Bolzano's Lehrjahre

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for Kevin Mulligan

Abstract

Bernard Bolzano (1781-1848) was born into a Prague catholic family (his father was Italian) with cultural interests. His younger years took place under the auspices of the reforms of the Austrian emperor Joseph II, which resulted in the modernisation of the society and the awakening of sciences. Bolzano spent the most important formative years in the Class of philosophy, where he became interested in mathematics and philosophy. His mathematical notebooks show his extensive knowledge of the literature of the 17th and 18 th centuries and contain entries about current mathematical problems and also reflections about methodology and critique of some fundamental concepts. In 1800, he decided to study theology, but at the same time he prepared his first publication on the Euclidean postulate of parallels (published in 1804). Having finished his studies, he participated in the contest for the chair of "science of religion", newly founded by the Emperor Franz to fight atheism and the ideals of the French revolution, and another for the chair of mathematics. He won both and the commission appointed him for the "science of religion". Bolzano did not perceive the larger political context and thought that this chair could be a forum for spreading his own ideas of the reform of the society. At the end of 1819, the contradiction between the intentions of the Emperor and Bolzanos own representation of an ideal society led to the dismissal of Bolzano from the university of Prague.

1781, the year Bernard Bolzano was born, was crucial for the Hapsburg Monarchy as well as for philosophy, and the two main events that took place influenced both his life and his thinking. In 1791, Joseph II started his fundamental reforms of the Austrian feudal regime and the same year Kant published his *Critique of pure reason.*

The reforms of Joseph II whose main expression was the "patent of tolerance" introduced religious freedom suppressed since 1620, suppressed serfdom and the feudal privileges of the nobles and for a time also censorship, forbade torture and reformed justice, opening the space for free expression of thought and learning.

Science, so long neglected, began to flourish. Already in 1770 the geologist Ignaz Born and the Count F. J. Kinsky founded the first learned society in the Hapsburg Empire called later the Royal Society of sciences of Bohemia (the Academy of Science of Vienna was created only in 1847). The most important scientists of Bohemia became its members: the technologist Franz Joseph von Gerstner, the founder of the Bohemian Polytechnics, the "patriarch of Slavic studies" Josef Dobrovsky, the physician Jiri Prochazka, one of the discoverers of the theory of reflex, the physiologist Jan E. Purkyne, known for his studies on cell theory, perception and dreams, the historian Frantisek Palacky author of the monumental *History of the Czech nation*, the physicist Christian Doppler specialised in wave theory and of course also Bernard Bolzano.

Bolzano was born in an Italian-German merchant family: his father came from the region of the Como Lake, but lived from early youth in Bohemia and worked as a businessman. His mother was the daughter of a German Prague merchant. There could not be a sharper contrast between the two parents: on the one hand, an austere and calculative farther, sixteen years older than his wife, on the other hand an emotional mother educated in a nunnery and living exclusively for her children[1]. Both devoted Catholics. Bolzano lived his boyhood with his almost always pregnant mother surrounded by small children, twelve altogether, who died one after another. Only Bernard and his elder brother Johann survived till full age. Almost all his life, Bolzano

[1] «Not only did she prefer to live in the circle of her children, but here only, she feel at her best, and when separated from her dearest, she could not stay even for an hour calm ». *Lebensbeschreibung*, p. 7.

felt sick; in his adolescent years he began to suffer tuberculosis which accompanied him for all his life. The deaths of his sister, of his mother, and of his benefactress Anna Hoffmann put him for long months in a state close to death.

In spite of the suffocating family atmosphere, on Sundays, books were read at home: mainly the Bible, different religious books, preaches, Hermes' *Handbuch der Religion*, and also *Opere drammatiche* of Metastasio, the works of Gellert, Gessner, Iffland, Engel and others. Besides, Bolzano read classical Greek and Latin authors, Klopstock, Schiller and Torquato Tasso. In the family circle he learned to love books and this love never abandoned him.

He was born at 224 Platnerska, which is today part of the Municipal Library and is opposite to Clementinum, today National Library, in his time the Charles-Ferdinand University; the family moved to 25 Celetna when he was five years old. Because of his fragile health, he began his instruction with private teachers at home and only at the age of eight he went to the German *Hauptschule* at the Tyn church at the Old City square, a hundred meters from his home at Celetna. At the age of ten, he attended the Piarist gymnasium at Prikop where he studied for five years. He might be among the first eight or nine pupils, but his achievement fell short of his master's recognition. Things changed when he entered he university.

