the airplane, a , is flying over the bird, b , so that the following is true:

(1) a is above b .

In this case, there is a fact consisting of the relation *above* proceeding from the airplane to the bird, i.e., the relation C_{d_1} , exemplified by an ordered set with the airplane and the bird, in that order, as members:

(1#) $C_{d_1}\langle \mathbf{a}, \mathbf{b} \rangle$,

and there is also another fact, conveyable by

(1') b is below a ,

consisting of the relation *below* proceeding from the bird to the airplane, i.e., the relation C_{d_2} exemplified by a different ordered set, with the airplane and the bird in the opposite order as members:

(1'#) $C_{d_2}\langle \mathbf{b}, \mathbf{a} \rangle$.

Here I have used boldface fonts to highlight the intention to represent a state of affairs, or more generally, an entity at the ontological level of truthmakers.² When deemed useful, I shall follow this convention in the following as well.

Directionalism presents an *ontic* hurdle, we may say, for it is of course very hard to make sense of the idea that objects are exemplified in an order (van Inwagen 2006; MacBride 2020, sec. 1). In *TK*, however, Russell abandons directionalism not so much for this hurdle but because he comes to privilege

2 I am assuming there are both propositions and states of affairs (or facts), with true propositions made true by states of affairs and false propositions lacking a corresponding state of affairs. In *POM*, Russell does not distinguish between states of affairs and propositions and takes the distinction between true and false propositions as indefinable. Hence, from his *POM* perspective, we should say that (1#) and (1'#) are two true propositions rather than two states of affairs. However, we can neglect this for present purposes.

the ontic argument while downplaying the semantic argument (1984, 84). As we can see from the above example, by distinguishing converses via directions, directionalism invites us to assume that there are two distinct facts, (1#) and (1' #), where we should think there is only one fact. To avoid this multiplication of facts, Russell comes to favor positionalism, in which relations have no intrinsic directions, thereby leaving no room for distinguishing converses in the way directionalism does. In Fine's (2000, 10–11) terminology, positionalist relations are “neutral or unbiased” and, correlatively, the relations with sense of directionalism are “biased.” However, such neutral relations are exemplified in different ways by relata, depending on the different “positions,” or “argument-places,” that the relata have with respect to the relation. For example, (1) and (1') are different representations of one and the same fact consisting of a neutral relation, N, jointly exemplified by the airplane and the bird, in such a way that the former has one position, say P_1 , with respect to the relation, and the latter has another position, say P_2 . In contrast, if it were the bird to be above the airplane, N would be exemplified by the airplane and the bird in such a way that the former would have position P_2 and the latter position P_1 . Fine (2000, 11) puts it as follows: “Exemplification must be understood to be relative to an assignment of objects to argument-places,” and also suggests that we can view positions as holes of different shapes and exemplification with respect to positions, or assignment to argument-places, as the filling of such holes by relata; in *TK*, Russell proposes a different picture in terms of the hooks and eyes of goods-trucks (1984, 86). Useful as these metaphors may be for illustrative purposes, they must be ultimately set aside in favor of a more precise characterization of what exemplification of a relation with respect to a position amounts to. We shall deal with this in the next section. For the time being, let us follow the hole metaphor and assume that in our case the holes are [] and (), with the airplane filling the former and the bird the latter. Then, the unique fact represented by both (1) and (1') can be represented thus:

(1c) **N(a)[b]**.

This fact exists if (1) and (1') are true. The writing order in this approach should not be taken to convey any information. Thus, (1c) and

(1d) **N[b](a)**

are one and the same fact.³

Even though, as noted, first-order logic and its set-theoretical semantics may be viewed as implicitly embodying directionalism, the current scenario seems to be more favorable to the anti-converses option, as the recent works cited above testify. This may be due to the fact that the focus has been on the ontological level, while the semantic level has been neglected. However, both levels deserve consideration. I shall now turn to the ontological level and then move to the semantic level.

2 Positionalist Relations as Sparse Attributes

As traditionally understood, sparse attributes account for the objective resemblances of things and for their causal powers, and with empirical science we try to individuate them a posteriori. They have coarse-grained identity conditions based on necessary equivalence. To illustrate, among sparse attributes we admit there are properties accepted by current science such as *negative charge* or *spin up*, but we now rule out that there is *caloric* or *unicorn*. We also admit a property such as (*made of molecules of*) H_2O , but we do not see this as a property over and above the property *water*. They are one and the same property on account of the fact that, necessarily, whatever is water is made up of molecules of H_2O . I am taking for granted here what Schaffer (2004) calls the “scientific conception” of sparse attributes, according to which they include not only the fundamental attributes of microphysical reality, but also attributes from all layers of reality: macro-physical, chemical, biological, psychological. Hence, H_2O counts as a sparse property. And, as this example shows, sparse attributes need not be simple, for H_2O is a complex property involving, *inter alia*, the further properties *hydrogen* and *oxygen*.

As Schaffer (2004, 99) notes, sparse attributes should be invoked when we look at reality as a source of truthmakers for true sentences or propositions. Following Armstrong (1997), we may view truthmakers as states of affairs consisting of the exemplification of attributes by objects, where the attributes

³ As Fine (2000) makes it clear, both directionalism and positionalism can be seen as different explanations of *differential application* (or *relational order*, in Hochberg’s (1987, 443) terminology), i.e., that relations can be exemplified by the same relata in different ways; e.g., *loving* is exemplified in one way by Romeo and Juliet insofar as Romeo loves Juliet and in another way, insofar as Juliet loves Romeo. Beside considering the problems posed by converses, current approaches to relations are quite sensitive to those raised by relational order (MacBride 2020, sec. 4). It seems to me that directionalism is not fully successful in accounting for it (see Orilia 2008, sec. 6), but we need not insist on this for present purposes.

in question are sparse attributes. In *monadic* states of affairs, there is simply an object exemplifying a sparse property, whereas in *relational* states of affairs, there are objects jointly exemplifying a sparse relation. Let us consider some examples. Suppose we focus on *c*, a certain amount of water in a glass before us, and make the following claims:

- (2) *c* is water;
- (2') *c* is made of H₂O molecules.

