THE TRUTH ON PREDICATES AND CONNECTIVES

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In his rich *"The Truth Predicate vs. the Truth Connective. On taking connectives seriously."* Kevin Mulligan [14] starts an inquiry into the logical form of truth ascriptions and challenges the prevailing view which takes truth ascriptions to be of subject predicate form, that is a truth predicate applied to a name of a proposition or sentence. Rather than appealing to a truth predicate, Mulligan argues, we should account for the logical form of truth ascriptions using the "truth connective". To this end Mulligan, in his genuine and original style, brings forward and merges arguments stemming from syntax theory, semantics, metaphysics and Bolzano to substantiate his dictum that it is not the truth predicate but the truth connective "which wears the trousers" (cf. Mulligan [14], p. 567).

In this piece we shall first discuss Mulligan's proposal from the perspective of linguistics and, especially, syntax theory. Even though theory of syntax provides little evidence for Mulligan's view, we shall argue that this does not disqualify the thesis that it is a truth connective (or operator as we shall frequently say) which figures in the logical form of truth ascriptions. This view can be supported by distinguishing between the grammatical and the logical form of a sentence. Moreover, as Mulligan notes there is a similarity between truth and modal ascriptions, where in most formal treatments their logical form is very different, that is, truth is commonly treated as a predicate but the modalities are conceived as operators. We think that these notions should be treated in an uniform way, either both as predicates or both as operators.

The prevailing philosophical view is that if truth and the modalities are treated as predicates, paradox will arise, though nothing of the like will arise when we opt for an operator treatment. We shall argue that the question of whether paradox will arise is somewhat orthogonal to whether we treat truth and the modalities as operators or as predicates. Rather it is the expressive power of the framework which is at stake, when it comes to the paradoxes.

1. Language

In fact, Mulligan's inquiry is seemingly even more ambitious than outlined above as he does not ask the question what the underlying form of truth ascription is, but identifies the expression

(O) It is true that ___.

as an operator or connective. That is, Mulligan claim that O figures as a unit in English sentences and is an expression which takes sentences as arguments to form new sentences. On

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the contrary, according to Mulligan, the expression

(P) ___ is true.

takes names or terms to form new sentences and therefore should be considered as a predicate. This presupposes that it makes sense to classify natural language expressions into the categories "predicate" and "operator" which leads Mulligan to stipulate *"that the categorial grammar of formal languages applies also to natural languages"*[1], especially English.

Now, this assumption comes at a cost, namely if we wish to avoid trivialization, we'd better come up with some theory or principled account of which expressions (within a grammatical sentence) are of which category and this account should provide an analysis of all English sentences or at least all sentences in which expressions of the type considered occur.

If we assume that (explicit) truth ascriptions in English employ either the truth predicate P or the truth operator O, Mulligan needs to argue that (i) all truth ascriptions employing the truth predicate P can be accounted for by (or reduced to) some truth ascription using the truth operator O and (ii) that there is an principled analysis or theory of English grammar which takes the expression O to be a member of a category of expressions and which analyzes the grammatical function of O in a way which licenses the claim that O is a one-place sentential operator.[2]

Mulligan brings forward a battery of examples and considerations which purport to show (i), namely Mulligan intends to show that for every sentence employing the truth predicate P there is a sentence employing the truth operator O which in some salient way is metaphysically prior to the former sentence.[3] Whereas we do not feel competent to comment on the metaphysical aspect of this thesis, it seems to involve the claim that we can translate every sentence of English employing the truth predicate into some sentence of English employing the truth operator where the latter sentence implies the former in some relevant sense. If this is right, however, sentences involving quantification into the argument position of the truth predicate and sentences where the truth predicate is applied to what Vendler [23] called perfect nominals[4] are serious trouble for his claim. For in order to account for these sentences, it seems that Mulligan would need to argue that there are expressions of English which act like propositional variables and in the case of the quantified statements, quantifiers binding these variables. And it's less than clear whether such expressions exist in English. These problems, however, are well known from

[1]Cf. Mulligan [14], p. 565.

[2]Of course, in principle one needs to do the same with respect to the truth predicate P, however, we take it that Mulligan is not bothered by the question whether P is indeed a truth predicate.