In 1796 he joined the three-year Class of philosophy which took place in the building of the university. "A new world opened before me when I entered the Philosophy classes"[2]. Nevertheless, "in the first weeks, I paid no attention to the subject that captivated me most later, namely to mathematics, because I was offended by a somewhat rude behaviour of the at that time professor of this science Vydra, in other respects full of merit". It's a paradox that he also had difficulties to understand philosophy, especially logic, all disciplines in which he later made decisive discoveries.

It was not in the lectures of his professors that Bolzano found some replies to his questions concerning the fundamental concepts of mathematics, but rather in the works of a second rate mathematician Abraham Kästner. In fact, the most important mathematicians of the 18th century were not genuinely interested in the foundations of mathematics. But Bolzano very early felt the necessity to reform, redefine mathematical concepts considered by everybody as sufficiently clear and engaged mathematics in the examination of its first concepts and its methods. "My preference for mathematics was properly directed towards its purely *speculative* part, in other words, in mathematics, I appreciated only what is at the same time *philosophy*."[3] He found such treatment of his subject in the well known *Anfangsgründe der Arithmetik, Geometrie, ebenen und sphärischen trigonometrie und Perspektiv* of Kästner (5th. ed. 1792):

"When I opened by chance a page in Kästner's *Treatise*, the small stars which indicate a note excited my curiosity and I decided immediately to study mathematics, hoping to find in this science what I have sought in vain since a long time."[4]

He begins to read the great authors of his time and his extensive reading list contains the names of Kant, Euler, d'Alembert, Carnot, Johann Schultze, Wallis, du Bouguet, Klügel, Legendre, Newton, Lagrange, Laplace, Barrow, Clairault, Boscovitch, Segner, later also Lacroix, Wronski, Gauss, Cauchy, with many annotations and comments. The long series of his mathematical notebooks called first *Adversaria*

[2] *Ibid*., 18.
[3] *Ibid.*, 72.
[4] *Ibid.,* 19.

mathematica **and later** ***Miscellanea mathematica*** **yields an invaluable source for the study of history of mathematics in the 18th and the first half of the 19th centuries. The first notebooks contain many explanations of concepts and tentative solutions of mathematical and physical problems with long calculations, e.g. on the balance, on friction, on the obstacles of movement, on trigonometric functions, on spherical triangles, on fluids and solids, on infinitesimal calculus, on the pressure of a roof on the walls of a building, on he movement of a hammer, on the comparison of spheric trigonometry with plain trigonometry, on conic sections, on the parallelogram of forces (in his later years he wrote an article about it), on the first concepts of mechanics, movement, force, etc. In 1803, he presents a definition of continuum hat can be found in later works and even in the** ***Paradoxes of the Infinite*****. I agree with Bob van Rootselaar that it was a current definition in the contemporary literature.**

The notebooks contain many excerpts of mathematical literature, e. g. Wolff's ***Elementa matheseos universae*****. An exceptionally long document is the summary of Lagrange's** ***Théorie des fonctions analytiques*****: carefully written 64 pages of one of the milestones of mathematical literature, which has just appeared in 1797. A note concerning Descartes'** ***Dissertatio de methodo*****: "About the advice:** ***to doubt about everything*** **should merit a proper treatment in logic". The young student plans already a dissertation about the doctrine of the mathematical magnitude.**

In 1799, Bolzano finished the three your Philosophy class with distinction. He hesitates about the choice of profession, thinking about becoming a priest. His parents try to dissuade him, showing strong disagreements at the obligation of celibacy. They all agree on the following compromise: Due to excellent results of final examinations, Bolzano got a grant and took a free year to deepen his knowledge of mathematics and to think more about his future. At the end of the academic year, he will take a decision. He took advantage of his freedom to accomplish in one year the two-years course on higher mathematics by Gerstner, studying at the same time physics, chemistry and philosophy. He also spent many hours writing about the choice of a profession. At the end of the academic year, he decided to study theology.