They are both true, since *c* is in fact water and thus also a liquid made of H₂O molecules. However, as noted above, there is just one sparse property, call it **W**, somehow characterizable as both water and H₂O. Accordingly, there is just one fact making (2) and (2') true, namely:

- (2*) **W(c)**.

Imagine we now focus on a triangularly-shaped object, *d*, and assert:

- (3) *d* is trilateral;
- (3') *d* is triangular.

They are both true, but there is only one sparse property that can be invoked to account for their truth, i.e., a certain shape, call it *T*, which *d* exemplifies, somehow characterizable as both triangular and trilateral. And thus, there is just one fact that makes both of them true:

- (3*) **T(d)**.

Let us now go back to (1) and (1'). Just as for the pairs (2)-(2') and (3)-(3'), it is natural to assume that there is just one truthmaker, and thus, one should think, only one relation should be invoked in putting forward such a truthmaker. Directionalism offers us two distinct relations, whereas positionalism is content with just one. Clearly, the latter is favored at the ontological level that we are now considering. It is an approach that offers us just one relation when different ways of thinking and speaking might suggest there are two relations, pretty much as in each of the above examples, we get one property instead of two.

However, as we saw, positionalism calls for a clarification of what the exemplification of a neutral relation with respect to positions amounts to. This can hardly be done without dwelling in turn on the nature of positions.

Fine (2000, 10) tells us that they are *specific entities*; what sort of entities? I think the best course is to take positions to be properties that are exemplified by the relata of a relational state of affairs inasmuch as, or insofar as, such relata jointly exemplify the relation: when the relata jointly exemplify the relation, by the same token they also exemplify the positions in question (Orilia 2011, 2014).⁴ Which properties work as positions and which relation is the neutral relation in our case?

What we find in reality is a certain spatial configuration with two items vertically aligned with respect to the earth's surface, and the configuration is such that one of the two items is closer to such a surface and the other is further away from it, so that one's location is higher than the other's. Thus, the neutral relation is a relation of vertical alignment (cf. MacBride 2007, 34) with respect to the earth's surface, call it **V**, and the positions could be characterized as *superior* and *inferior*. Hence, the single truthmaker for (1) and (1') postulated by positionalism turns out to be as follows:

(1*) **V(superior(a), inferior(b))**.

Again, the writing order should not be taken to convey any information: (1*) is the same fact as

(1**) **V(inferior(b), superior(a))**.

This notation is meant to highlight that the exemplification of the neutral relation **V** by the two relata, **a** and **b**, goes hand in hand with the exemplification of the properties **superior** and **inferior** by the relata in question, so that the existence of (1*) involves the existence of two further facts consisting of the exemplification of the two positions by the relata, namely **superior(a)** and **inferior(b)**. It is important here not to be misled by the fact that we are used to read formulas of first-order logic of the form " $R(x, y)$ " as telling us that the relation R holds between entities x and y ; for (1*) and (1**) do *not* tell us that the relation of vertical alignment, **V**, holds between the two entities **superior(a)** and **inferior(b)**. It rather tells us that this relation holds between **a** and **b** *insofar as* there are also the facts **superior(a)** and **inferior(b)**.⁵

4 Expressions such as "insofar as" or "by the same token" are counterparts of Latin expressions such as "quatenus" or "et eo ipso" used by Leibniz in his analyses of relations Orilia (2008).

5 More generally, a *relational* formula of the type " $R(p_1(a_1), \dots, p_n(a_n))$," where " R " stands for a neutral relation, each " p_i " stands for a position and each " a_i " stands for a relatum, tells us that the relation R holds between the relata insofar as each relatum a_i exemplifies the correspond-

Starting from Russell himself, positions have typically been considered entities somehow rigidly associated with one specific relation (Russell 1984; Hochberg 1987; Fine 2000; Gilmore 2013; Dixon 2018). For example, there are positions *lover* and *beloved* associated with *loving* and to no other relation; *hater* and *hated* associated only with *hating*; *giver*, *given*, and *givee* associated only with *giving*; and so on. Positions as so conceived are, we may say, *idiosyncratic*. In contrast with this, I have argued (2011, 2014) that positions had better be considered as *inter-repeatable*, i.e., multiply associated with different relations, for this may reflect objective resemblances in the real world, “similarities in arrangement” (2011, 5), which we should want to capture in our conceptualization. For example, there is something in common in the nice situation of someone loving someone else and in the nasty situation of someone hating someone else, namely that in both cases we can distinguish an active role, exemplified by the lover or by the hater, and a passive role, exemplified by the beloved or the hated. This can be captured by associating the same positions, *agent* and *patient*, to the different relations *loving* and *hating*. Similarly, e.g., the same positions, *source*, *theme*, and *destination*, can be associated with both *walking* and *running*, as triadic relations involving an item moving from one place to another. I have called positions as so conceived *onto-thematic roles*, in short, *o-roles* (2011), as they could be seen as ontological counterparts of the thematic roles postulated in linguistics, which I shall briefly discuss in the following.⁶ Thus, for example, the state of af-

ing position \mathbf{p}_i . Each “ $\mathbf{p}_i(\mathbf{a}_i)$ ” in this formula could be called a *positional term*. The structure ...(\dots , \dots , \dots) of this notation, where the first gap is meant to be filled by a term for a neutral relation, and the gaps within the parentheses by positional terms, could be taken to correspond to the Leibnizian notion *insofar as*, which I have invoked to explain how the exemplification of a neutral relation should be understood. The irrelevance of the writing order can be made explicit by a general identity law. Given a formula A of the type “ $\mathbf{R}(\mathbf{p}_1(\mathbf{a}_1), \dots, \mathbf{p}_n(\mathbf{a}_n))$,” call *positional permutation* of A either A itself or any formula that results from A by writing in a different order the positional terms in A . (Clearly, if there are n positional terms in A , there are $n!$ positional permutations of A .) Then the identity law is:

(IS) For any two positional permutations P_1 and P_2 of $\mathbf{R}(\mathbf{p}_1(\mathbf{a}_1), \dots, \mathbf{p}_n(\mathbf{a}_n))$, $P_1 = P_2$.