[3]To be more precise, Mulligan's claim is that for every sentence S employing the truth predicate, there exists a sentence S' employing the truth operator only, such that

If S, then S because S'.

For more details see Mulligan [14], pp. 567-570.

[4]These are nominalized sentences in which the verb is dead and has become a noun as in 'Goldbach's conjecture'. Cf. Vendler [23], pp. 122-46.

the "Prosentential Theory of Truth"[5] and we shall not discuss them here though propositional quantification will be of some importance in the remainder of the paper..

Still, to even get off the ground Mulligan needs to establish (ii), i.e. he needs to argue for a parsing of an English sentence as in

(1) $$\underbrace{\text{It is true that}}_{\text{O}}\ \underbrace{\text{Kevin is wrong}}_{\text{S}}$$

That is Mulligan needs to provide a grammar which acknowledges O to be a member of a syntactic category which is or can be, analyzed as a constituent of sentences like (1) and, moreover, the grammatical function of O should come out to be something like an operator.

For example, if one were to argue that the English word 'and' is, when used to conjoin sentences, a (two-place) sentential operator, one could substantiate this claim by arguing that 'and' belongs to the lexical category of conjunction words, and if 'and' is used to conjoin two sentences to form a new sentence, it is considered as a constituent of the latter sentence.[6] Moreover, since 'and' takes arguments of the same category (e.g. sentences) to form a new member of this category, its grammatical function can be considered to be an operator. More specifically, if 'and' is used to conjoin sentences, it can be taken to be a sentential operator.

Unfortunately, no argument of the latter kind is forthcoming in standard theory of syntax with respect to Mulligan's truth operator, that is the expression 'it is true that' does not belong to a syntactic category and thus, *a fortiori* it cannot be a constituent of a sentence. Moreover, it also seems that even grammars which are not based on constituent-structure analysis don't attribute a grammatical function to the expression 'it is true that' and thus do not recognize it as a truth operator.[7] Rather, in most theories of syntax[8] "that Kevin is wrong" would be considered as a unit, namely as a complementizer phrase (CP). We shall not discuss the grammatical analysis of sentence like (1) in detail, but to our knowledge all standard accounts analyze the grammatical function of 'true' or 'is true' to be that of a predicate, that is it takes terms as arguments to form sentences.[9]

Mulligan is well aware of the fact that mainstream theory of syntax does not provide any support for his view. He states:

> "Modern linguistics has no place for a category of pure connective expressions such as 'It is true that...'. " (See Mulligan [14], p. 582).

This, however, overstates the case and seems to equate "modern linguistics" with "theory of syntax". Where, as mentioned, it is true that pure syntactic analysis does not provide any evidence for the existence of a natural language expression which merits to be called a "truth

[5] Cf. Grover, Camp and Belnap [9], and Grover [8].

[6] Cf. Van Valin [22], pp. 130-31.

[7] See Sells [17] and Van Valin [22] for more on syntax theory.

[8] That is, in all constituent-structure grammars which accept the mainstream categories of constituents amongst which we have CP. All the different versions of Chomsky count as standard theory of syntax.

[9] It is important that 'term' is not understood semantical, i.e. as a referential expression. Whether a certain "term" is referential is a completely different issue.

operator", this does not exclude the possibility that certain natural language expressions should be treated as "truth operators" within categorial grammar. Differently put, the possibility of having "truth operators" within categorial grammar is ruled out only, if it is assumed that theory of syntax can be presented as a categorial grammar, that is the data produced by the theory syntax is assumed to fit the framework of categorial grammar without further modification.

However, this is a very strong assumption and probably a too strong of an assumption, as it seems sensible to distinguish between pure grammatical form and logical form. E.g. Higginbotham [11] argues at some length for this distinction and states:

> "Linguistic structure is a matter of grammar in the narrow sense; that is, a matter of what licenses certain combinations of words and other formatives as constituting a sentence of a language. But the concern of logical form is with the recursive structure of reference and truth. In distinguishing logical form from grammatical form we post a warning against the easy assumption that the referents of the significant parts of a sentence, in the ways they are composed so as to determine truth conditions, line up neatly with the words, in the way they are composed so as to make the whole wellformed." (See Higginbotham [11], pp. 173-74)