He changed the faculty and began to study at the faculty of Theology, without taking the final decision of becoming priest. For a long time, he continued to have doubts about the validity of religious doctrines and eventually only an incidental remark of Mika, teacher of pastoral theology, convinced him about the justification of the catholic religion: "a doctrine is justified if it is possible to show that our belief in it provides certain benefits". Bolzano comments: "Suddenly, it became indubitable for me that in matters of religion, namely concerning the divine revelation, there is no question about how the things are in themselves, but on the contrary, which representation of them is the most edifying."[5] This pragmatic attitude helped him to interpret different religious doctrines in a metaphorical or symbolic sense and, in a way, it also became the guideline for theoretical decisions in his ***Theory of science.***

The ***Adversaria mathematica 1*** **(1799?–1803) begin with considerations about physics, about the definition of a straight line, about the infinite, absolute and relative in geometry, but also contain a note about suicide and about freedom vs. divine predictions. A note answers a question treated later in Bolzano's** ***Contributions to a better founded presentation of mathematics*****, in the** ***Theory of science*** **and elsewhere:**

"Has a proposition multiple proofs?

[5] *Lebensbeschreibung*, p. 27.

If one understands by the proof of a proposition not the manner how this proof is conducted in language, but the grounds on which it is founded, I think that a proposition cannot be founded on different grounds from which it could be derived. One often believes to have yielded two proofs, but when one analyses the propositions used in both proofs, one finds that eventually they can be reduced to the same principles (*Grundsätze*). One has only put in different order and differently separated the different principles that compose the proposition to be proven. In this way, the elements of the proof only have different names, but the whole proof contains the same fundamental parts (*dieselben Grundtheile*)"[6].

More about fundamental concepts, above all that of magnitude. In 1802, Bolzano believes that only Kantian philosophy can define it: "The definition of magnitude does not belong to mathematics; mathematics cannot achieve it. The definition of this concept belongs to the transcendental philosophy" (*ibid*., p. 45) and to try to explain it as that what can be diminished or augmented is no definition because these concepts already presuppose that of magnitude.

Here we also find the exhaustive critique of the defects of the mathematical concept, which will be integrated in the *Contributions* and in other writings:

"The first definitions in Arithmetics , Geometry and in other mathematical disciplines are not logically correct. In arithmetics, we lack a correct definition of positive and negative magnitude, of irrational magnitude, of imaginary magnitude (Nota bene: the irrational magnitudes are in a way also imaginary and the imaginary magnitudes could be generally reduced to such that are at the same time > and <a), of power, of zero, of 1/∞ and ∞. In geometry, we lack the definitions of length, surface, line, point, and the theory of parallels" (*ibid*., p. 219).

His notebooks contain many entries on the last problem. Since 1802 Bolzano works on the proof of Euclid's postulate of parallels. He knows already that such a proof can be constructed from the principle of similitude. This is what he does in his first published work, *Considerations of some objects of elementary geometry* (1804).

Methodology: "[…] all propositions in mathematics (and only there) must be proven with complete rigor, and thus: one does not establish a concept without showing its possibility. One does not form a judgement without proving its necessity from previous concepts or from undoubted intuitions"[7].

In 1804, Bolzano finished his study of theology. He always thought about reforming society and during his studies, he wanted to become rather a teacher of religion than a priest. But a new event changed his mind: the establishment of a new chair of the "science of religion", ordered by the Emperor Franz. After the death of Joseph II, many of his reforms were revoked and the spirit of Enlightenment almost vanished, replaced by the catholic Restoration. For the Emperor, the creation of the new chair should help to educate obedient citizens of the State and to eradicate the ideas of the French *Lumières* and the ideals of the French revolution. Bolzano did not perceive this larger political context and thought that this chair could be a forum for spreading his own ideas on the reform of the society. The dismissal of Bolzano from his chair at the end of 1819 put an end to the contradiction between the goals determined by the imperial decree and Bolzanos own representation of an ideal society. Bolzano was made silent.

[6] *Adv. math. 1,* manuscript, Österreichische Nationalbibliothek, n° 3451, p. 9.
[7] *IV. Adv. math. 2,* Allerlei matematische Gedanken, p. 8r-9v.

But in 1804, he won the contest at the same time as the contest for the chair of mathematics; in spite of the proposition of Gerstner to win Bolzano for mathematics, the final decision of the commission nominated him for the chair of science of religion. One can only speculate what would be the history of mathematics if he would become professor of this science in Prague and would found a school. Instead of this, in haste, Bolzano accomplished two necessary steps for his nomination: on April 5, he graduated as doctor of philosophy and two days later, he was ordained priest. On April 19, he was introduced into his office at the university. In any case, in spite of the disfavour of fate or rather because of it, to humanity, he offered one of the most precious gifts: his *Theory of science.*