For example, “ $\mathbf{V}(\mathbf{superior}(\mathbf{a}), \mathbf{inferior}(\mathbf{b}))$ ” and “ $\mathbf{V}(\mathbf{inferior}(\mathbf{b}), \mathbf{superior}(\mathbf{a}))$ ” are positional permutations of each other, and thus (IS) certifies that this identity holds: $\mathbf{V}(\mathbf{superior}(\mathbf{a}), \mathbf{inferior}(\mathbf{b})) = \mathbf{V}(\mathbf{inferior}(\mathbf{b}), \mathbf{superior}(\mathbf{a}))$.

6 Positions had better be conceived of, not only as inter-repeatable, but also as *intra-repeatable*, i.e., as capable of being associated more than once with the same relation in a given state of affairs (Orilia 2014, sec. 3). I take it for granted that o-roles, as well as the c-roles to be discussed in the next section, are not only inter-repeatable but also intra-repeatable.

fairs of Romeo's loving Juliet is *loving* exemplified by Romeo insofar as he exemplifies *agent* and by Juliet insofar as she exemplifies *patient*, which more formally could be put as **L(agent(r), patient(j))**. Similarly, the state of affairs of Romeo's father, Montague, hating Juliet's father, Capulet, is *hating* exemplified by Montague insofar as he exemplifies *agent* and by Capulet insofar as he exemplifies *patient*, or **H(agent(m), patient(c))**. We may call this approach *role positionalism*.⁷

Going back to our airplane and bird example, from a role-positionalist perspective, we should view the *superior* and *inferior* positions as o-roles, and thus we should see whether there are similarities in arrangement that they capture. If we look at directions in a sufficiently general way, not confined to spatial directions, there is room for noting a generality that is relevant here. There is a direction from lower to higher locations as we move in space away from earth, but similarly, there is a direction from earlier times to later times or from lower to higher magnitudes. We may thus see *superior* and *inferior* as o-roles that can be associated not only with spatial relations such as *vertical alignment* but also with relations of degrees of magnitudes, **D**, and of temporal succession, **T**. For example, we could acknowledge that the fact that makes it true that the height of Peter, h_1 , is more than that of Mary, h_2 , is something like **D(superior(h_1), inferior(h_2))**, and that the fact that makes it true that the battle of Waterloo, b_1 , is before the battle of Stalingrad, b_2 , is something like **T(inferior(b_1), superior(b_2))** (since the time that has already elapsed when the former battle has taken place is more than the time that has already elapsed when the latter battle has taken place).⁸

To the extent that role positionalism distinguishes neutral relations and o-roles that can be associated with different neutral relations, it should similarly distinguish between a neutral relation as such, the bare neutral relation, so to

7 Since Castañeda (1967) commented on Davidson's theory of events, o-roles have been typically viewed as relations linking events, states of affairs, or the like to participants in them (see, e.g., Parsons 1990—I speak simply in terms of states of affairs, as for present purposes nothing hinges on this). I prefer my line in which o-roles are properties, since it grants a positionalist account of differential application (see Orilia 2011, sec. 5). Role positionalism has been endorsed by Paolini Paoletti (2016, 2021b), who, however, takes o-roles to be modes rather than properties understood as universals, as in my approach.

8 Alternatively, instead of invoking *superior* and *inferior*, we could appeal to the o-roles *source* and *destination*, respectively, as suggested in Orilia (2014, sec. 8). The corresponding thematic roles are, in fact, commonly used to indicate a directionality. However, this directionality is always taken to involve an object (typically classified as *theme*) moving (possibly in a metaphorical sense) from the source to the destination. In contrast, in the cases discussed above, there is no moving object.

speak, and a neutral relation as endowed with o-roles, which could be called an *embellished* relation.⁹ We can conveniently represent embellished relations by allowing for blank spaces after the symbols corresponding to o-roles. To illustrate, the state of affairs **L(agent(r), patient(j))** involves, on the one hand, the neutral loving relation, **L**, and, on the other hand, the following embellished relation: **L(agent(), patient())**. Similarly, **H(agent(m), patient(c))** involves, on the one hand, the neutral hating relation, **H**, and, on the other hand, the following embellished relation: **H(agent(), patient())**. In appealing to this notation, it is important to emphasize once more that writing order is not significant in this context, so that, e.g., **L(agent(), patient())** and **L(patient(), agent())** are the same relation.^{10,11}

9 Fine (2000, 11) implicitly makes a similar distinction within positionalism between neutral relations as considered independently of positions and neutral relations as endowed with positions and points out the analogous difference in directionalism between biased relations, involving a content and a direction, and the pure contents somehow implicit in biased relations.

10 We can convey this point in a general fashion with this identity law for sparse, embellished relations:

$$(IR) \text{ For any two role permutations } P_1 \text{ and } P_2 \text{ of } \mathbf{R}(r_1(), \dots, r_n()), P_1 = P_2,$$

where a role permutation in a formula A of the kind " $\mathbf{R}(r_1(), \dots, r_n())$ " is either A itself or any formula that results from A by writing in a different order the *role terms*, " $r_i()$," in A . For instance, "**L(agent(), patient())**" and "**L(patient(), agent())**" are role permutations of each other, and thus, by (IR), **L(agent(), patient())** = **L(patient(), agent())**. (IR) is analogous to the identity law for states of affairs (IS) (see footnote 5). When considering the latter, however, I had not yet dwelled on viewing positions as o-roles, and thus (IS) was presented in terms of positions rather than o-roles.

11 Partially symmetric relations such as *arranged clockwise in a circle* (Fine 2000, n. 10) and *playing tug-of-war* (MacBride 2007, 42) may appear to be problematic for positionalism. As a response, Donnelly (2016) has developed *relative positionalism*, according to which positions are understood as *relative*, i.e., as properties possessed by relata relative to other relata. Dixon (2019) defends this approach and notes that in order to handle similarities in arrangement, it could be turned into a form of relative role positionalism, which adopts relative inter-repeatable o-roles, rather than relative idiosyncratic positions (see his 2019, n. 11). I am using here my terminology (Dixon does not refer to my view in this context). However, if positions, whether idiosyncratic or inter-repeatable, are understood as relative, they appear to presuppose relatedness, which is what positionalism tries to explain in terms of positions (MacBride 2020, sec. 4). It thus seems to me a better course to tackle these problematic partially symmetric relations on a case-by-case basis, so as to show that they reduce to more primitive relations that can be understood in terms of o-roles without recourse to relative positions (Orilia 2011, 9, n.11).