Clearly, there is no easy answer to the question of how grammatical form and logical form, or, differently put, syntactic structure and semantic structure are related, but it seems reasonable to understand (or to adopt an understanding of) categorial grammar to be concerned rather with the semantic structure, i.e. the logical form, of natural language. However, if this view is adopted, then there seems room for a parsing of English sentences like (1) into a truth operator and a declarative sentence despite the alternative parsing arising from the analysis of the linguistic structure of (1). After all, the linguistic structure of the sentence

(2) 2+2 is not 4

does not match the standard account of its semantic structure according to which "not" is considered as a one-place sentential connective[10] and this has hardly convinced anyone to revise our common treatment and understanding of "not" as one place sentential operator for negation.

However, the fact that conceiving of 'it is true that' as a truth operator is not outruled by evidence to the contrary from theory of syntax does by no means establish that 'it is true that' is correctly conceived as such. This would require a principled analysis or theory explaining the transformations taking place in the transition from grammatical to logical form, or from syntactic to semantic structure. Consequently, to substantiate his view Mulligan would need to

[10]This is certainly true with respect to the analysis of the linguistic structure of (2) provided by constituent structure grammars, that is theory of syntax of the Chomskian making. The situation seems to be somewhat different in, e.g., "Role and Reference Grammar" where "not" is even called an "operator" in the analysis of the linguistic structure (see Van Valin [22], pp. 205-18). We are not sure whether this supports an understanding of 'not' as one-place sentential operator. It is worth noting that opposed to the Chomskian research program Rule and Reference Grammar does not stipulate the autonomy of syntax.

provide an analysis of this transition. For otherwise his thesis, i.e. that the expression 'it is true that' should be analyzed as a truth operator of English, remains some wild speculation without any evidence in its support.

Where Mulligan falls short from providing such a principled account, he correctly hints at an asymmetry in the treatment of modal notions and truth. On the face of it, e.g., necessity and truth behave alike from a linguistic perspective, that is, as in the case of truth we find linguistic constructions employing what Mulligan would call the necessity operator, i.e. 'it is necessary that' as well as the necessity predicate 'is necessary'.[11] This observation can be generalized to broad class of modalities and even propositional attitudes though the data is slightly different in this case. Still, in prominent systems of categorial grammar arising from Montague's work modalities are treated as sentential operators[12] and thus the question arises why then shouldn't we treat truth alike?

2. Logic

This asymmetry between the treatment of truth and the modalities within categorial grammar and logic does not only seem puzzling from a linguistic and philosophical point of view, it also causes several problems with respect to the logical form of principles connecting truth and the modalities. Thus, as we shall see, treating truth and the modal notions alike, whether as operators or as predicates, does not only even out the asymmetry between the two, it has real advantages compared to a heterogenous treatment of these notions and therefore, it is desirable from a philosophical and logico-semantical perspective.

For given a uniform treatment of truth and modalities, common puzzles with respect to the logical form of linking principles would disappear. Consider a sentence like

(3) If it is necessary that 2+2=4, then it is true.

If one takes necessity to be aptly treated as an operator, but conceives of truth as a predicate the semantic issue arise to what the pronoun 'it' in ´it is true' refers to. In this particular case the "it"-pronoun clearly seems to work anaphorically and if truth is conceived as a predicate the pronoun needs to refer to a previously designated object. However, if necessity is conceived as an operator no object will be designated in the antecedent sentence.

Similar and even more pressing problems arise when we consider principles involving quantification, as e.g. a generalized version of (a), namely

(4) Everything that is necessary, is true.

[11]Though there seems to be one difference between truth and the modalities which has been noted by Mulligan (cf. [14], pp. 676-77). For the modalities we can transform sentences appealing to what Mulligan would call operators into seemingly synonymous sentences using the adverbial counterpart of the operator (cf. 'it is necessary that p' and 'necessarily, p'). However, the adverbial counterpart of true, i.e. truly, does not seem to be synonymous with the truth operator.

[12]Even though this is essentially correct it oversimplifies the situation. See Thomason [18] for an exposition of Montague's work.