3 Distinct Converses as Abundant Attributes

Abundant attributes are assumed a priori as meanings of predicates and contributors to mental contents, i.e., accusatives of intentional attitudes such as beliefs. They exist, even if unexemplified. For example, *unicorn* can still be acknowledged among the abundant properties as meaning of the predicate “is a unicorn,” even though it has turned out that nothing exemplifies such a property. And we can have a mental content involving it; e.g., someone may correctly believe that nothing is a unicorn and someone else may incorrectly believe that something is a unicorn. Abundant properties have very fine-grained identity conditions, not reducible to necessary equivalence. For example, despite their necessary equivalence, *water* and H_2O are distinct abundant properties working as meanings of two distinct predicates such as “is water” and “is H_2O ,” respectively. One of these properties requires ordinary, commonsensical knowledge to be grasped, whereas the other requires some grasp of chemistry. And in fact, someone may have a mental content involving the former without thereby having a mental content involving the latter; e.g., someone could believe that *c*, the liquid in the glass, is water without believing that *c* is H_2O . Thus, sentences (2) and (2′) express two different propositions, i.e.,

(2a) water(*c*)

and

(2′a) H_2O (*c*).

And someone could believe the former without believing the latter.

Similarly, despite their necessary equivalence, *triangular* and *trilateral* are distinct abundant properties working as meanings of two distinct predicates such as “is triangular” and “is trilateral,” respectively, and in principle, someone could believe that the triangularly-shaped object, *d*, is triangular without thereby believing that *d* is trilateral, so that (3) and (3′) express different propositions, namely,

(3a) trilateral(*d*)

and

(3′a) triangular(*d*).

In the former case, the necessary equivalence in question can be known a posteriori via empirical investigation, whereas in the latter case, it can be known a priori via conceptual analysis. When the required conceptual analysis is simple and trivial, it may be hard to imagine that someone could have a belief involving a certain property *P* without having a corresponding belief involving a property *Q* that, by conceptual analysis, is equivalent to *P*. However, it becomes easier to see once we focus on cases in which the analysis is non-trivial and a fair amount of inferential effort is indispensable.

Now, just as “trilateral” and “triangular” appear to have distinct meanings and thus are taken to stand for different abundant properties, similarly, as Russell urges in his semantic argument, converse predicates such as “greater” and “less,” or “is above” and “is below,” appear to have distinct meanings and thus should be taken to stand, from this abundantist perspective, for distinct mutual converses. And in fact, we should acknowledge that someone might have a belief involving a certain abundant relation without thereby having a corresponding belief involving a converse of the relation in question.

Consider (1) and (1'), as well as these other pairs of sentences:

- (P1) (i) 4 is greater than 2;
(ii) 2 is less than 4;
- (P2) (i) Romeo loves Juliet;
(ii) Juliet is loved by Romeo;
- (P3) (i) Milan is north of Rome;
(ii) Rome is south of Milan;
- (P4) (i) the year 2019 is before the year 2020;
(ii) the year 2020 is after the year 2019;
- (P5) (i) Tom owns the car;
(ii) the car belongs to Tom;
- (P6) (i) John gives the ball to Richard;
(ii) Richard receives the ball from John.

It might be hard to imagine that someone could believe the proposition expressed by one member of one of these pairs without believing the proposition expressed by the other member of the pair. And yet, it should be granted that some amount of inferential effort, modest as it may be, is necessary to convince oneself that the sentences in each pair express necessarily equivalent propositions. So that, before this inferential effort, one could believe any of these propositions without believing their necessarily equivalent mates. In sum,

we should make room for the pro-converses option so as to allow converse predicates to have different meanings.

One way to do this is by buying role positionalism. Arguably, it is a peculiarly interesting and plausible way, since the appeal to roles appears to be fruitful in linguistics in accounting for a wide range of phenomena (see, e.g., Davis 2011, 400), and, as noted, it aims at capturing existing generalities. It is then worth seeing how the pro-converses option can be accommodated at the abundant level from a role-positionalist perspective. Before doing it, some clarifications are in order.

The thematic roles invoked in linguistics, in short, *t*-roles, can be seen as properties that noun or prepositional phrases implicitly have in the context of the sentences in which they occur. Phrases can come to have the *t*-roles they happen to have in a variety of ways, depending on different languages, and to understand which *t*-roles are in play is crucial to understanding a sentence and translating it into a language that exploits different conventions in assigning *t*-roles. Consider, for example, these equivalent English and Latin sentences:

- (E) Mark kills Antony with the sword;
 (L) Marcus Antonium gladio interficit.

According to a typical analysis, in (E) “Mark,” “Antony,” and “with the sword” have the *t*-roles *Agent*, *Patient*, and *Instrument* (following Davis 2011, I use an initial uppercase letter to indicate *t*-roles—this helps us to distinguish them from *o*-roles and from the *c*-roles to be considered in a moment). The expressions in question gain such *t*-roles, respectively, as follows: by preceding the verb, by following the verb, by containing the preposition “with.” Similarly, in (L), “Marcus,” “Antonium,” and “gladio” have the *t*-roles *Agent*, *Patient*, and *Instrument*. However, in this case, they acquire these *t*-roles by having appropriate case endings, namely, “-us,” “-um” and “-o,” respectively. It is essential to realize that, despite these different conventions, the same *t*-roles are involved in both sentences in order to understand them and see that they translate each other. Clearly, we grasp which *t*-roles phrases may have because we associate them with roles or functions that objects can play: objects can indeed act, undergo the effects of actions, or be used as tools.¹² There are then

¹² This may seem to conflict with taking *t*-roles to be properties of both noun phrases, e.g., “Mark,” and prepositional phrases, e.g., “with the sword” (as I have done). For whereas we typically take noun phrases to correspond to individuals that play roles in situations, there is not such a

meanings and mental contents corresponding to the t-roles. Since abundant properties are posited as meanings and mental contents, it is then natural to say that there are abundant properties such as *agent*, *patient*, *instrument*, and the like, which we grasp as concepts in recognizing the t-roles involved in the sentences we use and which occur as constituents of the propositions expressed by such sentences. We may call them *cognitive-thematic roles*, or, in brief, *c-roles* (Orilia 2011, 6). Thus, in order to understand a *relational* sentence expressing a *relational* proposition, we must grasp not only which neutral relation is expressed by the verb in the sentence but also the c-roles in question, and thus which embellished relation is expressed by the verb taken together with the t-roles. Grasping what such roles are, and which arguments they are associated to, goes hand in hand with grasping the embellished relation.

To illustrate all this and how c-roles occur as constituents of propositions, let us consider the proposition expressed by both (E) and (L), which I represent as follows:

(E/L) kill(agent(m), patient(a), instrument(s)).

It should be clear from this notation that, just as I viewed o-roles as sparse properties that are exemplified by *relata* inasmuch as such *relata* exemplify a certain neutral relation, I similarly assume that c-roles occur as abundant properties attributed to arguments of an abundant relation. In this case, *killing* is the abundant neutral relation, and m, a, and s are the arguments. In general, from a role-positionalist standpoint, a relational proposition, which attributes a relation to some arguments, involves, by the same token, the attribution of the relevant c-roles to the arguments in question. Thus, (E/L) is taken to entail these further propositions: agent(m), patient(a), instrument(s).

It is useful to note here that there are two senses in which we can identify a predicate in a basic sentence, such as (E) or (L). On the one hand, we can say that the predicate is the verb, “kills” in (E) and “interficit” in (L); we may call this the *verbal predicate*. The verbal predicate typically expresses a neutral relation, which can be seen as a constituent of the proposition expressed by the sentence in which the verb occurs. For example, both “kills” and “interficit” express the neutral relation *killing*, which is a constituent of the proposition (E/L). On the other hand, there is the predicate constituted

direct correspondence in the case of prepositional phrases: “with the sword” as such is not taken to correspond to an individual that plays a role in a situation. However, prepositional phrases typically contain noun phrases that correspond to individuals that play roles in situations, e.g., “the sword.” Hence, there is really no conflict.

by the verb and the t-roles implicitly present in the sentence, which we may call the *phrasal predicate*. We can make the phrasal predicate explicit by appealing to variables. For example, in (E) we have the phrasal predicate “*x kills y with z,*” and in (L) we have the phrasal predicate “*x-us y-um z-o interficit.*”¹³ The phrasal predicate expresses an embellished relation of the abundant level, which can also be seen as a constituent of the proposition expressed by the sentence in which the phrasal predicate occurs. We can appropriately represent the embellished relations of the abundant level by resorting to the lambda notation. Thus, for example, the embellished relation expressed by both the English and the Latin phrasal predicate that we are considering is $\lambda xyz \text{ kill}(\text{agent}(x), \text{patient}(y), \text{instrument}(z))$, which can be seen as a constituent of the proposition (E/L).

We are now ready to see how we can distinguish converses from this role-positivist point of view. The idea is that converse *phrasal* predicates express distinct embellished relations, typically involving different c-roles. Let us go back to (1) and (1') to illustrate this. In the first place, it is important to understand which propositions they express and, thus, in particular, which neutral relation is expressed by the verbal predicate and which c-roles are in play. It seems clear that the verbal predicate, “is,” expresses, in this case, a neutral relation such as *situated*. This suggests that a *theme* c-role is in play since the t-role *Theme* is typically attributed to the noun phrase working as subject in sentences with a verbal predicate of this sort, a noun phrase intuitively corresponding to an object situated in a location (see, e.g., Jackendoff 1983, chap. 9). Moreover, it appears that the “above” of (1) and the “below” of (1') correspond to two distinct c-roles. In keeping with the idea that c-roles are properties, we may say that the former corresponds to the property of being a boundary of a place extending upward (away from the earth's surface), the *abover* property, whereas the latter corresponds to the property of being a boundary of a place extending downward (toward the earth's surface), the *belower* property. In sum, an object that exemplifies *abover* is the lower boundary of some space, which counts as a place that some other object occupies, and similarly, an

13 Phrasal predicates sensitive to case endings must, of course, be managed with care because attention must be paid to the distinction between a case ending and the word root to which the case ending is attached; variables are taken to correspond to the latter. For example, in “*Maria Antonium amat*” (“Mary loves Antony”), there are word roots “*Mari-*” and “*Antoni-*” with nominative and accusative case endings, “*a*” and “*um,*” respectively. Accordingly, we get the phrasal predicate “*x-a amat y-um.*” Alternatively, one may invoke here traditional names of case endings and rather convey the phrasal predicate as follows (with obvious abbreviations): *x-nom amat y-acc.*”

object that exemplifies *below* is the upper boundary of some space, which counts as a place that some other object occupies (it should be noted that the *abover* object is the object that is below, the bird in our example, and the *below* object is the object that is above, the airplane in our example; this may sound counterintuitive, but it is in line with the fact that the preposition “above” precedes the noun phrase standing for the object that is below, and “below” precedes the noun phrase standing for the object that is above). Hence, the propositions *a is above b* and *b is below a*, expressed respectively by (1) and (1’), can be represented as follows:

- (1a) *situated*(*theme*(a), *abover*(b));
 (1’a) *situated*(*theme*(b), *below*(a)).

What (1a) conveys is that the airplane occupies a place by being situated within the space extending upward from the bird, whereas (1’a) tells us that the bird occupies a place by being situated within the space extending downward from the airplane. We know by conceptual analysis that these propositions are equivalent, indeed necessarily equivalent, as they simply offer different ways of conceptualizing the same spatial configuration; when two objects are vertically aligned, we can see one as placed in a spatial region delineated in the upward direction by the other object, or we can see the latter object as placed in a spatial region delineated in the downward direction by the former object. Thus, in general, we know that, necessarily, $\forall x \forall y (\textit{situated}(\textit{theme}(x), \textit{abover}(y)) \leftrightarrow \textit{situated}(\textit{theme}(y), \textit{below}(x)))$.