Conceiving of necessity and truth as predicates we can easily formalize (4) in first-order logic by the following

(5) $$\forall x(Nx \to Tx)$$

Similarly, if we work in an operator setting, that is we treat both necessity and truth as operators, then we can equally provide a straightforward formalization as long as we allow for propositional quantifiers in our language. (4) would then become

(6) $$\forall p(\Box p \to Tp)$$

But assuming necessity to be aptly formalized as an operator but truth as a predicate, we end up with

(7) $$\forall p(\Box p \to T\ulcorner p\urcorner)$$[13]

This, however cannot be considered as formal rendering of (4) as long as the quantifier is understood referentially since the quantifier does not bind the argument position of the truth predicate. Rather in the argument position of the truth predicate, we have a name of the propositional variable p. We end up in a similar muddle if we conceive of the necessity as a predicate but take truth to be an operator. Thus if we treat truth and modal notions in a non uniform way we need to give an account of the quantifier occurring in (4).

An obvious way to do this is to posit an overt '___expresses proposition___' relation and to posit that the natural language quantifier 'everything' actually triggers quantification over individual and propositional variables:

(8) $$\forall x \forall p(\Box p \wedge Expr(x,p) \to Tx)$$

An alternative is to introduce a device which provides a name for every proposition and thus introduces a standard name for every proposition. Let '$Q(*)$' be such a subnector[14], then (4) can be formalized as

(9) $$\forall p(\Box p \to TQ(p))$$

Where these are at least *prima facie* possibilities to account for quantification, if truth and the modalities are treated heterogeneously, they are completely *ad hoc* in character and there is a real issue how the introduction of these devices can be motivated. And this problem is even more pressing as there is a principled and motivated account of the logical form of these linking principles, namely to treat truth and the modalities in a uniform way. As we have seen this would resolve the problem of quantification and would equally make sense of the functioning of the 'it' pronoun in sentences like (3).

[13] $\ulcorner p\urcorner$ stands for a name of the propositional variable p.

[14] Belnap [2] introduces this terminology for operators which take propositions as arguments to produce terms of the language. In English expressions like 'that' or 'the proposition that' might be considered to be such subnectors.

2.1. **Operators, Predicates and Paradoxes.** Accordingly, there seems to be at least some motivation to revise the received view and to treat either truth as an operator or the modal notions as predicates.[15] And for many philosophers treating truth as an operator as proposed by Mulligan might then seem the right way to go. For Montague´s "Syntactical Treatments of Modalities" [13] is commonly considered as showing that predicate accounts of modality lead to paradox. Moreover, by treating truth as an operator, it seems that the semantical paradoxes with respect to truth are avoided likewise for Montague's theorem can be considered as a variant of Tarski's undefinability theorem: Where Tarski's undefinability theorem shows that for sake of inconsistency there can not be a predicate α for which the principle

$$(TB) \qquad \alpha(\overline{gn(\phi)}) \leftrightarrow \phi^{16}$$

comes out true, Montague showed that the right-to-left direction of the above biconditional could be replaced by the corresponding rule, that is he showed that no predicate α could be consistently characterized by

$$(T) \qquad \alpha(\overline{gn(\phi)}) \rightarrow \phi$$

$$(Nec) \qquad \frac{\phi}{\alpha(\overline{gn(\phi)})}$$

Once truth or the modal notions are treated as operators, at least *prima facie*, nothing alike these undefinability results is forthcoming. Put differently, if truth is conceived as a one-place sentential operator $\Box$, it can be governed by (TB),[17] i.e.

$$(10) \qquad \Box\phi \leftrightarrow \phi$$

and thus modal operators can be characterized by operator versions of (T) and (Nec). The reason for this asymmetry between operator and predicate is due to the fact that Gödel's diagonal lemma is applicable within the predicate setting only. In its parameter free version Gödel's diagonal lemma asserts that in any theory T extending elementary arithmetics, for every formula $\phi(x)$ with at most x free in ϕ, there exists a sentence δ such that

$$(11) \qquad T \vdash \phi(\overline{gn(\delta)}) \leftrightarrow \delta$$

[15] There is of course a further option which has been propagated by Kripke[12], Reinhardt [16] and more recently Halbach and Welch [10] which takes modalities to be aptly formalized by modal operators and truth by a predicate, but takes the occurrence of 'necessary' in (4) to be short for 'necessarily true' or 'true necessarily', that is (4) would be formalized as

$$\forall x(\Box Tx \rightarrow Tx)$$

Obviously, this equally resolves the so-called quantification problem. See Halbach and Welch [10] for more on this strategy.

[16] $gn(*)$ is the function that assigns to every expression of the language its Gödel number, $\overline{n}$ is the numeral of a number n.

[17] And trivially so, for e.g. read '$\Box$' as '$\neg\neg$'

By applying Gödel's diagonal lemma to the formula $\neg\alpha(x)$ we may obtain the formal liar sentence which asserts of itself that it is not true (i.e. that it is not α)

$$(\lambda) \qquad \neg\alpha(\overline{gn(\delta)}) \leftrightarrow \delta$$

λ is clearly inconsistent with (TB) and, as Montague [13] showed, so it is with (T) and (Nec). The application of the diagonal lemma to the formula $\neg\alpha(x)$ is possible, since the argument position is a term position and not a sentential position as it is, if we conceive of truth and the modal notions as operators. Nothing like the diagonal lemma is forthcoming within the operator setting and thus at least *prima facie* the operator approach to truth and modalities seems to be on the safe side when it comes to the semantical paradoxes like the liar paradox.

To be sure this feature has been brought up in favor of accounts inspired by Ramsey's redundancy theory of truth. Most of these accounts dispense of a truth predicate which allows for diagonalization and thus block the construction of the liar sentence. The prosentential theory of truth is but one example where from a formal point of view the truth predicate is substituted for propositional variables and propositional quantification. The prosentential theory of truth can essentially be considered as an operator conception of truth as it does away with the need of names for sentences of the language but instead introduces variables that occupy sentence position and this is essentially what happens within the operator account. According to Frápolli [6], e.g., prosentential theory of truth avoids the liar paradox[18] which she takes to be a strong point in favor the theory:[19]

> "The prosentential theory of truth accepts the paradox of the Liar for what it is, a linguistic muddle, and shows why it is not a real problem for a theory of truth."
> (See Frápolli [6], p. 132)

2.2. **Quantification and the paradoxes of indirect discourse.** However, avoiding the paradoxes has a price, namely that of severely restricting the expressive power of the framework. The operator approach avoids the paradox by virtually banning all "self-reference" from the language no matter whether the self-reference under consideration is of the vicious kind or not. In this respect the operator approach is suspect to the same critique Kripke [12] brought forward against Tarski's theory of truth. For, if we consider Kripke's example,

(12) All of Nixons utterances about Watergate are false.

then it seems that we should be able to, at least, formulate the sentence, no matter whether this sentence turns out to be paradoxical or not.[20] But in the operator framework paradoxical sentences cannot be formulated.

[18] Of course, any *reasonable* theory of truth should avoid the paradox for sake of consistency. However, in this case we cannot formulate the paradoxical sentences.

[19] Similar remarks can be found in Grover [8]. But see Grim [7] for a critical discussion. We shall comment in the same vein.

[20] At least, if we want to deal with truth in English, not some purified variant of English, and intend to provide a formal treatment thereof.

Kripke's example also raises the issue of quantification. First, in order to formulate sentences like (12) we need to introduce propositional quantifiers, which will move us beyond the first-order setting as propositional quantification is essentially second-order. Second, and more importantly, once we have propositional quantification at our disposal, it seems that Kripke's example suggests that an adequate treatment of the propositional quantifiers might reintroduce the paradox, for depending on what Nixon uttered the interpretation of the quantifier might depend on the truth and falsehood of the statement itself.

Basically, this observation was exploited by Prior [15] who discussed several paradoxes arising in modal operator languages equipped with propositional quantifiers. These paradoxes, even though closely related to the liar like paradoxes, are paradoxes of indirect discourse and therefore differ from the semantic paradoxes in their canonical presentation. In their simplest variant they follow the outlines of Epimenides' paradox. These paradoxes have not received as much attention as the paradoxes of direct discourse, that is the liar-like paradoxes, but have been discussed by Prior [15], Thomason[21], Burge [4, 5] and Asher [1].