We can now identify the converses *above* and *below* with the two embellished relations $\lambda xy \textit{situated}(\textit{theme}(x), \textit{abover}(y))$ and $\lambda xy \textit{situated}(\textit{theme}(x), \textit{below}(y))$. They have a common neutral relation, *situated*, and also a c-role in common, namely *theme*, but they crucially differ in that one involves the *abover* role and the other the *below* role. As the above discussion shows, we know that they are mutual converses by conceptual analysis, just as we know that the propositions (1) and (1’a) are necessarily equivalent.¹⁴

As we saw, when Russell, in *POM*, accepted the pro-converses option, he did this by endorsing directionalism. It should be clear at this point that this choice is in the way of a full understanding of how converse predicates may differ

¹⁴ It is worth noting that we need not take these c-roles as rigidly associated with the spatial relation *situated*. Just as with the **superior** and **inferior** o-roles discussed in the previous section, the c-roles *theme*, *location*, *abover*, and *below* could be seen as inter-repeatable and associated with relations of temporal succession and of degrees of magnitude (Jackendoff 1983, chap. 10).

in meaning. For directionalism makes it seem as if the difference between two converse relations has simply to do with the order in which the relata are given.¹⁵ This leads to the typical way in which, following *POM* (1903, sec. 28, §94), the distinction between a relation and a corresponding converse is introduced (Fine 2000, 3; MacBride 2020, sec. 1): a *converse* of a binary relation R is a relation R^* such that, necessarily, R holds between x and y whenever R^* holds between y and x . For example, *above* has *below* as its converse since the former holds between x and y , *in that order*, whenever the latter holds between y and x , *in that other order*. More generally, a converse of an n -ary relation R is a relation R^* such that, necessarily, R holds between x_1, \dots, x_n , just in case R^* holds between a permutation of x_1, \dots, x_n , e.g., $x_2, x_1, x_3 \dots, x_n$. For example, *giving* holds between x, y , and z (i.e., x gives y to z) whenever *receiving* holds of the permutation z, y, x (i.e., z receives y from x). More formally, in the familiar language of quantificational logic, one simply says “ $Rx_1 \dots x_n$,” or “ $R(x_1, \dots, x_n)$,” instead of “ R holds between x_1, \dots, x_n .”

In contrast with what directionalism suggests, thinking of the relata in a certain order seems neither necessary nor sufficient to capture the perceived meaning difference in members of pairs of converse predicates such as “is above”/“is below.” Turning again to Latin, wherein word order is less rigid than in English, allows us to bring this easily to the fore. For example, in Latin, we can say both “*Maria supra equo est*,” which we can literally translate in standard English as “Mary is above the horse,” and equivalently “*sub Maria equus est*,” which we can literally translate in not quite standard yet intelligible English as “below Mary, the horse is.” In both cases, we think first of Mary and then of the horse, and yet, in one case, we are thinking of them as related by *above* and in the other case as related by *below*. Hence, it does not seem that thinking order is sufficient to tell us which of these pairs of relations is involved. On the other hand, in Latin, beside “*Maria supra equo est*,” we can equivalently say “*supra equo Maria est*,” which we can literally translate into intelligible English as “above the horse, Mary is.” In one case we think first of Mary and then of the horse, and in the other case we think first of the horse and then of Mary, and yet it seems in both cases we think of them as related by *above*, not first by *above* and then by *below*. Thus, it seems that thinking order is not necessary to switch from one relation to its converse.¹⁶

15 This shortcoming of directionalism adds up to its problem with Russell’s ontic argument and its inadequacy in explicating differential application, mentioned in footnote 3.

16 In this discussion of directionalism, and perhaps elsewhere in the paper, I may give the impression that I take prepositions such as “below” and “above” as straightforwardly standing for relations.

Fortunately, as we have seen, we need not bind the pro-converses option to directionalism. By buying role positionalism, converses can be distinguished via different c-roles, independently of the sequential order by which we think of relata, as illustrated by the analysis of (1) and (1') provided above. However, from the fine-grained standpoint of the abundant conception, the sequential order emphasized by directionalism may well be significant, and if this is taken into account, we can somehow recover the standard way of distinguishing between a relation and its converse and find a grain of truth in directionalism. The point is that thinking is sequential, at least as far as it is exercised with natural language, which works sequentially (Castañeda 1975, 243): we think via propositions that we express with natural language sentences, which are constructed by concatenating words in a sequential order, and this order could be relevant in determining which propositions are expressed. Consider, for example, “John is nice and Mary is beautiful” and “Mary is beautiful and John is nice.” These two sentences differ merely in the order in which their sub-sentences are conjoined, and yet they could be taken to express two distinct, albeit necessarily equivalent, propositions that differ from each other in the order in which the conjuncts flank the conjunction (Bealer 1982, 54). After all, even in this case, some inferential effort is required to see the equivalence in question. Similarly, e.g., “a is above b” and “above b, a is” can be taken to express different, albeit necessarily equivalent, propositions: *situated(theme(a), abover(b))* and *situated(abover(b), theme(a))*, which differ from each other merely in the order in which the subconstituents, *theme(a)* and *abover(b)*, somehow occur in them. And accordingly, we should then also admit that there are two *above* embellished relations: a *theme first above*, namely $\lambda xy \textit{situated}(\textit{theme}(x), \textit{abover}(y))$, and a *theme second above*, namely $\lambda xy \textit{situated}(\textit{abover}(x), \textit{theme}(y))$. Clearly, the former holds between a and b just in case the latter holds between b and a, or, more formally, $\lambda xy \textit{situated}(\textit{theme}(x), \textit{abover}(y))(a, b) \leftrightarrow \lambda xy \textit{situated}(\textit{abover}(x), \textit{theme}(y))(b, a)$.¹⁷ This is in line with the stan-

In fact, as we have seen, I view them as standing for c-roles. Turning away from these prepositions and from Latin, a good example to illustrate how distinct converses may be evoked independently of thinking order is provided by the following pair of sentences: “the airplane is longer than the bird,” “the airplane is less short than the bird.”