Let us consider the language $L_\ddagger$ which is a propositional modal language with a truth operator T and one modal operator $\ddagger$, propositional variables $p, p', \dots$ and propositional quantifier $\forall$. For expository ease we read the modal operator as 'Onephrase asserts that'. We set up an hypothetical situation as follows.

(i) Onephrase asserts that everything Onephrase asserts is not true.
(ii) This is the only assertion Onephrase ever makes.

But given this setup (i) can be formalized in $L_\ddagger$ by means of propositional quantification as follows

$$\ddagger\forall p(\ddagger p \to \neg Tp) \tag{13}$$

and (ii) gives rise to the following assumption in $L_\ddagger$

$$\forall p(\ddagger p \to (p \leftrightarrow \forall p(\ddagger p \to \neg p)) \tag{14}$$

Assuming (13), (14) and the standard logic of quantification we can derive a contradiction:

Since by the operator version of (TB), i.e. (10), we can infer

$$\ddagger\forall p(\ddagger p \to \neg p) \tag{15}$$

from (13). We can then derive the inconsistency as follows:

1. $\forall p(\ddagger p \to \neg p) \to (\ddagger\forall p(\ddagger p \to \neg p) \to \neg\forall p(\ddagger p \to \neg p))$ (UI)
2. $\forall p(\ddagger p \to \neg p) \to \neg\forall p(\ddagger p \to \neg p)$ 1,(15)
3. $\neg\forall p(\ddagger p \to \neg p)$ 2
4. $\ddagger p \wedge p \to \forall p(\ddagger p \to \neg p)$ (14),(UI)
5. $\exists p(\ddagger p \wedge p) \to \forall p(\ddagger p \to \neg p)$ 4,(UI)
6. $\forall p(\ddagger p \to \neg p)$ 5

[21] Asher [1] as well as Belnap and Gupta [3] allude to a never published manuscript "Paradoxes of Intentionality?" as principle source of inspiration. More, recently joint work of Thomason with Tucker [20] entitled "Paradoxes of Intensionality", where the paradoxes of indirect discourse are discussed, is available online.

7. $\perp$ 3,6.

Whereas in the case of the Epimenides paradox, the paradox is for the most part blamed on the modal properties of the truth predicate, it is not clear whether in the present case there is any point in blaming the truth operator since

(i′) Onephrase asserts that everything Onephrase asserts is not the case.

seems to support (15) directly without appealing to (TB). But then, on the face of it, the paradoxical conclusion seems very puzzling as we haven't made any assumption on behalf of the truth or the modal operator and simply assumed the ordinary laws of quantification. One might take this to be a vindication of the predicate approach to truth and the modal notions, since in the predicate setting the liar like paradoxes depend crucially on the properties of the truth or the modal predicates, where in the operator setting it is quantification *simpliciter* that leads to paradox.

But this conclusion might be a bit premature as there are consistent modal logics[22] with propositional quantifiers. Whether propositional quantification will lead to inconsistency depends on whether we take the initial, hypothetical scenario to be a possible one which in turn relies on how fine grained we individuate propositions or, more generally, the objects of our modal attributions and to what extent the structure of these objects is transparent *within* the approach. If the hypothetical scenario is ruled out, we can consistently extend the modal logic under consideration by propositional quantification. Most prominently the individuation of propositions as sets of possible worlds allows for consistent modal logics with propositional quantification where these quantifiers range over sets of possible worlds. Similarly, approaches taking propositions to be entities *sui generis* and limiting the structural information available with respect to these entities will allow for propositional quantification.[23]

Still, where we might have some quarrels with respect to the above scenario, we should be careful trying to dissolve the paradox by dismissing the hypothetical situation as more plausible scenarios can be constructed and thus the dismissal has counterintuitive consequences. Asher [1] presents the following example:[24]

> Suppose Prior is thinking to himself:
>
> (Pr) Either everything that I am thinking at the present moment is false or everything Tarski will think in the next instant, but not both, is false.

Clearly, if Prior thinks (Pr) to himself at t_0 and Tarski thinks that $2 + 2 = 5$ to himself at t_1 there will be nothing paradoxical and thus the fact that Prior thinks (Pr) and nothing else to himself does not constitute a problem in this situation. But if Tarski thinks, e.g. that Snow is white to himself at t_1 we ended up in paradox. Still it seems counterintuitive to react toward

[22]From a formal point of view a logic with a truth operator governed by (TB) is nothing else than a modal logic. In fact, it is the trivial modal logic where the modal distinction collapses. Here and in what shall come the term "modal logic" is meant to include the truth operator logic.

[23]Cf. Thomason [19] for an approach along this line.

[24]The general pattern of the example is apparently due to Jean Buridan but was rediscovered and discussed by Prior in [15].

this paradox by stipulating that it is impossible that Prior thinks (Pr) and nothing else at t_0 where Tarski thinks that Snow is white and nothing else at t_1.[25] This suggests that we should take the paradoxes of indirect discourse seriously and not try to resolve them by dismissing the hypothetical scenario which we will call—following Asher [1]—*Prior situation*.

Intuitively, to properly evaluate Prior situations propositions need to be able to refer back to themselves as this is part of the content of (Pr), i.e. of what (Pr) asserts, and thus an adequate individuation of propositions should be capable of expressing self-reference. But if propositions are individuated appropriately in this respect propositional quantification, as argued, will have troublesome consequences. The reason for this is that the propositional quantifier is—and again we concur with Asher [1]—a surrogate of the truth predicate. That is, using propositional quantification we can quantify directly into sentence position and thus generalize over sentences. In a first-order setting this can be done only if syntactical predicates, i.e. predicates like the truth predicate that apply to names of sentences or propositions, have been introduced into the language, for instance, the truth predicate and we know that in the presence of syntactical predicates like truth care has to be taken in order not to run into the paradoxes of direct discourse. However, by means of propositional quantification we can generalize over sentence position without appeal to a truth predicate. For example, we can state a quantified version of the law of excluded middle in the following way:

$$\forall p(p \vee \neg p) \tag{16}$$

Moreover, when we analyze the role of the propositional quantifier in the paradox of indirect discourse, it becomes obvious that we face a similar problem as in the case of the liar paradox. For suppose we try to evaluate whether '$\forall p(\ddagger p \rightarrow \neg p)$' is true. Intuitively this sentence is true, if and only if, for all propositions $\mathcal{P}$, if Onephrase asserts that $\mathcal{P}$, then *the* proposition that $\mathcal{P}$ is false. But this seems to depend on whether *the* proposition that $\forall p(\ddagger p \rightarrow \neg p)$ is true unless there has been a proposition $\mathcal{P}$ to falsify '$\forall p(\ddagger p \rightarrow \neg p)$'. However, if *the* proposition that $\forall p(\ddagger p \rightarrow \neg p)$ were true, we would have found a proposition $\mathcal{P}$ which Onephrase asserts and which is true and '$\forall p(\ddagger p \rightarrow \neg p)$' would be false. Thus it seems as if we have ended in a circle similar to the one we encounter in connection with the liar sentence λ where the truth of $T\overline{gn(\lambda)}$ relies on whether λ, that is $\neg T\overline{gn(\lambda)}$, is true.

If this analysis is correct, it is not surprising that propositional quantification leads to contradiction provided the structure of the propositions is relevant with respect to their evaluation. Since propositional quantification appears to be a surrogate of the truth predicate, instantiating a universally quantified formula has a similar effect as disquotation in the case of the truth predicate. And we know that in the case of the truth predicate, we can't adhere to an unrestricted principle of disquotation, that is (TB), since in the standard setting self-referential statements

[25] Moreover, since given the temporal ordering this would imply that if Prior thinks (Pr) to himself at t_0 Tarski cannot think that Snow is white to himself at t_1 which seems an absurd consequence. For more on this see Prior [15] and Thomason and Tucker [20].

can be formulated. If the modalities are treated as operators the paradoxes of indirect discourse seem to suggest that we have to give up classical logic of quantification.[26]

Although clearly one might argue that in the case of truth and salient modal notions such as necessity we are not in need of a fine grained individuation and especially that there is no need for the structure of these entities to be transparent within the approach. Accordingly one might try to work with a more coarse grained individuation of propositions, but the need for a uniform treatment of truth and all modal notions suggests that strategies of the latter kind for avoiding the paradox do not amount to a viable solution.[27] The moral of this observation seems to be that there is no escaping from the paradoxes independently of whether truth and the modal notions are treated as predicates or operators as long as we can quantify into the argument position of truth and the modal notions and provide an account of quantification and truth and modality which is adequate from a natural language perspective. It also seems worth noting that the paradoxes of indirect discourse are a real threat to operator accounts of truth and modalities, and the operator accounts should have a good answer towards these paradoxes—exactly like accounts which conceive of truth and the modalities as predicates need to have a good answer towards the paradoxes of direct discourse, i.e. the Liar like paradoxes.