¹⁷ The formulas on the two sides of the biconditional are respectively equivalent, by lambda conversion, to two other formulas, namely, “*situated(theme(a), abover(b))*” and “*situated(abover(b), theme(a))*,” which should in turn be regarded as equivalent. The law of

ward way of presenting the distinction between a relation and its converse, and thus we could view λxy situated(theme(x), abover(y)) and λxy situated(abover(x), theme(y)) as converses. Their difference is however trivial, since it has to do simply with the order in which the c-roles involved in these relations occur. We should thus distinguish between *serious* converses, such as λxy situated(theme(x), abover(y)) and λxy situated(theme(x), belower(y)), which differ in some c-role, and *trivial* converses, such as λxy situated(theme(x), abover(y)) and λxy situated(abover(x), theme(y)), which differ merely in the order of the c-roles involved in them.¹⁸ Directionalism is, at best, fit to capture the distinction between trivial converses. However, since it is silent about roles, it cannot tell us anything about the more intriguing differences between serious converses.¹⁹

4 Relations in the Dualist View of Attributes

The sparse and abundant conceptions of attributes are typically viewed as rival (see Orilia and Paolini Paoletti 2020, sec. 3.2), and if one looks at them in this fashion, not much is gained by noting that the former favors the anti-

lambda conversion is typically assumed once one resorts to the lambda notation and goes as follows: $\lambda x_1 \dots x_n A(t_1, \dots, t_n) \leftrightarrow A(x_1/t_1, \dots, x_n/t_n)$, where $A(x_1/t_1, \dots, x_n/t_n)$ is the wff resulting from simultaneously replacing each x_i in A with t_i (for $1 \leq i \leq n$), provided t_i is free for x_i in A .

- 18 Once we freely appeal to variables and the lambda notation, we can generate different terms for relations by simply changing the order of the variables we choose. And given the importance attributed to order at the abundant level, one may think that these terms may well stand, at least in some cases, for further distinct converses. For example, in addition to “ λxy situated(theme(x), abover(y)),” there is “ λxy situated(theme(y), abover(x)),” and one may think that the latter stand for a converse of the relation expressed by the former term; after all, we should grant, by lambda conversion, that λxy situated(theme(x), abover(y))(a, b) \leftrightarrow λxy situated(theme(y), abover(x))(b, a). However, it does not seem wise to admit that distinct relations can be generated simply because we grant all this freedom in the choice of variables. We can avoid this result by using variables in a more regimented way in an effort to appropriately represent embellished relations. That is, we could conventionally assume that both the lambda variables (the ones following the lambda operator) and the variables in the open formula bounded by the lambda variables must always be used in alphabetical order (Orilia 2019, sec. 4). This rules out, as ill-formed, terms such as “ λyx situated(theme(x), abover(y)),” in which the lambda variables are not in alphabetical order, and terms such as “ λxy situated(theme(y), abover(x)),” in which the variables in the open formula are not in alphabetical order.
- 19 In Orilia (2019), I had already made room for the idea that there are distinct converses at the level of abundant attributes, but there I focused only on trivial converses without appealing to c-roles in order to investigate serious converses.

converses option and the latter the pro-converses option. We would still not know which option to pick. However, the two conceptions need not be viewed as rivals. Indeed, they should be considered as complementary, and in fact, the very promoters of the distinction accepted a hybrid view with both sparse and abundant attributes in order to account at the same time for the objective resemblances in the physical world and for matters of meaning and mental content. Following this line, we can accept both the anti-converses and the pro-converses options. Let us see how.

Abundant attributes can be taken to *correspond* to sparse attributes pretty much as the two Fregean senses of “Hesperus” and “Phosphorus” correspond to one and the same planet, or as the two Fregean senses of “the square root of 4” and “the even prime number” correspond to the number two, so that identity statements about properties can be taken to express the fact that two different abundant attributes correspond to the same sparse attribute (Orilia 1999).

The water/ H_2O and triangular/trilateral examples can illustrate how this works. Let us start with the former and go back to sentences (2) and (2'). We saw that there are good reasons to think there is only one state of affairs, (2*), which involves a certain sparse property, **W**, and makes (2) and (2') true. But we also saw that there are good reasons to think there are two distinct propositions, (2a) and (2'a), expressed by these two sentences, one involving the abundant chemical property H_2O and another involving the abundant commonsensical property *water*. Empirical investigation reveals that both properties correspond to one sparse property in the physical world, **W**. This correspondence may be expressed by an identity statement such as H_2O is *water* (or *to be H_2O is to be water*). However, in this perspective, the “is” of statements such as this should not be taken to express identity but the correspondence in question. It may be noted here that we shouldn't simply assert that water is H_2O , but that water is *reduced* to H_2O . This can and should be granted, of course, but it is quite compatible with the idea that we have two abundant properties corresponding to a single sparse property; we can grant that there is a reduction because the abundant property H_2O , by being embedded in a successful scientific theory with great explanatory and predictive power, reveals the hidden nature of the sparse property in question more conspicuously than the commonsensical abundant property *water*.²⁰

²⁰ We can then also say that the proposition that *c* is H_2O grounds the proposition that *c* is *water*, even though both have the same truthmaker.

Consider now the trilateral/triangular example and turn to sentences (3) and (3'). Again, we granted a single truthmaker, (3*), involving a certain sparse property, **T**, and also granted two different propositions, (3a) and (3'a), expressed by these sentences, involving the different properties *triangular* and *trilateral*. As in the water/H₂O case, there are two abundant properties that correspond to the single sparse property **T**. There are, however, important differences: in this case, it is conceptual analysis that reveals that the two abundant properties must correspond to one sparse property, and we have no reason to think that one of these abundant properties reveals more perspicuously than the other the real nature of the sparse property.²¹

Let us finally move to converse relations and thus to our paradigmatic above/below example and to sentences (1) and (1'). It seems to me that the difference between *above* and *below* is analogous to the difference between *triangular* and *trilateral*. We acknowledged that there is only one state of affairs that makes both (1) and (1') true, and hence we put forward a sparse neutral relation of vertical alignment, **V**, and the sparse o-roles **superior** and **inferior**, so that the state of affairs in question turns out to be (1*). We also admitted there are two propositions expressed by (1) and (1') and accordingly put forward the propositions (1a) and (1'a), involving two different embellished abundant relations: λxy situated(theme(x), abover(y)) and λxy situated(theme(x), belower(y)). These two relations can be taken to correspond to the same sparse embellished relation, **V**(**superior**(), **inferior**()), just as *triangular* and *trilateral* correspond to the same sparse property, **T**. In both cases, we know a priori by conceptual analysis that there is such a correspondence, and we have no reason to think that one of the abundant attributes in question reveals more perspicuously than the other the

21 Once we distinguish two abundant properties corresponding to one sparse property, as is the case with *water* and *H₂O*, or *triangular* and *trilateral*, then the following results: on the one hand, all sorts of distinct abundant attributes can be constructed from the abundant properties in question, and, on the other hand, the relevant sparse property is involved at the truthmaker level. Consider, for example, the two abundant relations *contains more water than* and *contains more H₂O than* (I take such relations to be embellished relations, thus involving c-roles, but for the sake of making this point, it does not matter which they are). The former should be taken to contain *water* as a constituent, whereas the latter should be taken to contain *H₂O* as a constituent, and accordingly, they are distinct just as *water* and *H₂O* are distinct. However, the true propositions involving them will have truthmakers that involve the same sparse property, **W**. Suppose, for example, that *a contains more water than b* and *a contains more H₂O than b* are true. Then, there will be a truthmaker for both involving **W**, a state of affairs such as **a contains more W than b** (which I take to involve appropriate o-roles, which is not important to specify for the sake of making this point).

real nature of the sparse attribute. It should be noted here, however, that we can conceive of an abundant embellished relation that corresponds to the sparse relation in a more revelatory way. We could express this with a predicate such as “ x and y are vertically aligned with x as superior and y as inferior” and take it to be λxy vertical-alignment(superior(x), inferior(y)). This abundant embellished relation has a distinct trivial converse, namely λxy vertical-alignment(inferior(x), superior(y)), which of course reveals the nature of the sparse relation **V(superior(), inferior())** just as well. In contrast, there is no converse for the sparse relation: **V(superior(), inferior())** and **V(inferior(), superior())** are one and the same, as emphasized in section 2.²² This sparse relation is involved in the truthmaker of (1) and (1’), namely (1*), which is the same as (1**).

Bealer (1982, 186) assumes there are primitive simple attributes, which are both sparse and abundant, wherefrom complex sparse attributes and complex abundant attributes are differently constructed: *condition-building operations* generate coarse-grained sparse attributes, and *thought-building operations* generate fine-grained abundant attributes. To illustrate, suppose P and Q are two primitive simple attributes, and $\&$ and \wedge are, respectively, a thought-building conjunction operation and a condition-building conjunction operation; then P and Q are both abundant and sparse attributes, and $P \& Q$ and $P \wedge Q$ are, respectively, an abundant attribute and a sparse attribute. Similarly, $Q \& P$ and $Q \wedge P$ are, respectively, an abundant attribute and a sparse attribute. However, abundant attributes are extremely fine-grained, and thus $P \& Q$ and $Q \& P$ are distinct. In contrast, sparse properties are coarse-grained, and thus $P \wedge Q$ and $Q \wedge P$ are one and the same attribute. If we followed this line, we could similarly say that abundant c-roles and neutral relations, at least to the extent that they are primitive and simple, could be identified with sparse neutral relations and sparse o-roles, respectively. We could say, for example, that the abundant *vertical-alignment*, *superior*, and *inferior* are identical to the sparse **V**, **superior**, and **inferior**. Alternatively, we could say that even

22 Of course, in our boldface notation conventionally adopted to represent sparse relations, we can distinguish the two terms “**V(superior(), inferior())**” and “**V(inferior(), superior())**,” which differ by the order in which the role terms are written. However, since there is no reason to think that in the realm of sparse attributes these two terms correspond to two distinct relations, we assume that **V(superior(), inferior())** = **V(inferior(), superior())**, so as to neutralize the wealth of options offered by writing order, and more generally, we assume the identity law (IR) of footnote 10. In contrast, we saw that thinking order makes a difference at the level of abundant attributes, and thus no law analogous to (IR) is assumed for the lambda terms that represent abundant embellished relations.

at the level of primitive simple attributes, we have correspondences between abundant and sparse attributes that fall short of identity, so that, e.g., the abundant *vertical-alignment*, *superior*, and *inferior* correspond, respectively, to the sparse **V**, **superior**, and **inferior** but are not identical to them. We may leave this open for present purposes, and similarly, we could leave it open whether there are complex sparse attributes built up from condition-building operations in the manner proposed by Bealer.

5 Conclusion


I considered in detail only one example of converses, but I expect it suffices to illustrate the general strategy and to indicate how other converses can be treated in an analogous manner. The role positionalism put forward here accommodates both Russell's ontic and semantic arguments and provides a way out of the dilemma they raise by rejecting converses at the level of sparse attributes and accepting them at the level of abundant attributes. It might seem, however, that it pays too high a price for this, since this strategy involves an ontological commitment to both sparse and abundant attributes. One might worry that lovers of desert landscapes would prefer only sparse attributes and lovers of jungles only abundant attributes, and that the combination of sparse and abundant attributes might be indigestible to both. However, the recourse to this dualism of attributes is independently motivated by the need to account simultaneously for matter and mind, or referents and meanings, and it is only by neglecting one or the other aspect that we can have the illusion of dispensing with either sparse or abundant attributes. And thus, it is quite legitimate to avail oneself of attribute dualism to resolve the dilemma about converses.

Even so, one could suspect that role positionalism has too many ontological commitments, for it is committed not simply to relations but to both neutral and embellished relations. In contrast, one could perhaps do with simply relations, as in the *primitivism* put forward by MacBride (2014) or in Fine's *anti-positionalism* (2000, sec. 4), further developed by Leo (2008, 2014), or even without relations, as in approaches that take all relations to be internal and do not consider internal relations as a real addition to being (Simons 2010; Lowe 2016). However, the distinction between neutral and embellished relations results from the appeal to roles, and roles, as we have seen, are needed to explicate how relations are exemplified by relata in ways that give rise to similarities in arrangements. Hence, having both neutral relations and

embellished relations is not a burden but a theoretical advantage, as it helps us to account for the relatedness we find in the world and in our thinking about the world. This relatedness, it seems to me, is simply not fully appreciated by those who deny that there are relations. On the one hand, external relations appear to be ubiquitous; for instance, the very existence of mechanisms and structures presupposes them (Paolini Paoletti 2021a, 2021b), and, on the other hand, it is far from obvious that internal relations are not additions to being (MacBride 2020, sec. 3).

Of course, to do full justice to these objections would take us too far afield. I trust, however, that I have done enough to motivate this *dualist role position-alism*, as we may call it. It is a view that needs much further research, for its full development requires an appropriate inventory of o-roles and c-roles. I hope that this paper may contribute to stimulate research in this direction.*

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