It seems that there is a more general lesson to be learned. Independently from quantification the capacity of referring back to certain assertion—if not the assertion itself—seems to be highly desirable from a natural language perspective. Natural language possesses devices as demonstratives, anaphora and, more generally speaking, pronouns which are designed to refer to other expressions of the language and sometimes to the very expression itself. These devices have the effect of reifying assertion, sentences or propositions. That is, these devices transform assertions, sentences or propositions into objects of discourse. Objects we can then speak about.

This view can be supported by the fact that the paradoxes are no isolated phenomena of formal languages but may be formulated within natural language as can be witnessed by the following reconstruction of the paradox of the knower.

> Consider the sentence
>
> "I don't know this sentence"
>
> and call it KN. Now, let's assume that I know KN. Then by the factivity of knowledge, i.e. the fact that everything that is known is the case, I can infer KN. But KN says that I don't know *this sentence*. But *this sentence* just is KN and hence I don't know KN. We have derived a contradiction starting from the assumption

[26]Asher [1] provides an inductive theory of propositional quantification which is based on Kripke's theory of truth and which leads to replacing the axiom of universal instantiation by the corresponding rule of inference. A less drastic move would be to opt for a free logic of propositional quantification, but it's not clear whether this really amounts to a viable alternative. Asher suggests that such a proposal would run into serious trouble with respect to anaphora (cf. pp. 22-23).

[27]Even if one were to allow for a heterogeneous treatment, the need to account for sentence similar to (4), i.e.

Everything Nixon asserts is false.

would force the introduction of an "expresses" relation or a "subnector" which would reintroduce the paradox anew.

that I know KN. Accordingly it seems sound to conclude that I don't know KN and even more it seems that I have just produced an impeccable proof to the effect that I don't know KN. But then, since I have proven that I don't know KN, I seem licensed to conclude that I know that I don't know KN. Thus I know the sentence that I don't know KN. But *the* sentence that I don't know KN is just KN itself and therefore I can conclude that I know KN and we have ended up in contradiction.[28]

This natural language reconstruction of the paradox seems to crucially involve the capacity of natural languages of naming, i.e. reifying, sentences using (demonstrative) pronouns. But clearly both the capacity of naming and the capacity of referring to previously introduced objects of discourse via pronouns play a crucial role within natural language and thus to deprive a formal account from similar resources is to seriously cripple the account.

The moral to be drawn, if we're not willing to take a revisionist stance towards natural language, seems to be that we should be suspicious towards any "solution" toward the paradoxes which comes at the price of limiting the expressive power of the framework. A more sensible approach would try to locate the source of the paradoxes not within language, but within reasoning.

3. Conclusion

Even if one is not convinced by Kevin Mulligan's view on truth and does not find his arguments compelling, one should appreciate that Kevin Mulligan has pointed toward an asymmetry in the way we conceive of modalities as opposed to truth which does not seem warranted by the data, be it from syntax theory or semantics (or maybe metaphysics).

In the absence of good arguments in favor of this asymmetry revising the received view which treats truth as a predicate but the modalities as operators seems an adequate strategy and conceiving of truth as an operator is one possible way to go. Conceiving of the modalities as predicates is another way.

It is sometimes thought that operator accounts of truth and the modalities are on the safe side when it comes to paradoxes but we have argued that this opinion is somewhat ill founded. If the aim is to provide an adequate account of truth and the modalities within natural language, especially English, any account, no matter whether it treats truth and the modalities as predicates or operators, will have to face the paradoxes at some stage. Therefore, the paradoxes should have no bearing on the decision of whether to treat truth and the modalities as predicates or operators. That is to say, the question of paradox is orthogonal to the question of whether it is the truth predicate or the truth operator which wears the trousers.

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